

Conditions for Monogamy of Quantum Correlation: Greenberger-Horne-Zeilinger versus W states

R. Prabhu, Arun Kumar Pati, Aditi Sen(De), and Ujjwal Sen
Harish-Chandra Research Institute, Chhatnag Road, Jhansi, Allahabad 211 019, India

Quantum correlations are expected to respect all the conditions required for them to be good measures of quantumness in the bipartite scenario. In a multipartite setting, sharing entanglement between several parties is restricted by the monogamy of entanglement. We take over the concept of monogamy to an information-theoretic quantum correlation measure, and find that it violates monogamy in general. Using the notion of interaction information, we identify necessary and sufficient conditions for the measure to obey monogamy, for arbitrary pure and mixed quantum states. We show that while three-qubit generalized Greenberger-Horne-Zeilinger states follow monogamy, generalized W states do not.

Introduction and main results. – Quantifying quantum correlations plays an important role in the development of quantum information science. In particular, entanglement has been successfully employed to interpret several phenomena which cannot be understood within classical physics [1]. It has also been identified as the basic ingredient for different quantum communication protocols [2] and quantum computational tasks [3]. Moreover, bipartite as well as multipartite entangled states are prepared and are being successfully used to implement different quantum information protocols.

However, in the past, several quantum phenomena have been discovered in which entanglement is absent. These include “quantum nonlocality without entanglement” – locally indistinguishable orthogonal product states [4], and the model of deterministic quantum computation with one quantum bit [5–7]. In view of this, it is important and interesting to quantify quantum correlations in a multiparticle quantum state, independent of the entanglement-separability paradigm. An important attempt in this direction is the introduction of quantum discord [8, 9] for bipartite systems. Discord is an information-theoretic physical quantity that characterizes quantum correlations. This arises because two classically equivalent definitions of mutual information do not match in the quantum level. Recently, the concept of quantum discord has been generalized to the multipartite scenario using e.g. the notions of dissonance [10] and dissension [11] (see also [7, 12–14]).

An important property that is satisfied by entanglement in multiparty quantum states is that it is monogamous – if two subsystems are highly entangled, then they cannot share a substantial amount of entanglement with other subsystems [15, 16]. In this paper, we ask whether such monogamous character is an intrinsic property of other quantum correlation measures, in particular, of quantum discord. We carry over the monogamy concept beyond the paradigm of entanglement, and ask: *Does the sharing of general quantum correlations follow the same broad guidelines that are followed by entanglement? In particular, does quantum discord satisfy the*

monogamy relation? We find that the answers in general are negative. For another instance where monogamy considerations have been taken to an information-theoretic quantum multiparticle measure, see Ref. [17]. Using the concept of interaction information, we identify necessary as well as sufficient criteria for quantum discord to be monogamous. The concept of interaction information in the context of classical information theory is an important one and it has many applications in various disciplines, including biophysics, medicine, data analysis, classical control theory, and Bell inequalities [18]. Here, we find that it naturally fits in the context of monogamy of quantum correlation, and it explains as to when it can be satisfied and violated. Interestingly, we find that the monogamy condition can be applied to distinguish two inequivalent classes of quantum states in a three-party situation. Specifically, we show that the generalized Greenberger-Horne-Zeilinger (GHZ) states [19] always satisfy the monogamy of quantum discord, while the generalized W states [20, 21] always violate the same. This therefore gives us a physically interesting method to detect the generalized GHZ against generalized W states. Three-qubit pure states can be divided into the GHZ and W classes, of which the generalized GHZ and W are respective proper subsets. Members of the two classes cannot be transformed into each other by local quantum operations and classical communication (LOCC), with a nonzero probability of success [21]. This has subsequently been applied in a wide variety of areas ranging from quantum information to black holes [22]. In course of our investigation, numerical searches in the GHZ and W classes have revealed that members of the GHZ class may or may not violate the monogamy relation, while those in the W class are always violating.

Measure of quantum correlation. – Quantum discord [8, 9] is defined as the difference between two classically equivalent expressions for the mutual information, when extended to the quantum regime: $D(\rho_{AB}) = \tilde{I}(\rho_{AB}) - I(\rho_{AB}) = S(\rho_{A|B}) - \tilde{S}(\rho_{A|B})$. (In [9], \tilde{I} and I are respectively denoted as I and J .) Here $\tilde{I}(\rho_{AB}) = S(\rho_A) - \tilde{S}(\rho_{A|B})$, is the quantum mutual information,

and can be interpreted as the total correlations in ρ_{AB} , where ρ_A and ρ_B are the local density matrices of ρ_{AB} . $S(\sigma) = -\text{tr}(\sigma \log_2 \sigma)$ is the von Neumann entropy of a density matrix σ . $\tilde{S}(\rho_{A|B}) = S(\rho_{AB}) - S(\rho_B)$ is the “unmeasured” quantum conditional entropy [23] (see also [24, 25]). On the other hand, $I(\rho_{AB}) = S(\rho_A) - S(\rho_{A|B})$ can be interpreted as the classical correlations in ρ_{AB} , where the quantum conditional entropy is defined as $S(\rho_{A|B}) = \min_{\{\Pi_i^B\}} \sum_i p_i S(\rho_{A|i})$, with the minimization being over all projection-valued measurements, $\{\Pi_i^B\}$, performed on subsystem B . Here $p_i = \text{tr}_{AB}(\mathbb{I}_A \otimes \Pi_i^B \rho_{AB} \mathbb{I}_A \otimes \Pi_i^B)$ is the probability of obtaining the outcome i , and the corresponding post-measurement state for the subsystem A is $\rho_{A|i} = \frac{1}{p_i} \text{tr}_B(\mathbb{I}_A \otimes \Pi_i^B \rho_{AB} \mathbb{I}_A \otimes \Pi_i^B)$, where \mathbb{I}_A is the identity operator on the Hilbert space of the quantum system that is in possession of A . A nonzero quantum discord signifies the presence of quantum correlations in a bipartite quantum state.

Monogamy.— Monogamy of quantum correlations is a property satisfied by certain entanglement measures in a multipartite scenario. Given a multipartite state $\rho_{A_1 A_2 \dots A_N}$ shared between N parties, the monogamy condition for a bipartite quantum correlation measure \mathcal{Q} assures that the bipartite quantum correlations in the multipartite state are distributed in such a way that the following relation is satisfied: $\mathcal{Q}(\rho_{A_1 A_2}) + \mathcal{Q}(\rho_{A_1 A_3}) + \dots + \mathcal{Q}(\rho_{A_1 A_N}) \leq \mathcal{Q}(\rho_{A_1 : A_2 A_3 \dots A_N})$, where $\rho_{A_1 A_2} = \text{tr}_{A_3 A_4 \dots A_N} \rho_{A_1 A_2 \dots A_N}$, etc. On the RHS, the quantity $\mathcal{Q}(\rho_{A_1 : A_2 A_3 \dots A_N})$ is the same quantum correlation measure obtained in the bipartition $A_1 : A_2 A_3 \dots A_N$ of the N parties. It was shown that certain entanglement measures satisfy the above monogamy inequality [15, 16]. In this paper, we ask whether quantum discord satisfy the same relation. In particular, we will consider whether quantum discord of a tripartite state ρ_{ABC} satisfy the following inequality:

$$D(\rho_{AB}) + D(\rho_{AC}) \leq D(\rho_{A:BC}). \quad (1)$$

Violation of the above inequality will imply that discord is polygamous for the corresponding state.

Necessary and sufficient criteria for quantum discord to be monogamous.— We will now present the conditions that signal whether a multi-site quantum state is monogamous in nature with respect to quantum discord. We begin with Theorem I, which will tell us as to when the quantum correlation will respect monogamy. Before proving the results, we need to present a few definitions about mutual information, conditional mutual information, and interaction information.

Definition I: The mutual information and conditional entropy for the two-party cases have already been defined. For a tripartite state ρ_{ABC} , the unmeasured conditional mutual information $\tilde{I}(\rho_{A:B|C}) = \tilde{S}(\rho_{A|C}) - \tilde{S}(\rho_{A|BC})$, and the interrogated conditional mutual information $I(\rho_{A:B|C}) = S(\rho_{A|C}) - S(\rho_{A|BC})$. Here, $\tilde{I}(\rho_{A:B|C})$

and $I(\rho_{A:B|C})$ are nonnegative, which follow from the fact that $S(\rho_{A|C}) \geq S(\rho_{A|BC})$ and $\tilde{S}(\rho_{A|C}) \geq \tilde{S}(\rho_{A|BC})$, which say that conditional entropy is nonincreasing when conditioned on more parties.

Definition II: We carry over the concept of interaction information from classical information theory [26] to quantum mechanics and define the (unmeasured) interaction information, $\tilde{I}(\rho_{ABC})$, as $\tilde{I}(\rho_{ABC}) = \tilde{I}(\rho_{A:B|C}) - \tilde{I}(\rho_{AB}) = S(\rho_{AB}) + S(\rho_{BC}) + S(\rho_{AC}) - [S(\rho_A) + S(\rho_B) + S(\rho_C)] - S(\rho_{ABC})$. Note here that since this interaction information is without “interrogation” (measurement), the conditional entropies are defined here as $\tilde{S}(\rho_{A|C}) = S(\rho_{AC}) - S(\rho_C)$ and $\tilde{S}(\rho_{A|BC}) = S(\rho_{ABC}) - S(\rho_{BC})$, so that no measurement is required for their definitions. Interrogated interaction information is defined by using conditional entropies that involve density operators of one subsystem given that a complete measurement has been performed on another subsystem. For the state ρ_{ABC} , and a given set of measurements, an interrogated interaction information is given by $I(\rho_{ABC})_{\{\Pi_k^B, \Pi_i^C, \Pi_j^{BC}\}} = I(\rho_{A:B|C})_{\{\Pi_i^C, \Pi_j^{BC}\}} - I(\rho_{AB})_{\{\Pi_k^B\}}$, where the suffix on $I(\rho_{ABC})_{\{\Pi_k^B, \Pi_i^C, \Pi_j^{BC}\}}$ is used to indicate that measurements are performed at B , C , and BC . Also, the suffixes on the other terms indicate the corresponding measurements that have been performed, so that $I(\rho_{A:B|C})_{\{\Pi_i^C, \Pi_j^{BC}\}} = S(\rho_{A|C})_{\{\Pi_i^C\}} - S(\rho_{A|BC})_{\{\Pi_j^{BC}\}}$ and $I(\rho_{AB})_{\{\Pi_k^B\}} = S(\rho_A) - S(\rho_{A|B})_{\{\Pi_k^B\}} \equiv S(\rho_A) - \sum_k p_k S(\rho_{A|k})$. Optimizing over the measurements, we have the interrogated interaction information for a tripartite state ρ_{ABC} .

Given a tripartite density operator ρ_{ABC} , the interaction information $I(\rho_{ABC})$ is the difference between the information shared by the subsystem AB when C is present, and when C is not present (traced out). In some sense, the interaction information measures the effect of a bystander C on the amount of correlation shared between A and B . One can interpret a positive interaction information by saying that the presence of C *enhances the correlation* between A and B . Similarly, negative interaction information will mean that the presence of C somehow *inhibits the correlation* between A and B .

Quantum interaction information has the following properties. (i) Even though the conditional mutual information and the mutual information are both positive, the interaction information can be either positive or negative. (ii) It is invariant under local unitaries. (iii) Under unilocal measurements, $I(\rho_{ABC}) \geq \tilde{I}(\rho_{ABC})$. To see (iii), one may check that the identity

$$I(\rho_{ABC}) - \tilde{I}(\rho_{ABC}) = D(\rho_{AB}) + D(\rho_{BC}) + D(\rho_{CA}) \quad (2)$$

holds, where in this case, $I(\rho_{ABC})$ is the optimized version of $I(\rho_{ABC})_{\{\Pi_i^A, \Pi_j^B, \Pi_k^C\}} = S(\rho_{A|B})_{\{\Pi_j^B\}} + S(\rho_{B|C})_{\{\Pi_k^C\}} + S(\rho_{C|A})_{\{\Pi_i^A\}} - S(\rho_{ABC})$, and $\tilde{I}(\rho_{ABC}) =$

$\tilde{S}(\rho_{A|B}) + \tilde{S}(\rho_{B|C}) + \tilde{S}(\rho_{C|A}) - S(\rho_{ABC})$, which imply (iii), as quantum discord is nonnegative. Remarkably, (iii) provides a necessary and sufficient condition for the vanishing of quantum discords for the local bipartite states of an arbitrary (pure or mixed) tripartite quantum state. As can be seen, discords for the bipartite reduced states vanish if and only if $\tilde{I}(\rho_{ABC}) = I(\rho_{ABC})$, under unilocal measurements.

Theorem I: For any ρ_{ABC} , quantum correlations captured by the bipartite discords will obey monogamy, i.e., $D(\rho_{AB}) + D(\rho_{AC}) \leq D(\rho_{A:BC})$ will hold, if and only if the interrogated interaction information is less than or equal to the unmeasured interaction information.

Proof. It can be checked that if discord respects monogamy then we will have

$$I(\rho_{A:BC}) - I(\rho_{AB}) \leq \tilde{I}(\rho_{A:BC}) - \tilde{I}(\rho_{AB}), \quad (3)$$

where the optimizations over measurement bases have already been performed.

Now we prove the converse. Assuming (3), we have $I(\rho_{A:BC})_{\{\Pi_i^C, \Pi_j^B\}} - I(\rho_{AB})_{\{\Pi_k^B\}} \leq \tilde{I}(\rho_{A:BC}) - \tilde{I}(\rho_{AB})$, and using the expressions for various mutual informations, we get $S(\rho_{A|B})_{\{\Pi_k^B\}} + S(\rho_{A|C})_{\{\Pi_i^C\}} - S(\rho_{A|BC})_{\{\Pi_j^B\}} \leq \tilde{S}(\rho_{A|B}) + \tilde{S}(\rho_{A|C}) - \tilde{S}(\rho_{A|BC})$. So we have $[S(\rho_{A|B})_{\{\Pi_k^B\}} - \tilde{S}(\rho_{A|B})] + [S(\rho_{A|C})_{\{\Pi_i^C\}} - \tilde{S}(\rho_{A|C})] \leq [S(\rho_{A|BC})_{\{\Pi_j^B\}} - \tilde{S}(\rho_{A|BC})]$. Consequently, we have $[\min_{\Pi_k^B} S(\rho_{A|B})_{\{\Pi_k^B\}} - \tilde{S}(\rho_{A|B})] + [\min_{\Pi_i^C} S(\rho_{A|C})_{\{\Pi_i^C\}} - \tilde{S}(\rho_{A|C})] \leq [\min_{\Pi_j^B} S(\rho_{A|BC})_{\{\Pi_j^B\}} - \tilde{S}(\rho_{A|BC})]$, leading to (1). \square

We note that for any pure tripartite state, the unmeasured interaction information, $\tilde{I}(\rho_{ABC})$, is zero. As a result, quantum discord is monogamous if and only if the interrogated interaction information, $I(\rho_{ABC})$, is non-positive. As a corollary of Theorem I, we can see that quantum discord will be polygamous if the unmeasured interaction information is negative, i.e., $\tilde{I}(\rho_{ABC}) < (\leq) 0$, and the interrogated interaction information is positive, i.e., $I(\rho_{ABC}) \geq (>) 0$.

This provides a physical insight as to why quantum correlation can be polygamous. Suppose that Alice (A) and Bob (B) are correlated due to the tripartite quantum state shared between A , B , and C . Oblivious of the presence of the third observer Charlie (C), the correlation between Alice and Bob is the quantum mutual information. When there is a third observer (Charlie), the interaction information quantifies the increase in correlations between Alice and Bob due to the presence of Charlie. The interaction information can be with or without interrogation on Charlie. Our results show that polygamy is satisfied in the following scenario: (i) When Alice and Bob find out about Charlie's presence, Alice may find that she is more correlated with Charlie, before interrogating him, and therefore is in comparison, less cor-

related with Bob – mathematically, this is summarized by a negative $\tilde{I}(\rho_{ABC})$. (ii) After interrogating Charlie, Alice finds that she is less correlated with Charlie, but more with Bob – this is captured by a positive $I(\rho_{ABC})$.

Monogamy detects SLOCC-incomparable tripartite classes. – We believe that the results obtained on monogamy of quantum discord will be useful in many areas in quantum information. As a first application of the results derived, we consider a game where a shopkeeper who sells quantum states promises to provide three-qubit pure states, which are either generalized GHZ [19] $|\psi_{GHZ}\rangle_{ABC} = \cos\Phi|000\rangle + \sin\Phi|111\rangle$, or generalized W states [20, 21] $|\psi_W\rangle_{ABC} = \sin\theta\cos\phi|011\rangle + \sin\theta\sin\phi|101\rangle + \cos\theta|110\rangle$. Here, $|0\rangle$ and $|1\rangle$ are orthonormal states of the corresponding system. The customer is given a large number of copies of the same state. The task is to find out whether the given state is a generalized GHZ or a generalized W state.

While there does exist other methods to get the answer, we show here that the monogamy inequality in (1) can be used to give the answer, that additionally provides an interesting perspective of the geometry and structure of tripartite quantum states. Among the present cases, the two-party local density matrices of $|\psi_{GHZ}\rangle$ are $\alpha^2|00\rangle\langle 00| + \beta^2|11\rangle\langle 11|$, whose discord vanishes for all α , so that the monogamy inequality (1) is always satisfied, as $D(|\psi_{GHZ}\rangle)$ is nonnegative for any bipartite partition, and for any α . In sharp contrast, we numerically find that the generalized W states always violate the monogamy for quantum discord. Numerical simulations were performed for about 2.5×10^4 random real sets of $\{\alpha_W, \beta_W\}$, and violation was obtained for all the cases. See Fig. 1.

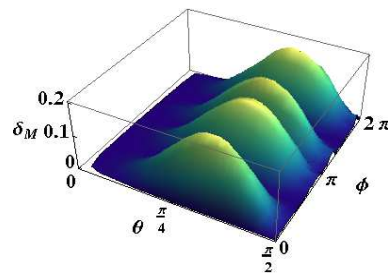


FIG. 1: (Color online.) Discord monogamy for generalized W. We plot δ_M (in bits) against (θ, ϕ) (dimensionless). Here, $\delta_M(\rho_{ABC}) = D(\rho_{AB}) + D(\rho_{AC}) - D(\rho_{A:BC})$.

The above result that distinguishes between generalized GHZ and generalized W states is intriguing because of the following fact. Generalized GHZ and generalized W states form proper subsets of the GHZ class and W class states. The latter encompasses the whole space of genuine three-party entangled states of three-qubit systems. Moreover, these two classes form inequivalent classes of three-party states in the sense that they cannot be transformed into each other even with stochastic LOCC (SLOCC) maps [21].

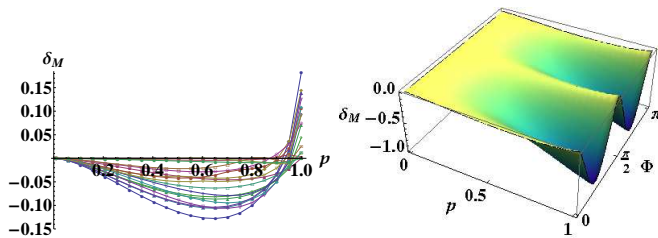


FIG. 2: (Color online.) Discord monogamy for mixed states. Left: δ_M (in bits) against the mixing parameter p (dimensionless), for different generalized W states. Right: δ_M is plotted against p and α (dimensionless) for all generalized GHZ states.

An arbitrary (unnormalized) state of the GHZ class is given by $|\psi_G\rangle = \cos\frac{\theta}{2}|000\rangle + |\psi_1\rangle|\psi_2\rangle|\psi_3\rangle$, where θ is real, and $|\psi_i\rangle = \alpha_i|0\rangle + \beta_i|1\rangle$, with α_i and β_i being complex numbers satisfying the normalization constraint $|\alpha_i|^2 + |\beta_i|^2 = 1$ ($i = 1, 2, 3$). For 25000 randomly chosen such states, approximately 24.49% violate monogamy. For an equal number of randomly chosen W class states, violation is obtain for *all* states, indicating that the monogamy relation can be used as an indicator for GHZ class states.

Next we ask whether the monogamy (or polygamy) behavior of given pure states is robust against noise admixture. To this end, we check the validity of monogamy of quantum discord, for pseudo-pure generalized GHZ and W states, given by $\rho_{GHZ} = (1-p)\mathbb{I}/8 + p|\psi_{GHZ}\rangle\langle\psi_{GHZ}|$ and $\rho_W = (1-p)\mathbb{I}/8 + p|\psi_W\rangle\langle\psi_W|$, where $0 \leq p \leq 1$, and \mathbb{I} is the identity operator on the three-qubit space. We perform numerical computations where we need optimizations over arbitrary two-qubit bases, for which we scan over the canonical entangled bases. The results are presented in Fig. 2, where we see that the monogamous or polygamous behavior persists for small admixtures of white noise. Moreover, admixing white noise to a W class state transforms it from being polygamous to being monogamous. This may be seen as due to the monogamous GHZ components present in white noise.

Conclusion.— Monogamy is an important aspect of entanglement, which tells us that this “costly” resource is not freely sharable. We have found that monogamy is not an intrinsic property of other quantum correlation measures. We have delineated necessary and sufficient conditions such that the monogamy can be satisfied. Using the notion of interaction information, we have proved that when interrogated interaction information is less than or equal to the unmeasured interaction information, then the quantum correlation obeys monogamy. Interaction information provides useful insights as to when quantum correlation will be monogamous and polygamous. In addition, we find that the monogamy conditions of quantum correlation can be used to distinguish generalized GHZ states from generalized W states. The results may have

applications in quantum information theory and quantum communication tasks.

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