Macroscopic Schrödinger Cat Resistant to Particle Loss and Local Decoherence

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The Schrödinger cat state plays a crucial role in quantum theory, and has important fundamental as well as technological implications, ranging from quantum measurement theory to quantum computers. The power of the potential implications of the cat state lies in the quantum coherence, as measured by the degree of entanglement, between its microscopic and macroscopic sectors. We show that in contrast to other cat states, it is possible to choose the states of the macroscopic sector in a way that the resulting cat state, which we term as the W-cat state, has quantum coherence that is resistant to the twin effects of environmental noise – local decoherence on all the particles and loss of a finite fraction of its particles. The states of the macroscopic sector of the W-cat state are macroscopically distinct in terms of their violation of Bell inequality.

I. INTRODUCTION AND MAIN RESULTS

The recent developments in computation and communication tasks have underlined the necessity to preserve quantum coherence in states shared by a large number of quantum systems [1]. Feynman proposed that complex and large quantum systems can be efficiently simulated only by using a quantum computer [2]. Shor's algorithm demonstrated that quantum algorithms can be used to efficiently solve problems that may not be possible with classical ones [3]. To build a viable quantum computer that can compile and implement a quantum algorithm, which outperforms the ones running on classical machines, requires quantum coherence preserved in a system of about 10^3 qubits [4]. Coherence in quantum states of a large number of particles is one of the essential ingredients for building a quantum communication network [5]. Such exciting developments on the theoretical front were accompanied by several experimental proposals and realizations, by using e.g. photons, ion traps, cold atoms, and nuclear magnetic resonance [6].

Since preserving quantum coherence in states shared between multiparticle quantum systems is one of the basic necessities for many communicational and computational tasks, it is important to build a macroscopic entangled state. Such an entangled state was first introduced by Schrödinger in his seminal 1935 paper [7] through the concept of the Schrödinger cat, which is an entangled state between a microscopic system and a macroscopic one. The microscopic system can be an atom, that can decay spontaneously, with the undecayed state |up| and the decayed state |down\rangle making up a two-dimensional complex Hilbert space (qubit). The macroscopic system was also conceived as a qubit made up of the alive and dead states of a cat, respectively denoted as |alive\) and |dead\|. The quantum state of the combined micro-macro system is considered to be

$$\frac{1}{\sqrt{2}} (|\text{up}\rangle|\text{alive}\rangle + |\text{down}\rangle|\text{dead}\rangle). \tag{1}$$

Apart from its significance in technological pursuits, it is also important for understanding the quantum measurement problem and the quantum-to-classical transi-

tion [8, 9].

The classic example of the Schrödinger cat is the Greenberger-Horne-Zeilinger (GHZ) state [10], given by

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle_{\mu} |0^{\otimes N}\rangle_{A_1...A_N} + |1\rangle_{\mu} |1^{\otimes N}\rangle_{A_1...A_N} \right).$$
(2)

The party denoted by μ is the microscopic part of the Schrödinger cat, while the parties A_1 through A_N make up the macroscopic portion of the same. The microscopic part is a single qubit, and is spanned by the orthonormal states $|0\rangle$ and $|1\rangle$. The macroscopic portion is built out of N qubits, denoted as A_1, A_2, \ldots, A_N , with each being spanned by the orthonormal states $|0\rangle$ and $|1\rangle$. Noise effects on the GHZ state have been studied by using a variety of models [11]. However, as is well-known, the GHZ state loses all quantum coherence if even a single qubit is lost.

In this paper, we propose a macroscopic state, which we call the W-cat state, shared between N+1 particles, given by

$$|H_C\rangle_{\mu A_1...A_N} = \frac{1}{\sqrt{2}} (|0\rangle_{\mu}|W_N\rangle_{A_1...A_N} + |1\rangle_{\mu}|0...0\rangle_{A_1...A_N}). (3)$$

Here, $|W_N\rangle$ is the N-particle W state [12–15], given by

$$|W_N\rangle_{A_1...A_N} = \frac{1}{\sqrt{N}} \sum |10...0\rangle_{A_1...A_N}, \qquad (4)$$

where the sum denotes an equal superposition of all N particle states consisting of a single $|1\rangle$ and (N-1) $|0\rangle$ s. We show that the W-cat state is robust, i.e. can preserve quantum coherence in the form of entanglement between its micro and macro sectors, against loss of a finite fraction of its particles and against local depolarizations on all its particles, and with the simultaneous action of both these noise effects. We then compare the robustness of this state with other macroscopic states, that can potentially be used as cat states. In particular, we find that for a finite number of particles in the macroscopic part, the W-cat state is more robust to local depolarizing noise than the GHZ state. Such investigations can potentially

be a step towards building quantum memory devices using macroscopic systems.

The paper is organized as follows. In Sec. II, we discuss the noise models that we have considered in this paper. The structure of the W-cat state that make it a Schrödinger cat is revealed in Sec. III. The entanglement measure used to measure the quantum coherence in the Schrödinger cats is briefly discussed in Sec. IV. The effect of noise models (local decoherence and particle loss) on the W-cat states are discussed in Secs. V, VI, and VII. In Sec. VIII, we compare the W-cat state with other potential cat states. Finally, we present a conclusion in Sec. IX.

II. ENVIRONMENTAL EFFECTS

It is of vital importance to investigate the effects of environmental noise on a Schrödinger cat, in understanding its fundamental as well as technological implications. Usually, the environmental effect that is considered for a Schrödinger cat is decoherence. Here we consider the coherence properties of the Schrödinger cat after it has been subjected, *simultaneously* as well as separately, to local decoherence channels, in the form of local depolarizing channels, on all its constituent particles (in the micro as well as the macro sectors) and to loss of a finite fraction of its particles (in the macro part).

The depolarizing channel destroys off-diagonal elements of a quantum density matrix, destroying quantum coherence in the state, and is the usual model for decoherence phenomena [16]. Decoherence has, for example, been used to understand the transition of a quantum system to an effectively classical system [9]. Protecting a quantum system from decoherence is one of the main challenges faced by quantum experimentalists and engineers. Mathematically, the action of a general depolarizing channel, denoted by D_p , on a qubit is given by [16]

$$|i\rangle\langle j| \to \frac{p}{2}I + (1-p)|i\rangle\langle j|,$$
 (5)

so that qubit remains unchanged with probability (1-p) and gets disturbed, by white noise, with probability p. Here, I denotes the identity matrix in the qubit Hilbert space. Note that this is a local noise model, which is a natural choice for multiparty experimental situations, and our interest is in analyzing the coherence retained after the action of this local noise on all the qubits building up a Schrödinger cat. The cat state is also simultaneously inflicted by loss of some qubits. The effects of local decoherence and particle loss, are also considered separately as special cases.

III. THE W-CAT STATE

In search for a cat state, that better withstands the environmental effects including particle loss, we propose

the W-cat state, $|H_C\rangle$. The states $|\text{alive}\rangle$ and $|\text{dead}\rangle$ of the original Schrödinger cat are certainly macroscopically different. In the "GHZ-cat" state, $|GHZ\rangle$, these are replaced by the states $|0^{\otimes N}\rangle$ and $|1^{\otimes N}\rangle$ respectively. They are macroscopically different with respect to their magnetizations, i.e. with respect to their average values for the operator $\frac{1}{N}\sum_{i=1}^{N}\sigma_z^i$, where $\sigma_z^i=|0\rangle\langle 0|-|1\rangle\langle 1|$. In case of the W-cat state, the states $|\text{alive}\rangle$ and $|\text{dead}\rangle$ of the original cat state are replaced respectively by the states $|W_N\rangle$ and $|0^{\otimes N}\rangle$. The latter are macroscopically different in terms of their violation of local realism.

It is well-known that quantum mechanics is inconsistent with the assumption of an underlying hidden variable ("realistic") theory that is also local. This is the statement of the celebrated Bell theorem [17]. An important quantity quantifying the amount of violation of local realism by a quantum mechanical state is the critical visibility beyond which the state violates local realism. Among the "alive" and "dead" cat states that are used to build the W-cat state, the state $|0^{\otimes N}\rangle$ certainly does not violate any local realism. However, the state $|W_N\rangle$ has a critical visibility, given by [15]

$$p_N^{crit} = \frac{N}{(\sqrt{2} - 1)2^{N-1} + N},\tag{6}$$

which tends to zero as $N \to \infty$. It is in this sense that the states $|W_N\rangle$ and $|0^{\otimes N}\rangle$ of the macroscopic part of the W-cat state are macroscopically different.

The initial density matrix, i.e. the density matrix of the W-cat state before it passes through the local depolarizing channels and is affected by particle loss, is denoted here by ρ_{N+1}^{in} , and given by

$$\rho_{N+1}^{in} = \frac{1}{2} [|0\rangle |W_N\rangle \langle 0|\langle W_N| + |0\rangle |W_N\rangle \langle 1|\langle 0...0| + |1\rangle |0...0\rangle \langle 0|\langle W_N| + |1\rangle |0...0\rangle \langle 1|\langle 0...0|].$$
 (7)

IV. ENTANGLEMENT

As mentioned before, it is important to understand the amount of environmental effect that a certain cat state can withstand. The "cat-ness" of a Schrödinger cat is in the quantum coherence that exists between the micro and the macro sectors of the state. We measure the quantum coherence between these two sectors by using an entanglement measure.

Entanglement of a two-party system shared between two parties A and B, like the micro-macro system in the cat states, is defined as the inability of a quantum state of that system to be expressed in the separable form

$$\sum_{i} p_{i} \rho_{A}^{i} \otimes \rho_{B}^{i}, \tag{8}$$

where $\{p_i\}$ forms a probability distribution, and ρ_A^i (ρ_B^i) are states of the party A (B) [18].

A convenient quantity to measure the entanglement in bipartite systems is the logarithmic negativity [19], defined, for a quantum state ρ of a two-party system, as

$$E_N(\rho) \equiv \log_2(2N(\rho) + 1),\tag{9}$$

where the "negativity", $N(\rho)$, is given by the sum of the absolute values of the negative eigenvalues of the partial transpose of ρ with respect to either of the two parties. If either of the two parties is a qubit, the maximal value of E_N is unity.

V. ENTANGLEMENT OF W-CAT STATE WITH PARTICLE LOSS

In this section, we investigate the situation where the system looses a certain number of particles from its macroscopic part. Suppose ρ_{N+1}^{in} looses m particles from among the N particles constituting the macroscopic portion of the system. The resultant density matrix will then be an (N-m+1) party system having the following form:

$$\rho_{N-m+1}^{L} = \frac{1}{2} \left[\frac{(N-m)}{N} |0\rangle\langle 0| \otimes |W_{N-m}\rangle\langle W_{N-m}| + \frac{m}{N} |0\rangle\langle 0| \otimes |0...0\rangle\langle 0...0| + \sqrt{\frac{(N-m)}{N}} |0\rangle\langle 1| \otimes |W_{N-m}\rangle\langle 0...0| + \sqrt{\frac{(N-m)}{N}} |1\rangle\langle 0| \otimes |0...0\rangle\langle W_{N-m}| + |1\rangle\langle 1| \otimes |0...0\rangle\langle 0...0| \right].$$
(10)

Here, the tensor product notation has been retained between the microscopic part (one qubit) and whatever has remained (N-m) qubits) after the loss of m particles from the macroscopic part. To investigate the effect of particle loss on the quantum coherence of the W-cat state $|H_C\rangle_{\mu A_1...A_N}$, we find the entanglement of the resultant state (after particle loss) in the $\mu:A_1...A_{N-m}$ bipartition. Note here that we have assumed, without loss of generality, that the particles $A_{N-m+1}, A_{N-m+2}, ..., A_N$ are lost. After taking the partial transposition with respect to the microscopic sector of the system, the partial transposed state of ρ_{N-m+1}^L is seen to be block-diagonal. The negative eigenvalue of the partial transposed state, denoted by λ_- , is given by

$$\lambda_{-} = -\frac{1}{2} \left(1 - \frac{m}{N} \right). \tag{11}$$

Therefore, the entanglement of W-cat state after the loss of m particles is given by

$$E_N(\rho) = \log_2\left(2 - \frac{m}{N}\right). \tag{12}$$

For large N, and for possibly large m satisfying $m \ll N$, the entanglement between the microscopic and macroscopic subsystems reaches unity, irrespective of the value of m, and hence the state is (nearly) maximally entangled in this bipartition, as is also clear from Fig. 1. The behavior of entanglement of the W-cat state with different

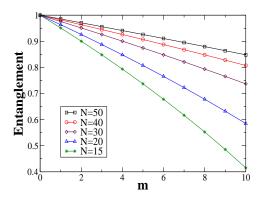


FIG. 1. (Color online.) Entanglement after particle loss in W-cat state. The horizontal axis represents the number of particles lost (m) from the macroscopic part of the W-cat state while the vertical one represents the entanglement between the micro and the macro parts of the W-cat state after particle loss. The entanglement with respect to m is plotted for different initial number of particles, N. The vertical axis is measured in ebits, while the horizontal one is in particles.

rates of particle loss and for different total numbers of particles, is depicted in Fig. 1. The logarithmic decrease of entanglement with increasing numbers of particles lost, as seen from Eq. (12), is also clearly visible in Fig. 1.

VI. ENTANGLEMENT OF W-CAT STATE UNDER LOCAL DECOHERENCE

This section is devoted to the investigation of the entanglement properties of the W-cat state under local decoherence effects on all the N+1 constituent particles. To this end, each particle, whether from the microscopic or the macroscopic sector, of the initial state ρ_{N+1}^{in} is fed to a the depolarizing channel, defined in Eq. (5). The output state, after this process, can be expressed as

$$D_p^1 \otimes D_p^2 \otimes \ldots \otimes D_p^{N+1} \rho_{N+1}^{in} \equiv \rho_{N+1}^{DP}, \tag{13}$$

where $D_p^1, D_p^2, \ldots, D_p^{N+1}$ are N+1 depolarizing channels acting on the N+1 particles in the initial state. The entanglement of the locally decohered W-cat state can now be analyzed in the micro: macro bipartition. The mathematical form of the entanglement will be presented in a more general context in the succeeding section, and so refrain from presenting it here. The results are depicted in Fig. 2, where we also present the corresponding curves for the GHZ states. Interestingly, we obtain that the W-cat state is more resistant to local decoherence than the GHZ state, and for example, for 10 particles in the macroscopic part, the W-cat state can preserve entanglement up to 44% of local decohering noise, while the GHZ state remain entangled until 28% of the same noise.

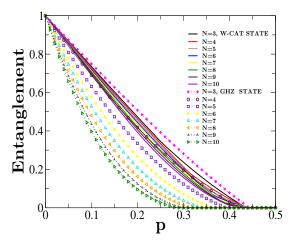


FIG. 2. (Color online.) Entanglement of GHZ and W-cat states against local decoherence. While the dotted lines are for the GHZ states, the continuous ones are for the W-cat. The horizontal axis represents the dimensionless (decohering noise) parameter p, and the vertical axis is the entanglement in the micro: macro bipartition (in ebits).

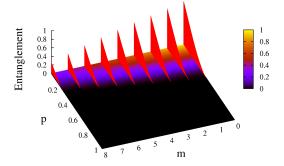


FIG. 3. (Color online.) Effect of local decoherence and particle loss on W-cat state. Entanglement (measured in ebits) in the micro: macro bipartition is plotted on the vertical axis against a base of the dimensionless depolarizing parameter p, and the number of lost particles (m). The W-cat state under consideration is of 11 qubits, so that N=10.

VII. ENTANGLEMENT OF W-CAT STATE UNDER PARTICLE LOSS AND LOCAL DECOHERENCE

We now consider the situation where the W-cat state is affected by local decoherence as well as by particle loss. We assume that m partcles are lost (from the macro part) and that the remaining N-m+1 particles are all affected by local decoherence as modelled by the depolarizing channel. The entanglement in the micro: macro bipartition is analyzed for the resulting N-m+1-party state. The two eigenvalues of the partial transposed state that make the maximum contribution are $\lambda_{-}^{(1)}$ and $\lambda_{-}^{(2)}$,

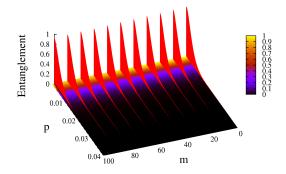


FIG. 4. (Color online.) Effect of local decoherence and up to 10% particle loss on a W-cat state of $10^3 + 1$ qubits. All other considerations except N is the same as in Fig. 3.

where $|\lambda_{-}^{(1)}| > |\lambda_{-}^{(2)}|$. These two eigenvalues are given by

$$\lambda_{-}^{(1)} = \frac{1}{4} \left\{ c + (N - m - 1)d + a - \sqrt{4(N - m)b^2 + (c + (N - m - 1)d - a)^2} \right\}, (14)$$

where

$$a = \gamma_1 \tilde{p} + \frac{m}{N} \tilde{p}^{N-m+1} + \frac{p}{2} \tilde{p}^{N-m},$$

$$b = \frac{1}{\sqrt{N}} (1-p)^2 \tilde{p}^{N-m-1},$$

$$c = \alpha_1 \frac{p}{2} + \frac{m}{N} \left(\frac{p}{2}\right)^2 \tilde{p}^{N-m-1} + \frac{p}{2} \tilde{p}^{N-m},$$

$$d = \frac{1}{N} \frac{p}{2} (1-p)^2 \tilde{p}^{N-m-2},$$
(15)

with $\alpha_1 = \frac{1}{N} \left(\tilde{p}^{N-m} + (N-m-1)(\frac{p}{2})^2 \tilde{p}^{N-m-2} \right), \ \gamma_1 = \left(\frac{N-m}{N} \right) (\frac{p}{2}) \tilde{p}^{N-m-1}, \ \tilde{p} = 1 - \frac{p}{2}, \ \text{and}$

$$\lambda_{-}^{(2)} = \frac{1}{4} \left\{ (a_1 - b_1 + f + \tilde{N}g) - \sqrt{4(\tilde{N} + 2)e^2 + (-a_1 + b_1 + f + \tilde{N}g)^2} \right\}, \quad (16)$$

where

$$a_{1} = \left(\frac{p}{2}\right)^{2} \tilde{p}^{N-m-1} + \frac{m}{N} \frac{p}{2} \tilde{p}^{N-m} + \gamma_{2} \tilde{p},$$

$$b_{1} = \frac{1}{N} (1-p)^{2} \tilde{p}^{N-m-1},$$

$$e = \frac{1}{\sqrt{N}} (1-p)^{2} \frac{p}{2} \tilde{p}^{N-m-2},$$

$$g = \frac{1}{N} (1-p)^{2} \left(\frac{p}{2}\right)^{2} \tilde{p}^{N-m-3},$$

$$f = \left(\frac{p}{2}\right)^{2} \tilde{p}^{N-m-1} + \frac{m}{N} \left(\frac{p}{2}\right)^{3} \tilde{p}^{N-m-2} + \alpha_{2} \frac{p}{2},$$
(17)

with
$$\alpha_2 = (1/N)[2\tilde{p}^{N-m-1}(p/2) + (N-m-2)(p/2)^3\tilde{p}^{N-m-3}], \quad \gamma_2 = \frac{1}{N}(\tilde{p}^{N-m} + (N-m-m-2))$$

1) $\left(\frac{p}{2}\right)^2 \tilde{p}^{N-m-2}$), $\tilde{N} = N-m-4$. The remaining eigenvalues make a contribution to the logarithmic negativity that is rather insignificant, and so for N=8, m=1 and p=0.1, their contribution to the entanglement is less than 10^{-2} . Note here that by setting m=0, we can obtain the entanglement expressions for the case considered in the preceding section.

The entanglements are plotted in Figs. 3 and 4. In particular, in Fig. 4, we consider the case when the macroscopic system is constituted out of $N=10^3$ particles, and we find that the entanglement in the micro: macro bipartition remains almost at its initial maximal value even with the loss of about 10% of its particles. Entanglement remains nonzero even when the remaining 90% particles are fed to local depolarizing channels until $p \lesssim .03$.

VIII. NOISE EFFECTS ON ENTANGLEMENT OF OTHER CAT STATES

In this section, we consider some other states which may potentially be considered as cat states, and compare their ability to withstand particle loss. We begin by considering the state

$$|\Psi_1\rangle_{\mu A_1...A_N} = \frac{1}{\sqrt{2}} \left(|0\rangle_{\mu} |W_N\rangle_{A_1...A_N} + |1\rangle_{\mu} |\widetilde{W}\rangle_{A_1...A_N} \right),$$

$$(18)$$

where

$$|\widetilde{W}_N\rangle_{A_1...A_N} = \sigma_x^{\otimes N} |W_N\rangle_{A_1...A_N},$$
 (19)

where $\sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|$. The state $|\Psi_1\rangle$ is the G state of Ref. [20]. This state is a cat state in the sense that the states $|W\rangle$ and $|\widetilde{W}\rangle$ are macroscopically distinct in terms of their σ_z -magnetizations, just like in the case of the GHZ state. This state, however, becomes separable if, for any N, we lose more than two particles. Another state that can be considered as the cat state is apparently quite similar to the W-cat state, with only the N-qubit W state state replaced by the state $|\widetilde{W}_N\rangle$. This state is

given, therefore, by

$$|\Psi_2\rangle_{\mu A_1...A_N} = \frac{1}{\sqrt{2}} \left(|0\rangle_{\mu} |\widetilde{W}_N\rangle_{A_1...A_N} + |1\rangle_{\mu} |0...0\rangle_{A_1...A_N} \right). (20)$$

This state is a cat state in the same sense as the W-cat state – the Bell inequality violations of $|\widetilde{W}_N\rangle$ and $|0\dots0\rangle$ are drastically different. Moreover, the states $|\widetilde{W}_N\rangle$ and $|0\dots0\rangle$ are also macroscopically different in terms of their σ_z -magnetizations. This state becomes separable if, for any N, we lose more than one particle. An interesting generalization of the GHZ state is the concatenated GHZ state [21], and is given by

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}} \left(|GHZ_l^+\rangle^{\otimes(N+1)} + |GHZ_l^-\rangle^{\otimes(N+1)} \right), \quad (21)$$
where

$$|GHZ_l^{\pm}\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle^{\otimes l} \pm |1\rangle^{\otimes l} \right).$$
 (22)

Here there are N+1 logical qubits, and each logical qubit is built by using l physical qubits. Loss of all physical qubits from a logical qubit renders this state separable, like the GHZ state. Also, loss of some physical qubits from different logical qubits leads to separable states.

IX. CONCLUSION

The concept of the Schrödinger cat is an important aspect of quantum physics with significance on the fundamental front as well as in useful applications. There is an ongoing effort towards experimental realization of such cat states in a variety of physical substrates. We have proposed a Schrödinger cat, whose "alive" and "dead" states are modelled by two quantum states that drastically differ by their amounts of violation of Bell inequalities. We show that this state is robust against loss of a finite fraction of its particles and simultaneously against local depolarizing channels, modelling a local decoherence mechanism. We compare our results with other potential cat states, including the Greenberger-Horne-Zeilinger state.

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