

FUZZY DATA ENVELOPMENT ANALYSIS FOR PERFORMANCE EVALUATION OF AN IRRIGATION SYSTEM[†]

K. SRINIVASA RAJU¹ AND D. NAGESH KUMAR^{2*}

¹*Department of Civil Engineering, Birla Institute of Technology and Science, Pilani, Hyderabad Campus, India*

²*Department of Civil Engineering, Indian Institute of Science, Bangalore, India*

ABSTRACT

Sixteen irrigation subsystems of the Mahi Bajaj Sagar Project, Rajasthan, India, are evaluated and selection of the most suitable/best is made using data envelopment analysis (DEA) in both deterministic and fuzzy environments. Seven performance-related indicators, namely, land development works (LDW), timely supply of inputs (TSI), conjunctive use of water resources (CUW), participation of farmers (PF), environmental conservation (EC), economic impact (EI) and crop productivity (CPR) are considered. Of the seven, LDW, TSI, CUW, PF and EC are considered inputs, whereas CPR and EI are considered outputs for DEA modelling purposes. Spearman rank correlation coefficient values are also computed for various scenarios. It is concluded that DEA in both deterministic and fuzzy environments is useful for the present problem. However, the outcome of fuzzy DEA may be explored for further analysis due to its simple, effective data and discrimination handling procedure. It is inferred that the present study can be explored for similar situations with suitable modifications. Copyright © 2013 John Wiley & Sons, Ltd.

KEY WORDS: data envelopment analysis; fuzzy; irrigation subsystems; performance; Spearman rank correlation

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RÉSUMÉ

Seize sous-systèmes d'irrigation du Mahi Bajaj Sagar Projet (Rajasthan, Inde) sont évalués et la sélection du meilleur compromis de performances est faite en utilisant l'analyse de développement de données (DEA) dans des environnements à la fois déterministe et flou. Sept indicateurs de performance connexes sont pris en compte, à savoir, travaux d'aménagement du territoire (LDW), approvisionnement à temps des intrants (STI), utilisation conjointe des ressources en eau (CUW), participation des agriculteurs (PF), conservation de l'environnement (CE), impact économique (EI) et productivité des cultures (CPR). Sur sept indicateurs, LDW, STI, CUW, PF, CE sont considérées comme des intrants alors que CPR et EI sont considérés comme des produits à des fins de modélisation de DEA. Les valeurs classées des coefficients de corrélation de Spearman sont également calculées pour différents scénarios. Il est conclu que la DEA à la fois dans l'environnement déterministe et flou est utile pour le problème posé. Cependant, les résultats du DEA flou peuvent être explorés pour une analyse plus approfondie des données, car les procédures sont simples, efficaces et discriminantes. On en déduit que la présente étude peut être utilisée pour des situations similaires, avec les adaptations appropriées. Copyright © 2013 John Wiley & Sons, Ltd.

MOTS CLÉS: analyse par enveloppement de données; environnement flou; sous systèmes d'irrigation; performance; corrélation classée de Spearman

INTRODUCTION

Performance evaluation of existing irrigation systems and improving the same is becoming a focal point in most developing countries mainly due to the high initial cost of new

irrigation systems, increasing concern about the environment and other related aspects (Raju and Nagesh Kumar, 2005). This situation necessitates cost-effective, sustainable and replicable alternative approaches that will enable us to improve the performance of existing irrigation systems. In addition, most of the indicators, for example land development works, timely supply of inputs, conjunctive use of water resources, participation of farmers, environmental conservation, economic impact and crop productivity, that

* Correspondence to: D. Nagesh Kumar, Department of Civil Engineering, Indian Institute of Science, Bangalore, India. E-mail: nagesh@civil.iisc.ernet.in

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are required to judge performance are subjective (lack of precise quantitative data availability is also assumed as subjective). Thus, more complexity is added to the planning problem. The present study addresses these issues by evaluating 16 selected irrigation subsystems of the Mahi Bajaj Sagar Project, Rajasthan, India, for prioritization so that those suitable among them will form the basis for further improvements. Data envelopment analysis (DEA) in both deterministic and fuzzy environments is employed for this purpose and comparative analysis is also made. The present paper comprises a literature review both with reference to DEA and the performance evaluation context, description of DEA in both deterministic and fuzzy environments, Spearman rank correlation coefficient, case study, results and discussion followed by summary and conclusions.

LITERATURE REVIEW

A brief literature review related to performance evaluation and DEA is presented below.

Bos *et al.* (2005) suggested guidelines for conducting performance assessment studies and related management. They discussed various facets of performance assessment such as identification of indicators, operational strategic performance assessment, diagnosing irrigation performance and data management. Gorantiwar and Smout (2005) presented two types of allocative measures, namely productivity and equity, and five types of scheduling measures, namely efficiency, sustainability, flexibility, reliability and adequacy, with the objective of efficient irrigation management. Molden *et al.* (2007) discussed performance assessment, poverty alleviation and suggested a number of ways where performance assessment can be used. Batt and Merkley (2010) emphasized the need for irrigation system improvement policies which are encouraging, realistic and support agriculture. Yakubov (2012) conducted surveys to understand farmers' views on irrigation service performance in three countries of Central Asia sharing the Fergana valley—Kyrgyzstan, Tajikistan and Uzbekistan. Conducted surveys helped to understand farmers' actual experiences, perceptions, priorities, and their satisfaction/ dissatisfaction with irrigation services. Ghazouani *et al.* (2012) undertook comprehensive surveys in a community-managed oasis in southern Tunisia and identified reasons for low irrigation performance.

Guo and Tanaka (2001) proposed two kinds of fuzzy DEA models for evaluating the efficiencies of decision-making units (DMUs) with fuzzy input and output data. They presented a perceptual evaluation method based on fuzzy DEA for solving a real-world problem. Saati *et al.* (2002) proposed a fuzzy extension of the Charnes, Cooper and Rhodes (CCR) model assuming that both input and output data follow triangular fuzzy numbers and employed

an α -cut level approach. They defined the DEA model as a possibility programming problem, and transformed it into an interval programming problem. A crisp linear programming model is used to provide efficiency for each given α -cut level and each DMU. They also developed an efficient algorithm to discriminate the DMUs when most of them are equally efficient (with a value of 1). Similar efforts were made by Lertworasirikul *et al.* (2003). Cook and Seiford (2009) provided a state-of-the-art review in DEA over the last three decades. Marbini *et al.* (2011) provided taxonomy and a review of fuzzy DEA methods and presented a classification scheme with four primary approaches, namely tolerance, α -level based, fuzzy ranking and possibility. They discussed various classification schemes and grouped the fuzzy DEA papers published over the past 20 years. However, very limited studies are reported in water resources or in irrigation planning scenarios.

Diaz *et al.* (2004) applied DEA to 35 irrigation districts of Andalusia, southern Spain, for three inputs, namely irrigated surface area, labour and total volume of water applied to an irrigation district, with agricultural production as output. It was concluded that DEA was found to be a useful technique for finding the efficiency of each irrigation district. Similar studies are reported by Srdjevic *et al.* (2005) for reservoir system performance in Brazil using DEA. Raju and Nagesh Kumar (2006) used DEA for ranking irrigation planning alternatives by using labour employment and agricultural production as inputs and net benefit as output for the case study of the Sri Ram Sagar Project, India, and showed the application potential of DEA in irrigation planning. Lilienfeld and Asmild (2007) determined the impacts of irrigation system type and other variables on irrigation water use efficiency for a sample of 43 irrigators in western Kansas between 1992 and 1999 using DEA. They determined the relationship between the magnitude of irrigation water excess/reduction potentials, and irrigation system type and number of other farm characteristics. Speelman *et al.* (2008) analysed the efficiency with which water is used in small-scale irrigation schemes in the North-West Province in South Africa using DEA. They considered farm size, type of irrigation scheme, landownership, crop choice, fragmentation and irrigation methods applied and showed that the smallholder irrigation farmers in the study area fail to reach their overall technical efficiency levels in water use.

Yilmaz *et al.* (2009) assessed irrigation efficiency in Buyuk Menderes Basin, Turkey, using DEA where 17 alternatives were evaluated on three indicators, namely water volume used, area irrigated and total production value. Frija *et al.* (2009a) employed two-stage analysis where DEA and Tobit models are applied to a case study of Cap Bon region, Tunisia, to determine overall, management, maintenance, and scale efficiency of the 45 Tunisian water user associations (WUAs). They used management expenditure,

maintenance expenditure and purchasing water cost as inputs and annual irrigated area and total annual irrigation water delivery per unit irrigated as outputs. Various aspects of WUAs including their performance, principal financial revenues and expenditures were also discussed. It was concluded that more analysis is required for understanding the functioning of the WUAs. Frija *et al.* (2009b) measured the technical efficiency of unheated greenhouse farms in Tunisia, and proposed a measure for irrigation water use efficiency using an alternative form of the DEA model. It is concluded that farmers' technical training in greenhouse management, investments in water-saving technologies and the existence of a fertigation technique on farm have a significant and positive effect on their level of irrigation water use efficiency, whereas it is significantly and negatively affected by the proportion of total farmland allocated to greenhouses.

Raju and Nagesh Kumar (2010) also discussed DEA in detail from various perspectives such as the cross-efficiency matrix and weight restrictions. To the authors' knowledge, fuzzy DEA is not used to evaluate irrigation systems with as many as seven indicators with subjective data based on field studies and interviews.

DATA ENVELOPMENT ANALYSIS

DEA considers the interrelationship between all the inputs and outputs simultaneously, resulting in a more consistent measure of efficiency, and uses linear programming to determine efficiency (Li and Reeves, 1999; Sarkis, 2000; Ramanathan, 2003). DEA is becoming an alternative methodology for solving multicriterion decision making (MCDM) problems, because of its comparison of efficiency of each planning alternative/decision-making unit to that of an ideal operating unit rather than to average performance, and its lower requirement of sensitivity analysis (Li and Reeves, 1999; Raju and Nagesh Kumar, 2010). In addition, in the case of MCDM approaches, it is assumed that all the indicators are independent even though they are related one way or another, whereas in DEA their relationship is considered in the form of input and output which is an added advantage.

In the present study, methodologies proposed by Charnes *et al.* (1978), namely the Charnes, Cooper and Rhodes (CCR) [deterministic form of DEA] and the fuzzy version of CCR proposed by Saati *et al.* (2002), are employed which are explained below.

Deterministic DEA

Charnes *et al.* (1978) proposed the CCR model to compute the efficiency of the decision-making unit. The model assumes that proportional change exists between inputs and outputs and does not consider the effects of external

factors (Boile, 2001). The model is expressed as follows: Maximum efficiency is defined as:

$$\text{Max}_{b_r, c_i} \sum_{r=1}^s b_r q_{rk} \quad (1)$$

subject to

$$\sum_{r=1}^s b_r q_{rj} - \sum_{i=1}^m c_i p_{ij} \leq 0; \\ j = 1, 2, \dots, k \dots, n \text{ [Constraint related to input - output]} \quad (2)$$

$$\sum_{i=1}^m c_i p_{ik} = 1; \text{ [Constraint related to input data]} \quad (3)$$

Bounds on weight vector:

$$b_r \geq \varepsilon_r; r = 1, 2, \dots, s \text{ [Bounds on output weights]} \quad (4)$$

$$c_i \geq \varepsilon_i; i = 1, 2, \dots, m \text{ [Bounds on input weights]} \quad (5)$$

where j = DMU index, $j = 1, 2, \dots, k, \dots, n$; r = output index, $r = 1, 2, \dots, s$; i = input index; q_{rk} = value of r th output for the k th DMU; q_{rj} = value of r th output for the j th DMU; p_{ik} = value of i th input for the k th DMU; p_{ij} = value of i th input for the j th DMU; b_r = weight of the r th output; c_i = weight of the i th input; $\varepsilon_r, \varepsilon_i$ = lower limit of weights for output r and input i .

A DMU with an efficiency value of 1 is viewed as the most preferred and no other DMU is more efficient than that DMU and the weights derived for that DMU are termed as optimal (Sexton *et al.*, 1986; Li and Reeves, 1999; Adler *et al.*, 2002). The problem of weak discriminating power that arose due to identification of too many DMUs as efficient (with value 1) can be minimized by use of the cross-efficiency approach (Sexton *et al.*, 1986) which is explained in the results and discussion section.

Fuzzy DEA

All the input and output data are assumed to be precise/ crisp in deterministic DEA. However, this may not be realistic in performance evaluation studies, where data related to inputs and outputs are ever-changing due to variation in field conditions from time to time, i.e. summer and winter, floods and droughts, subjective views of various stakeholders at different levels and at different times. Therefore, there is a necessity for the DEA model to be able to adequately represent impreciseness at micro to macro level. This aspect is addressed by the fuzzy DEA model. Marbini *et al.* (2011) in their study while discussing various fuzzy DEA models mentioned that the latter are very much suitable for real-world problems which are often experienced with imprecise

or vague data resulting from unquantifiable, incomplete and/or non-obtainable information.

The mathematical expression of the CCR model in a fuzzy DEA environment as proposed by Saati *et al.* (2002) is

$$\text{Max}_{b_r, c_i} \sum_{r=1}^s b_r q_{rk}^{\sim} [\text{Maximize efficiency}] \quad (6)$$

subject to

$$\sum_{r=1}^s b_r q_{rj}^{\sim} - \sum_{i=1}^m c_i p_{ij}^{\sim} \leq 0; \quad j = 1, 2, \dots, n \quad [\text{Constraint related to input - output}] \quad (7)$$

$$\sum_{i=1}^m c_i p_{ik}^{\sim} = 1 \quad ; j = 1, 2, \dots, n \quad [\text{Constraint related to input data}] \quad (8)$$

$$b_r \geq \varepsilon_r \quad ; r = 1, 2, \dots, s \quad [\text{Bounds on output weights}] \quad (9)$$

$$c_i \geq \varepsilon_i \quad ; i = 1, 2, \dots, m \quad [\text{Bounds on input weights}] \quad (10)$$

Fuzziness in inputs and outputs is represented with notation ' \sim '. Inputs and the outputs of the DMUs are assumed to follow a triangular membership function with (p_l, p_m, p_u) and (q_l, q_m, q_u) . Here l, m, u represent lower, middle and upper values respectively and 1^{\sim} represents $(1^l, 1, 1^u)$. In the present study an α -cut approach is used (Ross, 2010), in exploring α -cuts of objective function and constraints, and the resulting model is as follows:

$$\text{Max} \sum_{r=1}^s b_r [\alpha q_{rk}^m + (1 - \alpha) q_{rk}^l, \alpha q_{rk}^m + (1 - \alpha) q_{rk}^u] \quad (11)$$

subject to

$$\sum_i^m c_i [\alpha p_{ik}^m + (1 - \alpha) p_{ik}^l, \alpha p_{ik}^m + (1 - \alpha) p_{ik}^u] = [\alpha + (1 - \alpha) 1^l, \alpha + (1 - \alpha) 1^u] \quad \forall i \quad (12)$$

$$\sum_{r=1}^s b_r [\alpha q_{rj}^m + (1 - \alpha) q_{rj}^l, \alpha q_{rj}^m + (1 - \alpha) q_{rj}^u] - \sum_{i=1}^m c_i [\alpha p_{ij}^m + (1 - \alpha) p_{ij}^l, \alpha p_{ij}^m + (1 - \alpha) p_{ij}^u] \leq 0 \quad (13)$$

$$b_r \geq \varepsilon_r; \quad c_i \geq \varepsilon_i \quad \forall i, r \quad (14)$$

However, transformation is required in Equations (11)–(13) as these are in interval form. A nonlinear trend of the problem

resulting from the unknown nature of weights of the outputs and optimizing points can be linearized using a variable substitution approach (Saati *et al.*, 2002) resulting in

$$\text{Max} \sum_{r=1}^s \bar{q}_{rk} \quad (15)$$

subject to

$$\sum_{i=1}^m p_{ik}^{\bar{}} = 1 \quad (16)$$

$$\sum_{r=1}^s \bar{q}_{rj} - \sum_{i=1}^m p_{ij}^{\bar{}} \leq 0 \quad \forall j \quad (17)$$

$$c_i (\alpha p_{ij}^m + (1 - \alpha) p_{ij}^l) \leq p_{ij}^{\bar{}} \leq c_i (\alpha p_{ij}^m + (1 - \alpha) p_{ij}^u) \quad \forall i, j \quad (18)$$

$$b_r (\alpha q_{rj}^m + (1 - \alpha) q_{rj}^l) \leq \bar{q}_{rj} \leq b_r (\alpha q_{rj}^m + (1 - \alpha) q_{rj}^u) \quad \forall i, j \quad (19)$$

$$b_r \geq \varepsilon_r; \quad c_i \geq \varepsilon_i \quad \forall i, r \quad (20)$$

where $q_{rj}^{\bar{}} = b_r \hat{q}_{rj}; p_{ij}^{\bar{}} = c_i \hat{p}_{ij}$ with

$$\hat{q}_{rj} \in [\alpha q_{rj}^m + (1 - \alpha) q_{rj}^l, \alpha q_{rj}^m + (1 - \alpha) q_{rj}^u]$$

$$\hat{p}_{ij} \in [\alpha p_{ij}^m + (1 - \alpha) p_{ij}^l, \alpha p_{ij}^m + (1 - \alpha) p_{ij}^u]$$

α -cut values can be varied from 0 to 1 (0 for impreciseness/uncertainty and 1 for preciseness/certainty) and can be used for evaluating and comparing DMUs. Saati *et al.* (2002) also proposed a method for ranking the efficient DMUs (where efficiency value is one due to identifying too many DMUs as efficient) which is analogous to the cross-efficiency matrix (presented in the form of Equations 21–24):

$$\text{Min} \delta \quad (21)$$

subject to

$$\delta (\alpha p_{ik}^m + (1 - \alpha) p_{ik}^l) \geq \sum_{j=1}^n \beta_j (\alpha p_{ij}^m + (1 - \alpha) p_{ij}^u) \quad \forall i \quad (22)$$

$$\alpha q_{rk}^m + (1 - \alpha) q_{rk}^u \leq \sum_{j=1}^n \beta_j (\alpha q_{rj}^m + (1 - \alpha) q_{rj}^l) \quad \forall r \quad (23)$$

$$\beta_j \geq 0 \quad (24)$$

Detailed and informative discussion on fuzzy DEA is available in Saati *et al.* (2002).

Spearman rank correlation coefficient

The Spearman rank correlation coefficient R is useful to determine the strength of association between ranks achieved by different scenarios for a given set of alternatives and is expressed as (Gibbons, 1971):

$$R = 1 - \frac{6 \sum_{a=1}^N D_a^2}{N(N^2 - 1)} \tag{25}$$

where D_a is the difference between ranks X_a and Y_a achieved by the alternative a ; N is number of alternatives; R values vary between -1 and 1 .

CASE STUDY

The Mahi Bajaj Sagar Project is situated in the Banswara district in the southern part of Rajasthan state, India. Live storage capacity of the reservoir is 1830 Mm^3 (million cubic metres). An area of $57\,531 \text{ ha}$ has been explored for irrigation out of the cultivable command area of $80\,000 \text{ ha}$. There are three main canal systems, namely the left main canal (LMC), right main canal (RMC) and the Bhungra canal (BC). The main crops grown in the command area in the *kharif* and *rabi* seasons are paddy, cotton, wheat, gram, pulses. The area is classified as semi-arid. There are 16 irrigation subsystems (synonymously also termed decision-making units), namely Banka, Chhich, Gopinath Ka Gara, Parsoliya, Arthuna, Badliya, Udpura, Bhawarwad, Narwali, Jagpura, Karan Pur, Ganoda, Loharia, Badi Saderi, Asoda and Khodan. These are denoted D1 to D16. Among the 16, 11 come under the RMC and the remaining 5 under the LMC (MBSP Report, 2002; Vasana, 2005). The location and index map of the 16 irrigation subsystems are presented in Figure 1.

These irrigation subsystems are evaluated based on seven indicators (Bos, 1997; Raju and Pillai, 1999; Vasana, 2005; Raju and Vasana, 2007), namely land development works (LDW) which includes evaluation of land levelling, land shaping and consolidation of holdings which also depends on the original status of the land and crops grown; timely supply of inputs (TSI) which requires farmers' knowledge of the technology, developments in irrigated agriculture, timely supply of inputs such as seeds, fertilizers and other resources; conjunctive use of water resources (CUW) are essential to provide a more reliable supply of water to crops when needed. This also reduces waterlogging effects, if any; participation of farmers (PF) for specific assigned tasks is essential for the optimum utilization of resources, which determines the success of an irrigation project; economic impact (EI) is assessed by the economic status of the individual farmer or group of farmers; crop productivity (CPR) can be assessed by determining the yield of the crop in the command area; environmental conservation (EC) issues analysed after introduction of irrigation facilities are groundwater table and salinity level.

Two payoff matrices are formulated individually—one by the researcher (Vasana, 2005) and the second by farmers (payoff matrix for each farmer is formulated based on their views about the irrigation subsystem to which they belong and the average of the total farmers' response for each irrigation subsystem for each indicator is used as the basis) for the 16 irrigation subsystems for the above 7 indicators (Vasana, 2005). These formulations are based on a numerical scale of 0 to 100 [excellent (100), very good (80), good (60), fair (40), average (20), unsatisfactory (0)]. Flexibility is also provided to choose intermediate values other than those marked on the numerical scale to minimize subjectivity while assessing the indicator values. Analysis is based on views of individual farmers, suggestions from project authorities and irrigation management experts, inferences

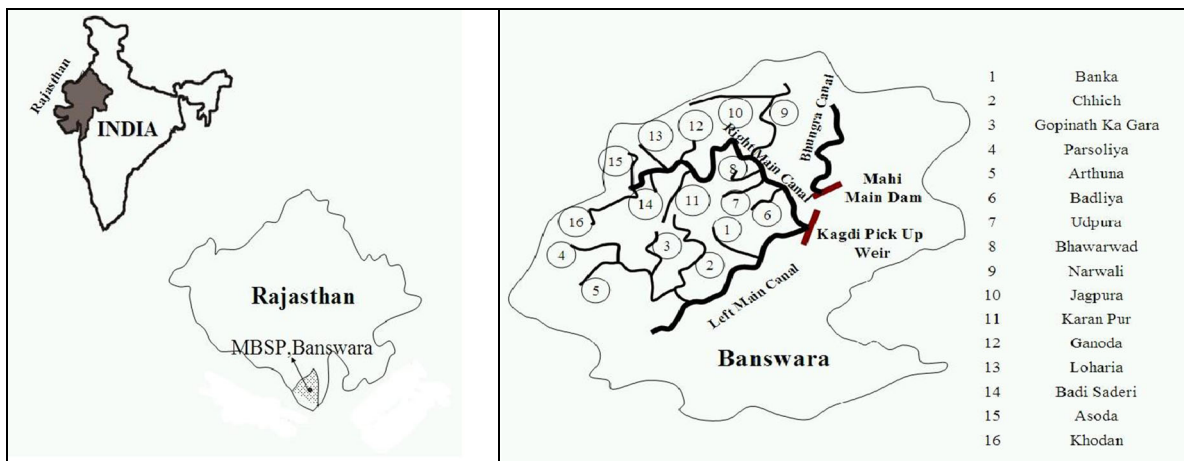


Figure 1. Location and index map showing 16 irrigation subsystems of the Mahi Bajaj Sagar Project

from interviews conducted, actual field conditions in the distributary from time to time, overall water required by farmers of the canal system, availability of water for irrigation, etc. (Vasan, 2005). Two values (for example d , f) presented for each indicator for each irrigation subsystem in Table I are the perception of the researcher and the group of farmers or vice versa as the case may be. For example, in Table I, LDW is rated as the (35, 37) for D1. Here 35 is the perception of the researcher and 37 is of the group of farmers. These are the basis for formulating the triangular membership function (d , e , f) which can be used as the basis for fuzzy DEA methodology. Here variable e represents the average of d and f which is also used as data for DEA in a deterministic environment. Table I presents a payoff matrix that was formulated after such process.

RESULTS AND DISCUSSION

In the present study, out of the seven indicators, LDW, TSI, CUW, PF, EC are considered as inputs whereas CPR and EI are considered as outputs for DEA purposes. Classification of indicators into input and output categories is based on extensive discussion with experts.

Fuzzy DEA

Fuzzy DEA is applied to compute efficiency of the 16 irrigation subsystems using an α -cut approach with various values of α : 0, 0.01, 0.02, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, (totalling 13 scenarios). It is noted that (results are not presented here) for α -cut values 0, 0.01, 0.02, 0.1, 0.2, 0.3,

0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1 and 16 irrigation subsystems under consideration, 15, 15, 15, 15, 14, 14, 14, 14, 14, 13, 11, 10, 10 irrigation systems achieved an efficiency value of 1 which means that irrigation subsystems are equally efficient. This may be due to fewer sample sizes chosen for evaluation (due to resource constraints) and restrictions imposed on efficiency values in fuzzy DEA formulation.

This may create ambiguity as irrigation planners may opine that irrigation subsystems are equal efficient and may be considered the basis for possible improvements simultaneously subject to resource availability. In this situation, the main purpose of ranking the irrigation subsystems is minimized and the advantage of utilizing DEA is nullified (Saati *et al.*, 2002).

In this regard, the approach proposed by Saati *et al.* (2002) that discriminates the equally efficient irrigation subsystems (Equations 21–24) is employed. The approach is developed such that if the efficiency of an irrigation subsystem is less than 1, it keeps the same value, whereas in equal efficiency irrigation subsystems, the efficiency value is shown as more than 1. This helps to differentiate equal efficiency irrigation subsystems, which is evident from Figure 2.

It is observed from Figure 2 that:

- Efficiency values for irrigation subsystems D1 for all α -cut values are less than 1. Efficiency values for D3, D4, D6, D7, D10, D12 to D16 are greater than 1 (except α -cut value of 1);
- Efficiency values decrease with increase in α -cut values;

Table I. Payoff matrix for 16 irrigation subsystems of the Mahi Bajaj Sagar Project

Sl. No.	Irrigation subsystem	Inputs					Outputs	
		LDW	TSI	CUW	PF	EC	CPR	EI
D1	Banka	(35, 37)	(74, 76)	(38, 40)	(65, 67)	(63, 65)	(44, 46)	(40, 45)
D2	Chhich	(33, 40)	(35, 69)	(45, 47)	(52, 55)	(57, 60)	(51, 55)	(39, 45)
D3	Gopinath Ka Gara	(20, 24)	(80, 84)	(25, 27)	(44, 50)	(68, 70)	(50, 57)	(54, 60)
D4	Parsoliya	(40, 42)	(40, 42)	(55, 57)	(55, 57)	(74, 80)	(54, 60)	(60, 68)
D5	Arthuna	(63, 65)	(45, 52)	(44, 50)	(44, 50)	(64, 66)	(50, 52)	(49, 51)
D6	Badliya	(45, 47)	(84, 86)	(30, 36)	(67, 70)	(54, 56)	(64, 70)	(47, 50)
D7	Udpura	(65, 69)	(83, 85)	(43, 45)	(35, 41)	(35, 37)	(30, 34)	(50, 52)
D8	Bhawarwad	(50, 55)	(75, 78)	(48, 55)	(55, 60)	(66, 70)	(45, 52)	(57, 70)
D9	Narwali	(44, 46)	(64, 66)	(44, 50)	(76, 80)	(45, 47)	(45, 47)	(53, 55)
D10	Jagpura	(35, 39)	(57, 60)	(35, 37)	(64, 66)	(68, 70)	(58, 60)	(55, 61)
D11	Karan Pur	(47, 50)	(75, 83)	(51, 60)	(52, 70)	(60, 62)	(62, 65)	(37, 40)
D12	Ganoda	(53, 55)	(77, 80)	(47, 50)	(69, 71)	(45, 49)	(59, 61)	(59, 65)
D13	Loharia	(47, 50)	(55, 57)	(25, 29)	(30, 35)	(70, 72)	(40, 43)	(45, 50)
D14	Badi Saderi	(51, 55)	(65, 67)	(39, 53)	(25, 29)	(69, 75)	(49, 51)	(35, 46)
D15	Asoda	(33, 35)	(58, 60)	(45, 47)	(47, 50)	(35, 39)	(58, 60)	(40, 53)
D16	Khodan	(25, 29)	(61, 65)	(30, 32)	(54, 60)	(46, 50)	(25, 29)	(50, 52)

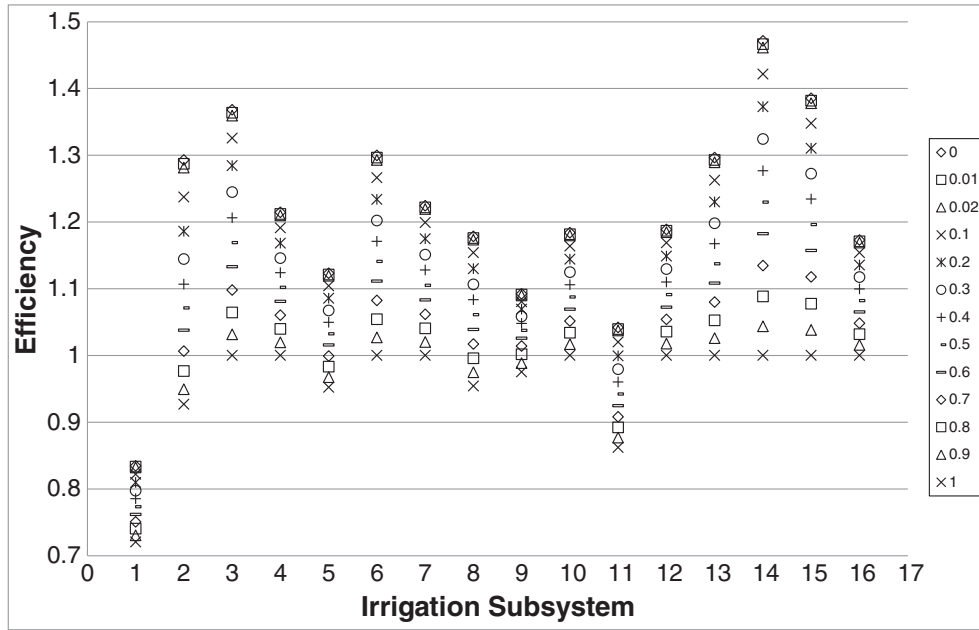


Figure 2. Efficiency values for each irrigation subsystem for various α -cut values

- Efficiency values more than 1 indicate that the algorithm is able to differentiate the efficient irrigation subsystems effectively for possible improvements;
- Lower and upper bounds of efficiency values for D1 to D16 are: (0.835, 0.721), (1.293, 0.927), (1.368, 1.000), (1.214, 1.000), (1.123, 0.952), (1.300, 1.000), (1.224, 1.000), (1.178, 0.954), (1.092, 0.975), (1.184, 1.000), (1.042, 0.862), (1.189, 1.000), (1.296, 1.000), (1.471, 1.000), (1.385, 1.000), (1.173, 1.000).

Figure 3 presents the ranking pattern of each irrigation subsystem for various α -cut values. It is observed from Figure 3 that:

- Ranking pattern is the same for different values of α , i.e. 0, 0.01, 0.02;
- D14 occupied the first position (for all α -cut values) whereas D15, D3, D6 occupied the second, third and fourth positions respectively (with the exception of α -cut value 1, In this case, those occupying first position as a tie are considered the same);
- D11 and D1 occupied 15th and 16th positions (for all α -cut values);
- Similar inferences can be drawn for the average ranking scenario as evident from Figure 3.

The Spearman rank correlation coefficient R is used in the present study to assess the strength of association (very high or high, etc.) between the ranking patterns obtained from various α -cut scenarios. For example, if

the R value is between 0.9 and 1, it is opined that a very high strength of association exists whereas 0.7–0.9 indicates high association (Raju and Nagesh Kumar, 2010). It is observed from Figure 4 that:

- R value of 1 indicates comparison of the same scenario, for example 0/0.01/0.02 against 0/0.01/0.02. Similar inferences can be drawn with comparison of the same scenarios;
- R value between scenarios 0/0.01/0.02 and 0.1/0.2 is 0.997, indicating a very high strength of association between these scenarios;
- R value between various scenarios (excluding α -cut value of 1) varying from 0.879 to 0.997 indicates a very high to high strength of association which indicates a similar ranking pattern in these scenarios;
- R values between $\alpha = 0-0.9$ and $\alpha = 1$ vary in the range 0.331–0.581. Careful selection of the α value is very important (as the α value gets closer to 1, the lower the degree of uncertainty, whereas an α value closer to 0 indicates a higher degree of uncertainty) which depends on the individual irrigation planner, the perception of the decision maker’s judgement and data availability aspects in the field. In case of $\alpha = 1$, efficiency remains same as in deterministic DEA.

Deterministic DEA

In deterministic DEA, Equations (1)–(5) provide the basis for efficiency computation and these values for irrigation systems D1 to D16 are 0.721, 0.927, 1.000, 1.000, 0.952,

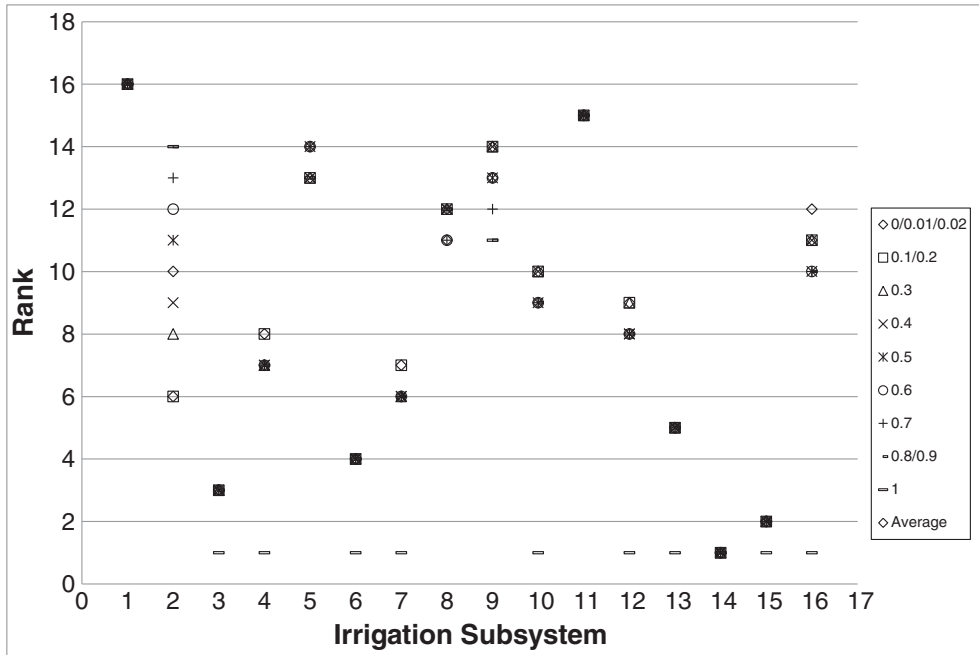


Figure 3. Ranking pattern of each irrigation subsystem for various α -cut values

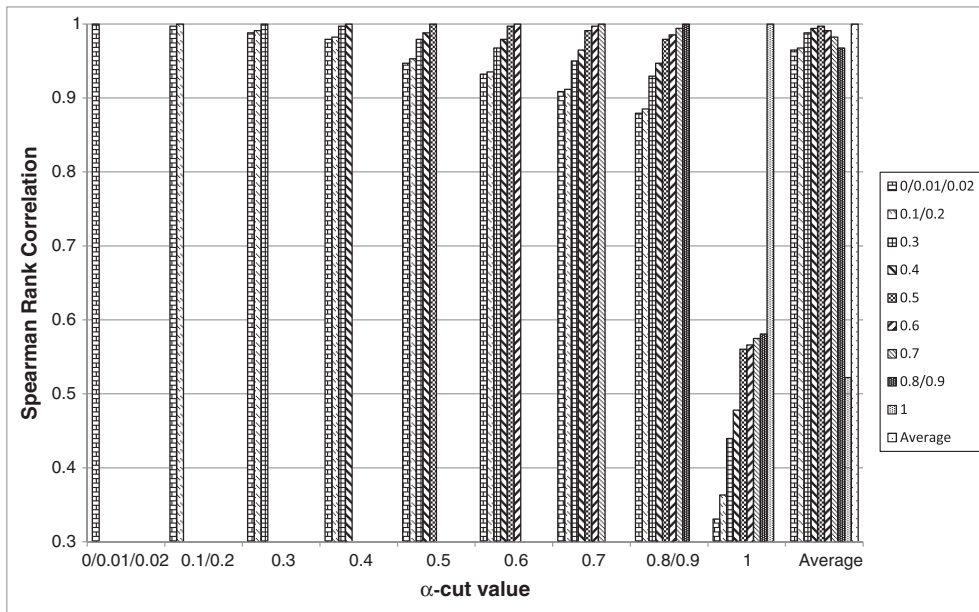


Figure 4. Spearman rank correlation coefficient values for various α -cut values

1.000, 1.000, 0.954, 0.975, 1.000, 0.862, 1.000, 1.000, 1.000, 1.000, 1.000 (these values are the same for $\alpha = 1.0$ in a fuzzy DEA environment which indicates preciseness). Out of the 16, 10 irrigation subsystems have an efficiency value of 1.000 due to which discrimination among the irrigation subsystems is difficult as explained in the fuzzy

DEA scenario. To minimize ambiguity and to improve discrimination ability, the cross-efficiency matrix approach is employed (Sexton *et al.*, 1986; Li and Reeves, 1999). The matrix contains not only irrigation subsystems' usual DEA efficiency (diagonal values in the cross-efficiency matrix obtained through Equations (1)–(5) for various

irrigation subsystems), but also irrigation subsystems' cross-efficiencies, rated based on the other 15 irrigation subsystems' optimal weights within the payoff matrix. In this process, sometimes the efficiency values in the cross-efficiency matrix (other than diagonal values) are more than 1. These efficiencies are then averaged for the irrigation subsystems under consideration and the resulting value becomes a new measure of efficiency for the irrigation subsystems (Raju and Nagesh Kumar, 2010). Table II presents the cross-efficiency matrix and average efficiency values for irrigation subsystems D1 to D16 along with the ranking pattern. It is observed that average efficiency values obtained after such a process are 0.767, 0.803, 0.982, 1.081, 0.883, 0.967, 0.782, 1.034, 0.900, 1.024, 0.834, 1.076, 0.807, 0.773, 0.896, 0.752 and the corresponding ranking pattern is 15, 12, 5, 1, 9, 6, 13, 3, 7, 4, 10, 2, 11, 14, 8, 16.

It is observed that there is a significant difference between the efficiency values and ranking pattern before and after the cross-efficiency matrix approach. It is noticed that the Spearman *R* value is negative (−0.296) for this combination. In the latter case it is more convenient to discriminate between the irrigation subsystems which may be used as the basis for improving irrigation subsystems on a priority basis if DEA is chosen for decision making purposes.

Comparative analysis

It is noted from the result pattern of Figure 2 and Table II that a vast difference in ranking pattern is observed between deterministic DEA (after the cross-efficiency approach) and fuzzy DEA, as expected, due to differences in data handling, discrimination and averaging procedure. In the case of fuzzy DEA, D14, D15, D3 and D6 are preferred (top four positions) whereas in the case of deterministic DEA, the top are D4, D12, D8 and D10. In the case of deterministic DEA only crisp data are utilized (average values) and discrimination analysis is performed using the cross-efficiency matrix. In the case of fuzzy DEA, a triangular membership function is employed to consider the impreciseness in the data. Discrimination methodology (Equations 21–24) proposed by Saati *et al.* (2002) is used in the present study which is different and simpler in terms of computational complexity as compared to the cross-efficiency approach.

The present study is first of its kind where deterministic and fuzzy DEA are compared in performance evaluation studies in a developing-country environment with as many as seven subjective indicators (five inputs and two outputs) supported by two different methodologies for discrimination among efficient alternatives. From the discussion with various stakeholders it is felt that the outcome of fuzzy DEA deserves to be explored for further analysis due to its simple, effective data (including the imprecision situation in the form of α -cut variation) and discrimination

Table II. Cross-efficiency matrix

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16	Ave.	Rank
D1	0.721	0.787	0.759	0.672	0.837	0.672	0.843	0.641	0.777	0.743	0.606	0.700	0.896	1.004	0.763	0.843	0.767	15
D2	0.761	0.927	0.752	0.656	0.965	0.791	0.824	0.626	0.759	0.736	0.714	0.700	0.875	1.035	0.898	0.824	0.803	12
D3	0.920	0.944	1.000	0.891	1.018	0.806	1.118	0.850	1.030	0.978	0.727	0.917	1.188	1.298	0.915	1.118	0.982	5
D4	1.010	0.997	1.119	1.000	1.085	0.851	0.980	0.954	1.156	1.094	0.768	1.023	1.333	1.435	0.966	1.255	1.081	1
D5	0.829	0.892	0.882	0.781	0.952	0.761	0.980	0.745	0.903	0.862	0.687	0.811	1.042	1.160	0.864	0.980	0.883	9
D6	0.920	1.172	0.884	0.766	1.209	1.000	0.961	0.731	0.885	0.865	0.903	0.826	1.021	1.238	1.136	0.961	0.967	6
D7	0.720	0.560	0.876	0.797	0.647	0.478	1.000	0.760	0.921	0.856	0.431	0.791	1.063	1.064	0.542	1.000	0.782	13
D8	0.959	0.857	1.109	1.000	0.955	0.731	1.255	0.954	1.156	1.085	0.660	1.009	1.333	1.388	0.831	1.255	1.034	3
D9	0.839	0.805	0.941	0.844	0.881	0.687	1.059	0.805	0.975	0.921	0.620	0.861	1.125	1.199	0.780	1.059	0.900	7
D10	0.961	1.032	1.022	0.906	1.102	0.881	1.137	0.865	1.048	1.000	0.795	0.941	1.208	1.345	1.000	1.137	1.024	4
D11	0.799	1.119	0.716	0.609	1.135	0.955	0.765	0.581	0.704	0.701	0.862	0.677	0.813	1.049	1.085	0.765	0.834	10
D12	1.008	1.049	1.089	0.969	1.128	0.896	1.216	0.924	1.120	1.065	0.808	1.000	1.292	1.419	1.017	1.216	1.076	2
D13	0.753	0.735	0.838	0.750	0.801	0.627	0.941	0.716	0.867	0.820	0.566	0.766	1.000	1.072	0.712	0.941	0.807	11
D14	0.732	0.875	0.733	0.641	0.914	0.746	0.804	0.611	0.741	0.717	0.674	0.680	0.854	1.000	0.848	0.804	0.773	14
D15	0.849	1.032	0.842	0.734	1.074	0.881	0.922	0.701	0.849	0.823	0.795	0.783	0.979	1.156	1.000	0.922	0.896	8
D16	0.689	0.472	0.870	0.797	0.566	0.403	1.000	0.760	0.921	0.851	0.364	0.782	1.063	1.034	0.458	1.000	0.752	16

handling procedure. However, the α value is to be chosen carefully. Suitable irrigation subsystems identified from fuzzy DEA methodology and continuous improvement of all irrigation subsystems as the focus to reach excellence are used as the basis so that the performance of others/all can be improved to attain a similar status. Some of the improvements that can be explored in this regard are:

- Land development works are rated minimum in D3 and maximum in D7. Farmers should be trained in modern techniques of land levelling and shaping and more assistance can be provided by relevant agencies including NGOs. Effective land levelling and shaping can also minimize other detrimental effects;
- Timely supply of inputs is rated minimum in D4 and maximum in D6. A coordinated supply of inputs such as seeds, fertilizers and bank loans at the right time always helps to improve the situation. However, water-saving technologies such as drip and sprinkler irrigation for some crops will reduce stress on the available water;
- Conjunctive use of water resources is rated minimum in D3 and maximum in D11. The combined use of surface and groundwater can be explored for a more reliable supply of water at the right time in adequate quantity. However, water quality considerations are to be addressed in detail in a sustainable manner;
- Participation of farmers is rated minimum in D14 and maximum in D9. This aspect can be improved with more informal meetings among farmers, bringing the ownership attitude on the subsystem and transferring the affordable technology to their doorstep. Regular meetings with various stake holders including project officials can also be arranged;
- Environmental conservation is rated minimum in D7 and maximum in D4. Education on detrimental effects of excessive waterlogging and salinity helps the judicious use of water. Here participation of NGOs and academics can also be explored;
- Economic impact is rated minimum in D11 and maximum in D8. Economic impact as a output indicator depends on the performance of all the input indicators. Improvements in the input indicators will enhance economic status;
- Crop productivity is rated minimum in D16 and maximum in D6. Crop productivity as a output indicator depends on the performance of all the input indicators. Improvements in the input indicators will enhance crop productivity.

The above outcome is based on the subjective perception of various stakeholders (which may vary from time to time), limited sampling size and chosen solution methodology, i.e. DEA/ fuzzy DEA. However, the methodology and

applicability of DEA/fuzzy DEA are clearly demonstrated, which is the main focus of the present study.

SUMMARY AND CONCLUSIONS

The application potential of DEA as a decision-making approach in both deterministic and fuzzy environments is explored in the performance evaluation of an irrigation system. A case study of the Mahi Bajaj Sagar Project, Rajasthan, India, is considered in order to rank 16 irrigation subsystems. Out of seven indicators, land development works, timely supply of inputs, conjunctive use of water resources, participation of farmers, environmental conservation are considered as inputs and economic impact and crop productivity are considered as outputs. Discrimination between irrigation subsystems is also performed both in fuzzy and deterministic DEA which make these approaches more suitable for ranking in limited data situations. The following conclusions are drawn:

- Fuzzy DEA can be adopted for performance evaluation of a system due to its simple, effective, impreciseness (in the form of α -cut variations) and discrimination handling procedure as compared to deterministic DEA;
- Effect of the α -cut is significant on the efficiency values and ranking pattern which indicates that careful selection of the α -cut value is essential;
- Suitably identified irrigation subsystems for the case-study using fuzzy DEA methodology (D14, D15, D3 and D6) can be explored for further improvement to achieve overall improvement of the entire system;
- Spearman rank correlation between various scenarios (excluding α -cut value of 1) varies from 0.879 to 0.997, indicating a very high to high strength of association which may be due to the similar ranking pattern in these scenarios;
- It is observed that there is a significant difference between the efficiency values and ranking pattern before and after the cross-efficiency matrix analysis in DEA.

Further studies can be carried out with more precise and quantitative data for each crop season for better performance evaluation.

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