

Characterizing Drought Using the Reliability-Resilience-Vulnerability Concept

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Abstract: This study borrows the measures developed for the operation of water resources systems as a means of characterizing droughts in a given region. It is argued that the common approach of assessing drought using a univariate measure (severity or reliability) is inadequate as decision makers need assessment of the other facets considered here. It is proposed that the joint distribution of reliability, resilience, and vulnerability (referred to as RRV in a reservoir operation context), assessed using soil moisture data over the study region, be used to characterize droughts. Use is made of copulas to quantify the joint distribution between these variables. As reliability and resilience vary in a nonlinear but almost deterministic way, the joint probability distribution of only resilience and vulnerability is modeled. Recognizing the negative association between the two variables, a Plackett copula is used to formulate the joint distribution. The developed drought index, referred to as the drought management index (DMI), is able to differentiate the drought proneness of a given area when compared to other areas. An assessment of the sensitivity of the DMI to the length of the data segments used in evaluation indicates relative stability is achieved if the data segments are 5 years or longer. The proposed approach is illustrated with reference to the Malaprabha River basin in India, using four adjoining Climate Prediction Center grid cells of soil moisture data that cover an area of approximately 12,000 km². DOI: 10.1061/(ASCE)HE.1943-5584.0000639. © 2013 American Society of Civil Engineers.

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Introduction

Drought is recognized as a prolonged below-normal supply of water. Due to its inherent complexity, variable spatial and temporal extent, and impact, a universally accepted definition of droughts is not possible (Heim 2002). The American Meteorological Society (1997) has classified droughts into four categories: meteorological, agricultural, hydrological, and socioeconomic. Meteorological drought is related to below-normal precipitation, agricultural drought is related to the shortfall of soil moisture, hydrological drought is related to the inadequacy of water resources to meet demands, and socioeconomic drought is based on ensuing economic consequences for the region. Considerable attention has been focused on quantifying drought through a series of derived attributes and is the focus of the research presented here.

In general, drought is quantified using indices that are derived from incident precipitation and evapotranspiration for the region. These hydrologic variables influence the soil moisture status, which is a primary input for agricultural droughts. Onset of agricultural droughts is indicated if the soil moisture falls below a

predefined threshold value, such as the permanent wilting point. Since the early twentieth century, several indices have been developed to characterize droughts (see Keyantash and Dracup 2002; Heim 2002; Mishra and Singh 2010, 2011; Dai 2011). A brief review is presented here with an emphasis on agricultural droughts, which is the main focus of this paper.

An approximate chronological development of meteorological drought indices includes Munger's index (Munger 1916), the precipitation effectiveness index (Thornthwaite 1931), Blumenstock's index (Blumenstock 1942), the antecedent precipitation index (API) (McQuigg 1954; Waggoner and O'Connell 1956), the Palmer drought severity index (PDSI) (Palmer 1965), the rainfall anomaly index (RAI) (van Rooy 1965), the drought area index (Bhalme and Mooley 1980), the standardized precipitation index (SPI) (McKee et al. 1993) and effective precipitation (Byun and Wilhite 1999). Thornthwaite (1948) also used precipitation minus evapotranspiration as a drought index. In API, both the amount and timing of precipitation are incorporated, and although it was originally designed to estimate soil moisture content for use in flood forecasting, more recent use has been to assess its impact on design floods (Pui et al. 2011). This index is computed on a daily basis and thus can be a measure of short-term drought. The PDSI, along with the Palmer hydrological drought index (PHDI) and the Palmer moisture anomaly index (PMAI) (aka Z Index) (Palmer 1965), can be treated as the first breakthrough in the development of drought indices. Even though the PDSI is perhaps the most prominent meteorological index used (Heim 2002), several criticisms and limitations have been noted in the literature (Alley 1984). Keyantash and Dracup (2002) showed that the PDSI is more of a hydrological index, and the time scale of drought addressed by the PDSI is unclear. More recent assessments of drought have utilized SPI as a probabilistic means of characterizing rainfall anomalies (Keyantash and Dracup 2002).

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McGuire and Palmer (1957) developed the moisture adequacy index (MAI) to measure agricultural drought. The MAI is expressed as a percentage ratio of actual evapotranspiration and potential evapotranspiration. The PMAI (aka Z Index), one of the three Palmer indices, is an intermediate term in the computation of PDSI (Keyantash and Dracup 2002). Palmer (1968) also developed the Crop Moisture Index (CMI) to monitor the weekly change of soil moisture condition. The value of this index changes very rapidly, and the index is suitable only for very short-term drought assessment. Bergman et al. (1988) developed another index known as the Soil Moisture Anomaly Index (SMAI). The SMAI fluctuates at a moderate rate as compared to the CMI (Bergman et al. 1988). The Vegetation Condition Index (VCI) (Kogan 1995, 1997; Liu and Kogan 1996) allows drought detection by monitoring vegetation health. However, it is useful for summer seasons. During the cold season, when vegetation is mostly in a dormant state, its utility is limited (Heim 2002; Mishra and Singh 2010). Among the different existing agricultural drought indices, the Computed Soil Moisture (CSM) (Huang et al. 1996) was identified as the most suitable one by Keyantash and Dracup (2002) using predefined subjective weightage factors to different desirable aspects of a drought index.

Historical development of different indices for hydrological droughts include the PHDI (Palmer 1965), the Total Water Deficit (TWD) (Dracup et al. 1980), the Surface Water Supply Index (SWSI) (Shafer and Dezman 1982), and the Cumulative Streamflow Anomaly (CSA) (Keyantash and Dracup 2002). Detailed review of these indices is left out as our focus is on the agricultural drought indices. As a summary, an overall comparison between different indices by Keyantash and Dracup (2002) indicates that the suitability of agricultural drought indices (also hydrological drought indices) is inferior with respect to that of meteorological drought indices.

It can be further concluded from the overall review of different drought indices, that most of these drought-related indices indicate the ongoing (current) status of the drought, as characterized using a single metric. In this study, we attempt to quantify long-term characteristics of drought of a given area, which may change slowly (on say a decadal time frame) owing to low-frequency variability and change in the climate. In this context, a new index is proposed, which seeks to quantify the long-term drought characteristics of a given area, providing catchment managers with a tool that assesses the frequency (reliability), vulnerability, and ability to recover (resilience) from a drought. The reliability, resilience, and vulnerability concept was used earlier in the context of water resources management. In this study, these measures are used in the context of drought; thus, the name of the index is proposed as the drought management index (DMI). The proposed index ranges from 0 to 1, with higher values indicating higher drought proneness of the area and vice versa.

Traditionally, most drought-related indices are based on one or two attributes of the hydrologic time series used in their formulation (i.e., time series of soil moisture in case of agricultural drought, time series of precipitation in case of meteorological drought, and time series of streamflow/reservoir storage in case of hydrological drought). And, in most of the cases, these particular attributes fall below a predefined threshold during drought events, which can be analogous to failure of the system. However, a system failure should be simultaneously characterized by its reliability, resilience, and vulnerability, as is done in the context of water supply management with single or multiple reservoir systems. The joint behavior of these parameters should be considered while characterizing the failure (drought). Whereas the reliability and resilience behave in a similar way (Hashimoto et al. 1982), the

interrelationship between resilience and vulnerability or reliability and vulnerability should be jointly used to characterize the drought.

The theory of copula is used to obtain the joint distribution between resilience and vulnerability. In the recent past, copulas have been used for various hydrological analyses (De Michele and Salvadori 2003; Favre et al. 2004; De Michele et al. 2005; Grimaldi and Serinaldi 2006a, b; Zhang and Singh 2006, 2007; Renard and Lang 2007; Gebremichael and Krajewski 2007; Kao and Govindaraju 2007a, b; Maity and Nagesh Kumar 2008; Villarini et al. 2008; Zakaria et al. 2010; Vandenberghe et al. 2010). Application of multivariate joint distribution between drought variables has gained popularity in the recent past (Bonaccorso et al. 2003; Nadarajah 2009; Mishra et al. 2009; Vangelis et al. 2011). For instance, Vangelis et al. (2011) used bivariate probability analysis to assess the severity of drought episodes assuming normal distribution of precipitation and potential evapotranspiration. Probabilistic analyses of drought duration, intensity, and return period also have been undertaken by many other researchers (Fernandez and Salas 1999; Chung and Salas 2000; Cancelliere and Salas 2004; Nadarajah 2009), mostly using the time series of precipitation or streamflow as the key variable. With improved understanding of the impacts of oceanic phenomena on drought characteristics, hydrologists have also studied the variation and characteristics of hydrological droughts in relation to El Niño-Southern Oscillation (ENSO) events (Ryu et al. 2010; Wong et al. 2010). Meteorological and hydrological droughts are in turn linked to agricultural droughts. Even though the vulnerability of a region to agricultural droughts has been studied (Wilhelmi and Wilhite 2002), a comprehensive analysis quantifying the risk of system failure (drought event) is lacking. In this context, the new index, DMI, is proposed here to measure the extent of agricultural drought risk over a catchment. Resilience and vulnerability of the soil moisture series are computed to obtain the new index. Previous research shows that the dependence of correlated stochastic variables involved in droughts can be modeled successfully through copulas (Shiau 2006; Shiau et al. 2007; Kao and Govindaraju 2008; Shiau and Modarres 2009; Serinaldi et al. 2009; Song and Singh 2010a, b; Kao and Govindaraju 2010; Mikabari et al. 2010; AghaKouchak et al. 2010; Wong et al. 2010). For instance, Kao and Govindaraju (2008) demonstrated the suitability of Plackett copula (Plackett 1965) for both positive and negative dependence while analyzing extreme rainfall events. Song and Singh (2010b) used meta-elliptical copulas to model the dependence of drought duration, severity, and interarrival time. Wong et al. (2010) studied drought characteristics (intensity, duration, and severity) conditioned on different ENSO states using Gumbel-Hougaard as well as t-copulas. Thus, copulas provide flexibility in terms of selecting suitable marginals and dependence structure of variables. This makes copula an attractive tool for modeling the joint distribution between reliability, resilience, and vulnerability, which is used in this study. The proposed method and computation of the DMI is illustrated in and around Malaprabha River basin considering four adjoining CPC (Climate Prediction Center) grids of soil moisture data that covers an approximately 12,000 km² area.

Methodology

The overall methodology can be broadly divided into three parts: (1) assessing reliability-resilience-vulnerability (RRV) for the hydrological time series, (2) fitting a suitable copula to obtain the joint probability distribution between these measures, and (3) developing a copula-based DMI as a measure of long-term drought characteristics over the region. These parts are elaborated

separately but in relation with each other in the following subsections.

Reliability-Resilience-Vulnerability of Soil Moisture Time Series

The concept of RRV was introduced by Hashimoto et al. (1982) in the context of water resources systems. In this paper, the concept of RRV is used in the context of agricultural drought through the analysis of temporal variation of soil moisture. The failure or unsatisfactory stage is considered as the depletion of soil moisture below the Permanent Wilting Point (PWP), which is an indicator of agricultural drought as mentioned earlier. The PWP is the minimum amount of soil moisture required for the plants not to wilt (Taiz and Zeiger 1991). If the soil moisture falls below this limit, the plants can no longer come out of their drooping stage and eventually die. PWP depends on the integrated effects of plant, soil and atmospheric conditions at a particular location. Let X_1, X_2, \dots, X_n be the time series of soil moisture having a data length n to assess the RRV. If $X_t \geq \text{PWP}$, it is considered a satisfactory stage, denoted as S and if $X_t < \text{PWP}$, it is considered an unsatisfactory stage, denoted as F .

Reliability

Reliability is defined by the probability that a system is in a satisfactory state (Hashimoto et al. 1982). In the context of soil moisture, reliability may be defined as the probability that the soil moisture is above a certain threshold (here PWP). Thus, the reliability α is stated as

$$\alpha = P(X_t \in S) \quad (1)$$

where S = the satisfactory stage as stated before. From the time series, α can be computed as

$$\alpha = \lim_{n \rightarrow \infty} \frac{Lt}{n} \sum_{t=1}^n Z_t \quad (2)$$

where $Z_t = 1$ if $X_t \in S$ and $Z_t = 0$ if $X_t \in F$.

Resilience

Resilience is a measure that indicates how quickly the system can return to a satisfactory stage after it has fallen below the satisfactory threshold. This can be defined as the ratio of the probability of transition from the unsatisfactory to the satisfactory stage and the probability of failure, i.e.,

$$\gamma = \frac{P(X_t \in F, X_{t+1} \in S)}{P(X_t \in F)} \quad (3)$$

where S and F are as defined earlier. The numerator, probability of transition from the unsatisfactory to the satisfactory stage is denoted as ρ . In the long run, the number of times the system transforms from the satisfactory to the unsatisfactory stage and from the unsatisfactory to the satisfactory stage will be same. Thus, it can be eventually expressed as $\rho = P(X_t \in F, X_{t+1} \in S) = P(X_t \in S, X_{t+1} \in F)$. From the time series, ρ can be computed as

$$\rho = \lim_{n \rightarrow \infty} \frac{Lt}{n} \sum_{t=1}^n W_t \quad (4)$$

where W_t = the event of transformation from the satisfactory to the unsatisfactory stage (or vice versa) and $W_t = 1$ if $X_t \in S$ and $X_{t+1} \in F$ and $W_t = 0$ otherwise. The denominator of

Eq. (3) can be expressed as $P(X_t \in F) = 1 - P(X_t \in S)$. Again, $P(X_t \in S)$ is expressed as reliability α as explained before. Thus, Eq. (3) can be expressed as

$$\gamma = \frac{\rho}{1 - \alpha} \quad (5)$$

Vulnerability

Vulnerability is a measure of severity of a failure event, once it has occurred. It is defined as

$$\nu = \sum_{j \in F} s_j e_j \quad (6)$$

where s_j = the numerical indicator of severity for an observation x_j , which belongs to the unsatisfactory state; e_j = the probability of that x_j , corresponding to s_j , which is the most unsatisfactory and severe outcome that occurs from the set of unsatisfactory states. In the context of soil moisture, vulnerability is a probability weighted average of the soil moisture deficits (with respect to the PWP of the location) of failure events. Thus, the shortfall of the available soil moisture below the PWP is the severity indicator, and vulnerability is measured in terms of the mean soil moisture deficit caused during the failure events, assuming that deficits of different magnitudes are equiprobable.

Interrelationships between Reliability-Resilience-Vulnerability and the Role of Copula

The interrelationship between reliability, resilience, and vulnerability needs to be considered while assessing drought characteristics. In order to do this, the theory of copulas is utilized. Many researchers have successfully used copulas to perform multivariate hydrologic analysis as reported earlier. Previous literature suggests that negative association between random variables can be effectively captured by various copulas, which are explained in this section.

Joint Probability Distribution Using Copulas

A copula is a function that joins or couples together univariate marginal distributions to form a multivariate joint distribution (Nelsen 2006). Let us consider that X and Y are two continuous random variables with marginal Cumulative Distribution Functions (CDFs) $F(x)$ and $G(y)$, respectively, and joint distribution function $H(x, y)$. Sklar's (1959) theorem states that, for a joint distribution function H with margins F and G , there exists a copula C for all (x, y) in the extended real line \bar{R} such that

$$H(x, y) = C[F(x), G(y)] \quad (7)$$

If F and G are continuous, then C is unique; also C is unique on the simultaneous range of F and G . In the context of RRV analysis, X represents either reliability or resilience, Y represents vulnerability, and the objective is to develop their joint distribution. Recognizing the fact that these pairs possess negative association (shown later), Gaussian copula, Frank copula, and Plackett copulas are initially selected to compare their suitability to capture the joint behavior between reliability-vulnerability and resilience-vulnerability. "Copula-Based Drought Management Index" provides a brief mathematical background of these copulas.

Gaussian copula belongs to the class of elliptical copula, which is able to take care of entire range of positive and negative association between random variables. The bivariate Gaussian copula is defined as

$$C_G(u, v) = \int_{-\infty}^{\varphi^{-1}(u)} \int_{-\infty}^{\varphi^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \exp\left[-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right] dy dx \quad (8)$$

Dependence parameter of this copula ρ is related to the Kendall's tau, τ , by

$$\tau = \frac{2}{\pi} \sin^{-1}(\rho) \quad (9)$$

Among the popular Archimedean family of copulas, the Frank copula (also a few others copulas) is able to capture the entire range of dependence. The Frank copula is defined as

$$C_p(u, v) = \frac{[1 + (\theta_p - 1)(u + v)] - \sqrt{[1 + (\theta_p - 1)(u + v)]^2 - 4uv\theta_p(\theta_p - 1)}}{2(\theta_p - 1)} \quad (12)$$

where θ_p = the dependence parameter of this copula, which is the cross product ratio between the random variables. θ_p can be estimated by the pseudo likelihood function (Genest et al. 1995). It can also be estimated directly from the observations and sample medians (Mardia 1970). The sample medians divide the observations into four quadrants: (1) where observations of both the variables are greater than their respective median values, (2) where observations are less than the median for the first marginal and greater than the median for the second marginal, (3) where observations of both the variables are less than their respective median values, and (4) where observations are greater than the median for the first marginal and less than the median for the second marginal. If $\hat{\theta}_p$ is the cross product ratio estimated from observations, then it can be expressed as Eq. (13):

$$\hat{\theta}_p = \frac{n_{00}n_{11}}{n_{01}n_{10}} \quad (13)$$

where n_{11} , n_{01} , n_{00} , and n_{10} are the number of observations in the first, second, third, and fourth quadrants, respectively. The possible range of $\hat{\theta}_p$ is between zero and infinity. A value less than one indicates a negative association, whereas the positive association is indicated by the values greater than or equal to one. For a detailed theoretical derivation, readers may refer to Kao and Govindaraju (2008). Their findings showed that the theory of constant cross product ratio, on which the Plackett copula is based, is applicable for both discrete and continuous random variables. In this study, a bivariate Plackett copula model is also adopted due to its ability to capture negative association between variates.

Copula-Based Drought Management Index

Based on the joint distribution between resilience and vulnerability, an index is to be developed that will convey the simultaneous information of resilience and vulnerability. It is clear from the previous discussion that more favorable conditions can be indicated with the increase in resilience. Here, drought is identified as an unfavorable phenomenon. On the other hand, conditions are less

$$C_F(u, v) = -\frac{1}{\theta_F} \ln \left[1 + \frac{(e^{-\theta_F u} - 1)(e^{-\theta_F v} - 1)}{(e^{-\theta_F} - 1)} \right] \quad (10)$$

where θ_F = the dependence parameter, which is related to the Kendall's tau, τ , by

$$\tau = 1 + \frac{4}{\theta_F} [D_1(\theta_F) - 1] \quad (11)$$

where D_1 is the first-order Debye function, which is defined as $D_1(\theta_F) = \int_0^{\theta_F} [t/(e^t - 1)] dt$ for $\theta_F > 0$ and $D_1(-\theta_F) = D_1(\theta_F) + (1/2)$ (Genest 1987; Zhang and Singh 2006).

Another copula, which is also able to capture both the positive and the negative association between variates is known as Plackett copula (Plackett 1965; Nelsen 2006). It is defined as

favorable with the increase in vulnerability. Thus, the proposed DMI should increase with the increase in vulnerability and with the decrease in resilience and vice versa. This can be achieved by a joint measure of probability that indicates exceedence in resilience and nonexceedence in vulnerability. Thus, the DMI is defined as

$$DMI = P(R > r, V \leq v) \quad (14)$$

where $P(\dots)$ = probability of the event (\dots), R = resilience, V = vulnerability, and r and v = the reduced resilience and reduced vulnerability (see the appendix for further information), respectively, calculated from the observed soil moisture series using a suitable threshold. As mentioned earlier, this threshold cannot be adopted randomly. The PWP is a suitable threshold based on the soil-crop combination at a particular location. To obtain actual values of DMI for a location, the PWP pertaining to that location must be used.

Assessment of DMI for Malaprabha River Basin

Study Area and Data Used

Soil moisture data from the CPC from 1961 to 2010 for four grid points in and around the Malaprabha River basin (up to the Malaprabha reservoir, Fig. 1) are used for this analysis. For the purpose of testing the newly proposed index, soil moisture series for five other locations (26°15'00'' lat × 72°15'00'' long, 22°15'00'' lat × 78°15'00'' long, 30°15'00'' lat × 76°15'00'' long, 12°15'00'' lat × 75°45'00'' long, 26°15'00'' lat × 90°45'00'' long), having widely different climate regimes, have also been used. Monthly gridded soil moisture data has been reconstructed by Fan and van den Dool (2004) with a spatial resolution of 0.5° × 0.5°. The monthly data set is averaged soil moisture equivalent to standing water height. CPC soil moisture data (in millimeters) is provided by the NOAA/OAR/ESRL PSD, Boulder, Colorado, USA, from their website at <http://www.esrl.noaa.gov/psd/>. Readers may note that this is not reanalysis data, and it is argued that the soil moisture data from reanalysis

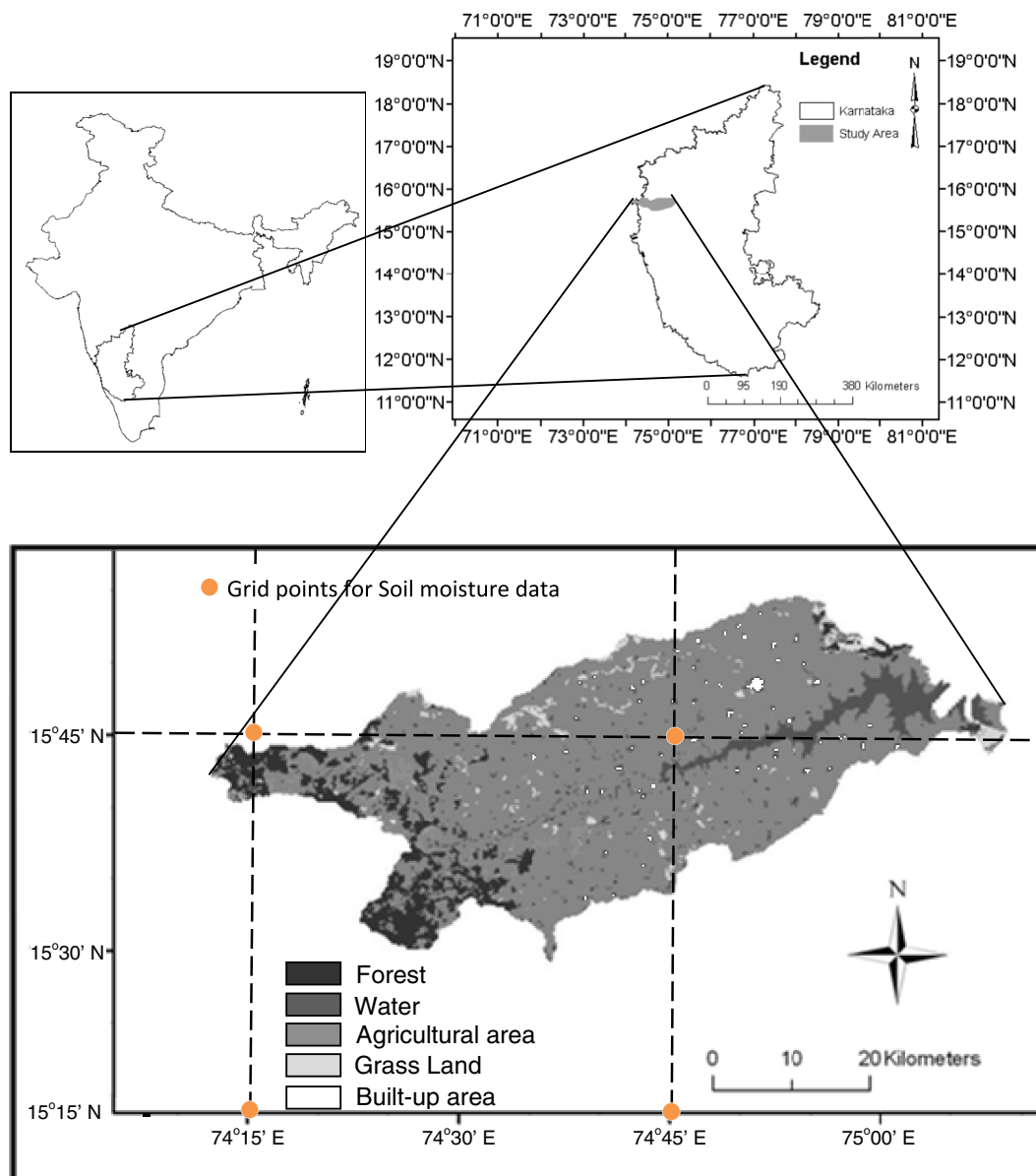


Fig. 1. Location of Malaprabha River basin along with locations of grid point for soil moisture data

data are not reliable due to the bias in precipitation (van den Dool et al. 2003; Fan and van den Dool 2004).

Reliability, Resilience, and Vulnerability of Soil Moisture Data

The 50-year (1961–2010) soil moisture time series obtained from the CPC is split into shorter durations of 5 years for each of the four grid points in the study region. The effect of the data length is investigated and reported in later sections. Based on soil type and crops grown in the study area, the PWP is taken as 250 mm. However, it can vary from one location to another depending on the soil-crop combination. When DMI is to be computed for a particular location, the specific PWP for that location must be used. The effect of different values of PWP (in case of different soil-crop combinations) is investigated and reported in later sections of this paper.

Reliability, resilience, and vulnerability are assessed for each 5-year-long series from all the grid points. These are computed using Eqs. (1)–(6) as explained in the methodology. The values of

reliability, resilience, and vulnerability computed from the set of 5-year time series are found to conform to an approximate Gaussian distribution (using standard tests of distributional fit). Moreover, resilience and vulnerability values were found to have better fit as compared to reliability (as measured by the Kolmogorov-Smirnov test resulting in p -values equal to 0.132 for reliability, 0.551 for resilience, and 0.998 for vulnerability).

Pairwise scatter plots are prepared to understand their interrelationship. These pairwise scatter plots are shown in Fig. 2. It is noticed that reliability and resilience exhibit a well-defined monotonic nonlinear relationship that allows the specification of one given the other. This observation is not new, rather it is in agreement with the observation of Hashimoto et al. (1982) in case of water resources systems that “Resilience generally shows the same trend as reliability.”

On the other hand, the relationships between reliability and vulnerability and also resilience and vulnerability are found to be scattered with a negative association. As mentioned before, both reliability or resilience and vulnerability of the events should be

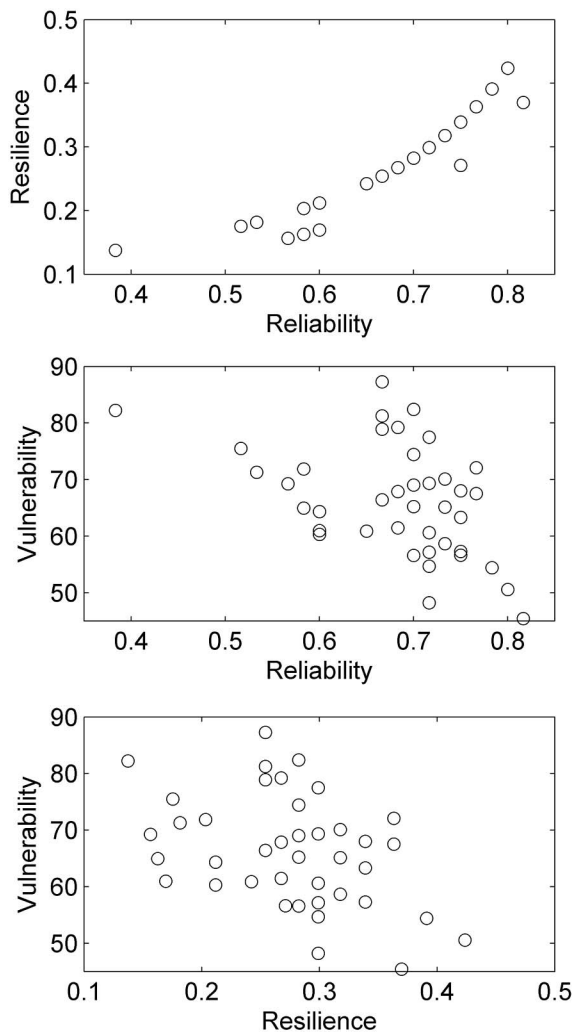


Fig. 2. Pairwise scatter plot between reliability, resilience, and vulnerability

considered simultaneously while assessing the drought characteristics. Thus, a joint specification of reliability-vulnerability or resilience-vulnerability is proposed here as a more versatile basis for characterizing drought. As it is found that reliability and resilience follow a similar trend and have an almost deterministic relationship, any one of the pairs reliability-vulnerability or resilience-vulnerability can be considered. In this study, resilience-vulnerability is considered for developing the new index for long-term drought characterization.

Joint Distribution between Resilience and Vulnerability of Soil Moisture Data

Different copulas are initially selected to derive joint distribution between observed resilience and vulnerability. Kendall's tau between resilience and vulnerability is found to be -0.285 , which confirms the negative association between them. The dependence parameter of the Gaussian copula, ρ , is found to be -0.432 . Similarly, dependence parameters of Frank copula (θ_F) and Plackett copula (θ_p) are also estimated as described in "Methodology." θ_F is estimated to be -0.893 , and θ_p is estimated to be 0.347 . A comparison between Gaussian, Frank, and Plackett copulas is performed to assess their suitability in characterizing the joint distribution (please refer to the appendix). It is noticed from the results (Table 1) that statistical measures (S_n , T_n) are lowest for Plackett

Table 1. Performance Measures for Different Copulas

Copula	S_n	T_n
Gaussian	0.0685	0.7766
Frank	0.0889	0.6142
Plackett	0.0406	0.5156

copula, indicating it to be more suitable than Gaussian and Frank copulas. So, the Plackett copula is selected to obtain the joint distribution between resilience and vulnerability. The joint probability density function (pdf) and joint CDF between reduced variables of resilience and vulnerability are represented by a contour plot in Fig. 3(a). Probability density values at specific observed pairs of resilience and vulnerability are also computed and shown as colored points in the same plots of joint pdf and CDF. Reliability and vulnerability are also negatively associated as is evident from the sample estimate of the cross product ratio, which is obtained as 0.273 . The contour plots [Fig. 3(b)] representing the joint pdf and joint CDF between reliability and vulnerability are very similar to those of resilience and vulnerability as expected. Thus, any one of the two negatively associated pairs between resilience-vulnerability and reliability-vulnerability may be further analyzed for the computation of DMI. The CDF of the former pair is used in this study to compute the DMI for a given series by computing its resilience and vulnerability values using Eq. (14).

Computation of DMI for Different Types of Soil Moisture Series

DMI estimates for different types of soil moisture time series are investigated in order to test performance. Having a specific range of DMI (0 to 1) should reveal the difference between extreme (either very dry or very wet) series in terms of its numerical value, even if these types of series were not encountered during model development. So, it is considered to test this with some series that might be unforeseen during the development period. These series should represent different climatic conditions such as desert and wet areas and should have diversity in terms of reliability, resilience, and vulnerability combinations. For this purpose, soil moisture series from different climatic parts of India are selected. We have selected soil moisture series from five different locations in India (Fig. 4). These locations have widely differing climatic patterns with respect to the considered threshold (PWP for the study area), which are reflected in their reliability, resilience, and vulnerability combinations. For instance, the location in a desert area (marked by +) is very dry, and, hence, a high DMI value must result if an extreme dry situation, similar to this desert area, occurs in Malaprabha basin. On the other hand, soil moisture conditions similar to that of the location in a high rainfall zone (marked by •), when occurring in Malaprabha basin, must produce low DMI. Similarly, soil moisture conditions similar to locations at northern and central part of India will lead to some intermediate DMI values. However, as the developed index depends both on resilience and vulnerability, the value of drought index should reflect this aspect.

Soil moisture time series from these five locations are shown in Fig. 5. It can be visually noticed that the time series in Fig. 5(a) (desert area) shows the driest condition (unfavorable), whereas the series in Figs. 5(d) (west coast) and 5(e) (high rainfall zone at northeast part of India) shows most wet condition (favorable) case. Maximum variation is observed in the series located in the west coast region. Resilience and vulnerability (with respect to the PWP of the study area) are computed for each time series, and the DMI values are computed as explained earlier. As can be noticed, the DMI is very high for the series from the desert area

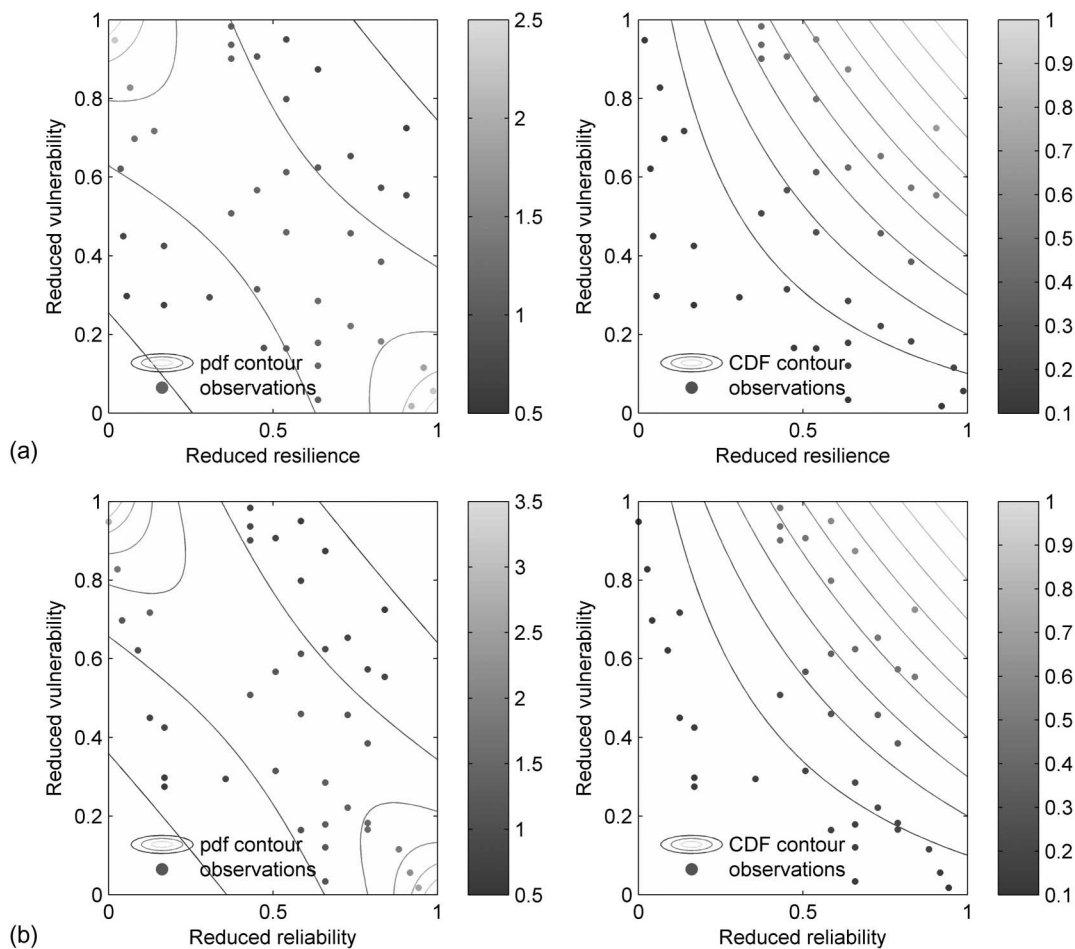


Fig. 3. Joint pdf and joint CDF: (a) between reduced resilience and reduced vulnerability; (b) between reduced reliability and reduced vulnerability

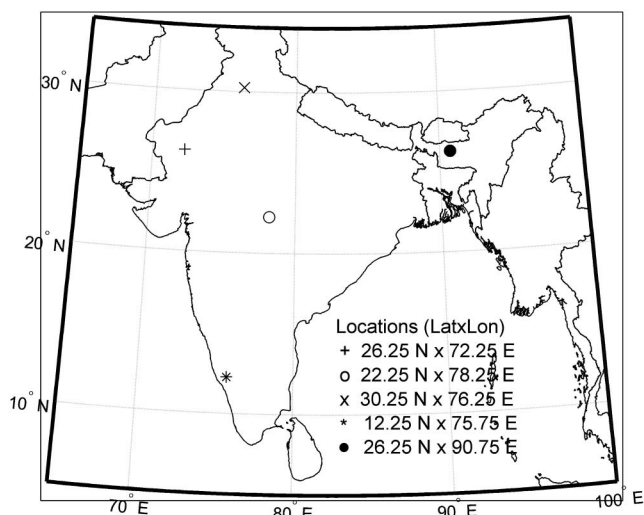


Fig. 4. Location map of soil moisture series used for testing with latitudes and longitudes and approximate coastal line

(marked as + in Fig. 4) and zero for the series from the northeast part of India (marked as • in Fig. 4). Other series have different DMI values depending on their respective resilience and vulnerability values. For instance, the series located at the northern part of India (marked as × in Fig. 4) is having very low resilience and very high vulnerability. So even if the reliability of this series is

much higher as compared to the series at the desert area (marked as + in Fig. 4), in terms of their ratios, the DMI of the series located at northern part of India is as high as that of desert area. On the other hand, the soil moisture series from the central part of India (marked as o in Fig. 4) and that from the west coastal part (marked as * in Fig. 4) are found to have similar reliability (and resilience) values. However, the former has an almost 150% higher vulnerability value. So, the computed DMI value is much lower (approximately 2%) at the latter location than that at the former location. Thus, the reliability (or resilience) estimates may differentiate the drought characteristics in an indiscernible scale. This would have been acceptable if the vulnerability was same for these series. However, as shown, this is not the case. Vulnerability varies considerably between these series. Thus, computed DMI also differs considerably from each other. This was the motivation behind the development of this new drought-characterizing index, which reflects the effective use of vulnerability information along with resilience (or reliability) information.

Sensitivity of DMI for Time Length and Threshold Level of Soil Moisture

Sensitivity analysis for length of data considered is investigated for its possible impact on the final computation of DMI. Data lengths ranging from 2 to 10 years are considered separately, and the Plackett copula-based DMI is developed for each case. The DMI for the five different soil moisture series is computed for each case, and its variation with respect to different data lengths

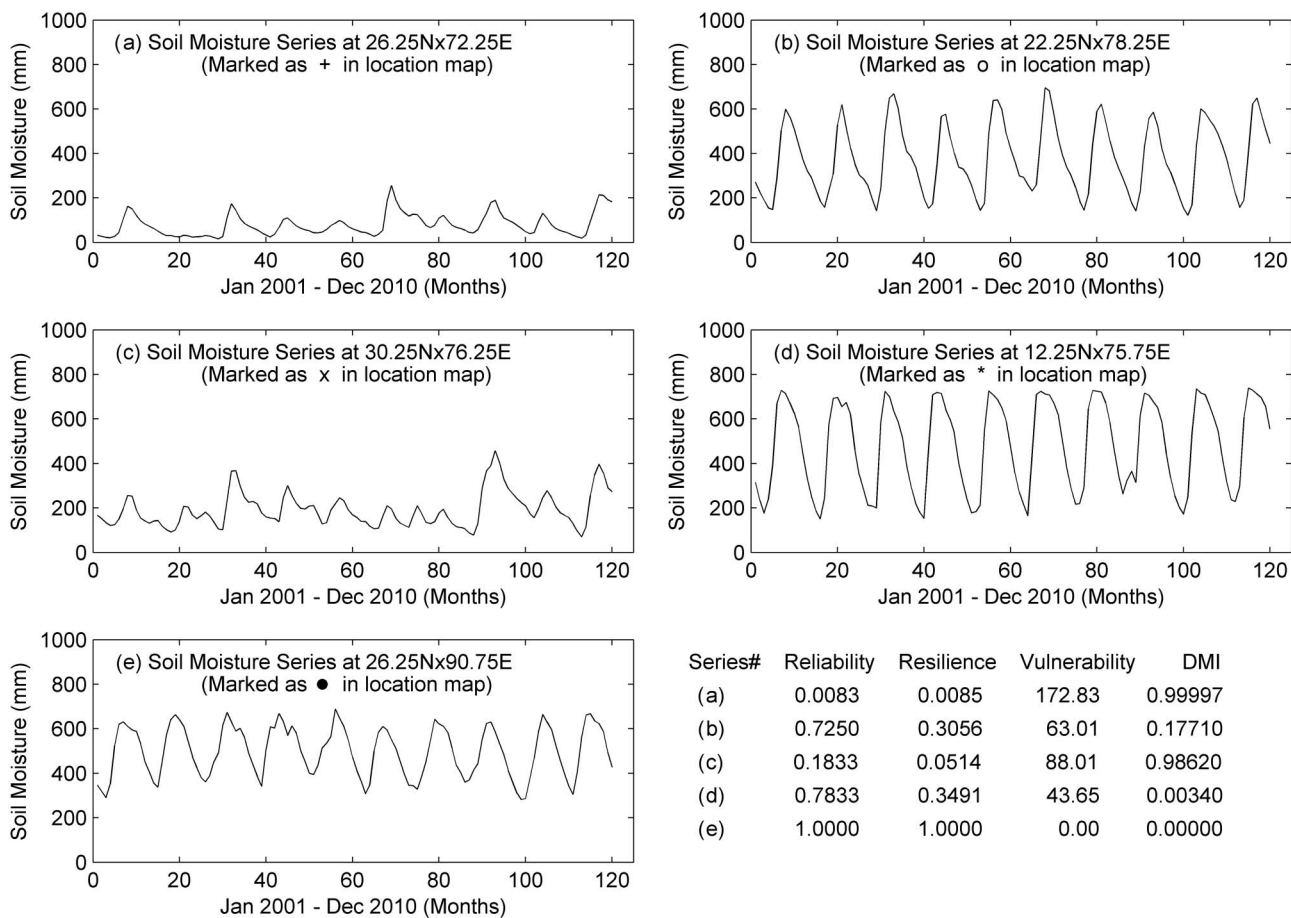


Fig. 5. Time series of soil moisture from five different locations across India with their reliability, resilience, vulnerability, and DMI values: (a) soil moisture series at $26^{\circ}15'00''$ lat \times $72^{\circ}15'00''$ long; (b) soil moisture series at $22^{\circ}15'00''$ lat \times $78^{\circ}15'00''$ long; (c) soil moisture series at $30^{\circ}15'00''$ lat \times $76^{\circ}15'00''$ long; (d) soil moisture series at $12^{\circ}15'00''$ lat \times $75^{\circ}45'00''$ long; (e) soil moisture series at $26^{\circ}15'00''$ lat \times $90^{\circ}45'00''$ long

is studied. Results are shown in Fig. 6. It is observed that the computed DMI more or less stabilizes beyond the data length of a 5- or 6-year period. To obtain a stable index as well as finer temporal resolution to assess the temporal variation of DMI, it is desired to select the minimum possible length of data from which a reasonably stable estimate of the index can be obtained. Hence, a data length of 5 years is selected because the computed index is more or less stable for a data length longer than a 5-year period. Thus, for computation of DMI, time scales should be 5 years or more.

The threshold value for failure is not a simple number but a predefined physical parameter depending on the type of drought considered. For example, the PWP may be considered for agricultural drought as indicated earlier, which can vary from one location to another depending on the soil-crop combination. This threshold may also vary for a particular location for different crops. If this threshold increases (decreases) for a particular soil moisture time series, the DMI value is expected to increase (decrease) for the same soil moisture regime. For the present study, the effect of varying the threshold is studied for all the time series (from different locations) considered for testing. Various threshold values are selected ranging from 175 to 400 with an increment of 25. Computed DMI are plotted in Fig. 7 for different values of threshold for the same time series. It is found that the DMI increases with the increase in threshold value as expected. It is further noticed that the index is bounded by 0 and 1, and the increase in DMI value can be observed for all the cases of time series with the increase

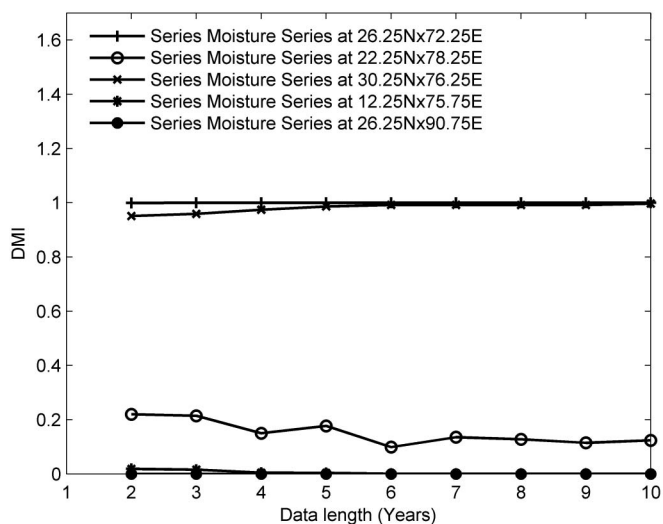


Fig. 6. Variation of DMI with respect to different data lengths

in the threshold value. Finally, the developed index, DMI, is found to reflect the characteristics of different types of soil moisture series. Thus, the use of vulnerability information along with resilience information can be concluded to be more effective.

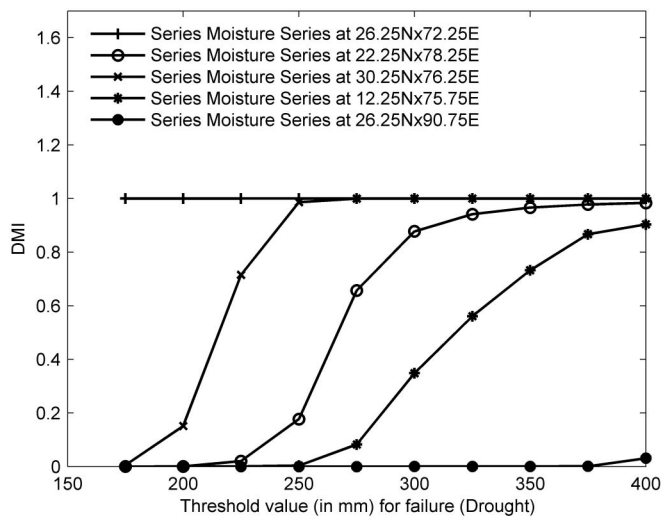


Fig. 7. Variation of DMI with respect to different threshold values

5-Year Variation of DMI over Last 50 Years for the Study Area Considered during Model Development

In “Sensitivity of DMI for Time Length and Threshold Level of Soil Moisture,” DMI was computed for the five different soil moisture time series from five different locations from India. It was found to represent all types of drought characteristics. In this section, DMI is computed for the soil moisture data for the area in and around Malaprabha basin using the copula function developed in “Joint Distribution between Resilience and Vulnerability of Soil Moisture Data” [refer to Figs. 3(a and b)]. As mentioned earlier, DMI values are found to stabilize when a data segment of at least 5 years is considered. Hence, the soil moisture data from the study region for the period 1961–2010 is segmented into 5-year windows and DMI is computed for each of them. The DMI is considered to assess the drought characteristics of the basin for each 5-year period starting from 1961. Results are shown in Fig. 8. It is observed that the DMI varies in a cyclic pattern. Either the rising or falling phase is approximately a decade long. An overall mild increasing trend is also observed with a rate of 0.0048 per 5-year period or approximately 0.01 per decade. This is from the equation of the overall trend line with the slope of 0.0048 as shown in Fig. 8. However, the cyclic pattern is more prominent as compared to the overall increasing trend. Thus, the drought characteristics of the study region follow a cyclic pattern with a slow increasing trend. It is further noticed that currently the basin is in the rising phase of DMI. It is expected that the same may be in the falling phase a decade later as is evident from Fig. 8.

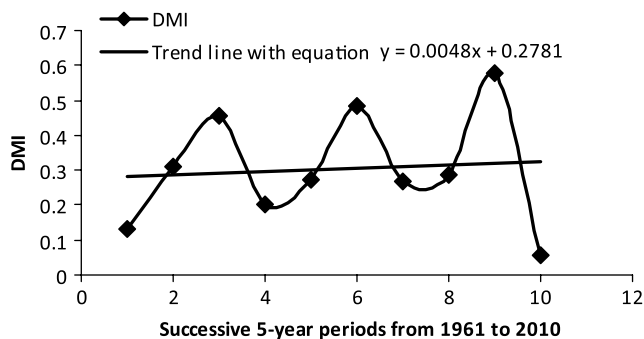


Fig. 8. Variation of 5-year DMI for the study region over last 50 years

Before concluding, it is worthwhile to mention here that the use of PWP as a threshold is analogous to standardization with respect to the soil moisture regime of that region since it allows a comparison of drought risks of different locations relative to their natural regimes. The regular standardization procedure followed in other drought indices such as the SPI is avoided here since it has certain drawbacks. For instance, even a small deficit in precipitation may be reflected as a large negative value for the locations with small variation in precipitation (or any other hydrologic variable for drought) (Mallya et al. 2011). However, DMI does not suffer from such shortcomings as the PWP is a specific quantity at a region for a particular crop. Moreover, the SPI is ineffective for longer time scales due to very high temporal overlap (Mallya et al. 2011). On the other hand, DMI is designed to capture the long-term drought characteristics.

Conclusions

The reliability, resilience, and vulnerability derived from soil moisture data are investigated in this paper. A new index to characterize the drought proneness of a catchment is developed using the joint information of resilience and vulnerability. Plackett copula is used to obtain the joint distribution of resilience and vulnerability of soil moisture time series. The developed index, DMI considers the information of both resilience and vulnerability. This is important from an effective drought characterization point of view as this index simultaneously considers the frequency/recovery period as well as the severity of droughts—once a drought has occurred. For five different soil moisture time series from different locations, it is observed that even for same resilience intervals, the obtained DMI values depend on their vulnerability values. DMI is found to stabilize over a time period of 5 years or more, suggesting the use of a 5-year period for assessment of DMI variability. While investigating the temporal variation of DMI for the study area, a cyclic pattern was observed with a slight increasing trend. Current status of Malaprabha was found to be in rising phase with an indication of falling trend a decade later. The study can be extended to investigate spatial variation of DMI, which will be useful for analysis over a larger area, say a country. However, soil moisture data at different spatial points, preferably gridded, is required. Further extensions to assess the changes due to global warming as simulated by General Circulation Models are in progress and will be reported in the future.

Appendix. Identification of the Appropriate Copula

For all the data points (reduced variable of resilience, u_1 and vulnerability, u_2), values of empirical CDF are obtained and denoted as C_n . For the bivariate case considered in this study, C_n is estimated as

$$C_n(\mathbf{u}) = \frac{1}{n} \sum_{i=1}^n \mathbf{I}(U_{i1} \leq u_1, U_{i2} \leq u_2) \quad (15)$$

where $\mathbf{u} = [u_1, u_2]$ is the reduced variable of resilience (u_1) and vulnerability (u_2), $\mathbf{I}(A) = 1$ if A is true, and $\mathbf{I}(A) = 0$ if A is false. Reduced variables are obtained after transforming through their marginal distribution, i.e., $u_1 = \phi^{-1}(R)$ and $u_2 = \phi^{-1}(V)$, where R is the resilience, V is the vulnerability, and ϕ^{-1} is the inverse of cumulative normal distribution. U_{ij} ($i = 1, \dots, n$ and $j = 1, 2$) are known as pseudo-observations. These are obtained as $U_{ij} = R_{ij}/(n + 1)$ where R_{ij} = the ranks of the data.

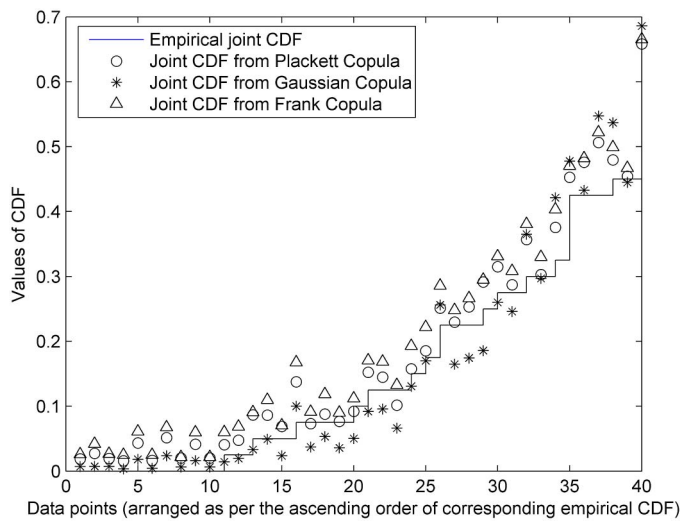


Fig. 9. Performance of different copulas to represent the empirical CDF and joint CDFs between resilience and reliability

Values of C_n are arranged in an ascending order and plotted as a solid line in Fig. 9. For each value of empirical CDF, values of CDF using Gaussian (C_G), Frank (C_F), and Plackett (C_P) copulas are also obtained for the corresponding data. These values are also plotted on the same plot (Fig. 9).

The suitable copula will produce the values of resulting cdf close to those obtained by empirical copula. Genest et al. (2009) suggested metrics, such as, $S_n = \int_{[0,1]^2} D_n(\mathbf{u})^2 dC_n(u)$ and $T_n = \sup_{u \in [0,1]^2} |D_n(\mathbf{u})|$, where $D_n(\mathbf{u}) = \sqrt{n}(C_n - C)$ and $n =$ the number of data points. Superscripts of C_{θ_n} are omitted to make it general (i.e., applicable to all the copulas). The values of S_n and T_n are given in Table 1.

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