

Study of dynamic behaviour of recession curves

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Abstract:

Since Brutsaert and Neiber (1977), recession curves are widely used to analyse subsurface systems of river basins by expressing $-dQ/dt$ as a function of Q , which typically take a power law form: $-dQ/dt = kQ^\alpha$, where Q is the discharge at a basin outlet at time t . Traditionally recession flows are modelled by single reservoir models that assume a unique relationship between $-dQ/dt$ and Q for a basin. However, recent observations indicate that $-dQ/dt - Q$ relationship of a basin varies greatly across recession events, indicating the limitation of such models. In this study, the dynamic relationship between $-dQ/dt$ and Q of a basin is investigated through the geomorphological recession flow model which models recession flows by considering the temporal evolution of its active drainage network (the part of the stream network of the basin draining water at time t). Two primary factors responsible for the dynamic relationship are identified: (i) degree of aquifer recharge (ii) spatial variation of rainfall. Degree of aquifer recharge, which is likely to be controlled by (effective) rainfall patterns, influences the power law coefficient, k . It is found that k has correlation with past average streamflow, which confirms the notion that dynamic $-dQ/dt - Q$ relationship is caused by the degree of aquifer recharge. Spatial variation of rainfall is found to have control on both the exponent, α , and the power law coefficient, k . It is noticed that that even with same α and k , recession curves can be different, possibly due to their different (recession) peak values. This may also happen due to spatial variation of rainfall. Copyright © 2012 John Wiley & Sons, Ltd.

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INTRODUCTION

Recession flows indicate the ability of river basins to retain rain water and release it gradually during dry periods. The study of recession flows therefore holds societal importance. Due to the problems in directly assessing subsurface systems, conceptual models are generally used for prediction of recession flows with limited success (e.g. Kroll *et al.*, 2004). An alternative route is to gather information on the subsurface flow processes indirectly by analysing recession curves. This can be achieved by identifying the patterns in the recession flows and linking them to the physical characteristics of basins by making use of suitable analytical tools (e.g. Brutsaert and Nieber, 1977; Brutsaert and Lopez, 1998; Tague and Grant, 2004; Biswal and Marani, 2010). A more accurate assessment of the subsurface systems through study of recession flows will help in developing parsimonious models for prediction purposes.

Brutsaert and Nieber (1977) introduced a radical change in recession analysis by eliminating the need to select a reference time by introducing $-dQ/dt$ as a function of Q , where Q is discharge at the outlet of a basin at time t . They found that $-dQ/dt$ versus Q curves of a basin typically follow a power law relationship of type:

$$-\frac{dQ}{dt} = kQ^\alpha \quad (1)$$

Recession flows in real basins with different geological, topographical and climatological characteristics tend to follow Equation (1). Thus, it is widely realized that there exists a physical process common to all real basins, which controls their recession flows. Many studies have therefore been undertaken in the past to explain the power law relationship between $-dQ/dt$ and Q . Brutsaert and Nieber (1977) found that the outflows from a homogeneous rectangular aquifer resting on an impermeable bed (the Dupuit-Boussinesq aquifer) generate $-dQ/dt$ versus Q profiles that follow Equation (1). Thus, they suggested that a basin can be assumed to behave as a single (linear or non-linear) reservoir during recession periods. This prompted them and several others to assume a unique relationship between storage and discharge, or a unique relationship between $-dQ/dt$ and Q , for a basin (e.g. Brutsaert and Lopez, 1998; Tague and Grant, 2004; Rupp and Selker, 2005; Harman *et al.*, 2009; Kirchner, 2009). Thus, in traditional practices, all data points from the $-dQ/dt$ versus Q curves of a basin have been plotted together, and the obtained values of α and k have been considered as the representative values for the basin.

The assumption of a unique $-dQ/dt - Q$ relationship for a basin can, however, be questionable if there are factors such as human influences (e.g. cities drawing water), evapotranspiration, observational and numerical errors, overland flows mixing with subsurface flows, and flow in vadose zone being taken into consideration (e.g. Federer, 1973; Brutsaert and Lopez, 1998; Sloan, 2000; Tung *et al.*, 2004; Rupp and Selker, 2006; Rupp *et al.*, 2009; Wang and Cai, 2009; Krakauer and Temimi, 2011). Biswal and Marani (2010) showed that in log-log plot $-dQ/dt$ versus Q curves of a basin show significant deviations from one another.

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Therefore, superposition of $-dQ/dt$ versus Q curves can give erroneous results. They found that though the power law exponent α of recession curves of a basin remains fairly constant, the coefficient k varies greatly from event to event. Thus, they suggested to analyse recession curves of a basin individually. After computing α of the recession curves of a basin separately, they considered the median of the distribution as the representative α for the basin. Interestingly, they found that α of a basin has links with its channel network morphology. Biswal and Marani ('Universal' Recession Curves and their Geomorphological Interpretation, submitted to Journal of Geophysical Research, 2012, henceforth referred to as submitted manuscript) found that k of a recession curve is linked to the characteristic discharge Q_n , mean discharge observed in the n -th day after the peak of the recession curve, through a power law relationship, hinting that the dynamic relationship between $-dQ/dt$ and Q has a physical basis.

In this study, the reasons for the dynamic relationship between $-dQ/dt$ and Q are investigated through the network morphological-based recession flow model proposed by Biswal and Marani (2010). Two main factors responsible for the dynamical relationship are identified: (i) degree of aquifer recharge and (ii) spatial variation of rainfall. It is shown that recession curves with same α and k can be different if their peak values are different, which can be also possible because of spatial variation of rainfall.

DYNAMIC PROPERTIES OF RECESSION CURVES: DATA AND DATA ANALYSIS

Daily streamflow data were obtained from USGS (available at: <http://waterwatch.usgs.gov/>) for four US basins: (i) Bull, near Walnut Shade, Missouri, 494.7 sq km, USGS gauge ID: 07053810 (data available from 1994); (ii) Arroyo Seco, near Pasadena, California, 41.4 sq km, USGS gauge ID: 11098000 (data available from 1910); (iii) Beaver, near Mason, Texas, 582.7 sq km, USGS gauge ID: 08150800 (data available from 1994) and (iv) Sipsey Fork, near Grayson, Alabama, 238.5 sq km, USGS gauge ID: 02450250 (data available from 1966). All the selected basins are free from significant human interventions (observation made with Google Earth). This condition is essential, as recession flow patterns can be considerably affected by human activities, such as the presence of dams, cities, etc. (e.g. Wang and Cai, 2009; Biswal and Marani, 2010). Furthermore, the basins are steep (mountainous), which means the geomorphological recession flow model can be suitably applied for them (Biswal and Marani, 2010).

Following Brutsaert and Nieber (1977), $-dQ/dt$ and Q were calculated as $(Q_i - Q_{i+1})/\Delta t$ and $(Q_i + Q_{i+1})/2$, respectively, from the discharge time series data of the study basins. The suffix i stands for the values at time t , and $i + 1$ for the values at time $t + \Delta t$. The time step Δt was 1 day in all cases. A recession curve here is defined as a continuously decreasing streamflow lasting for at least

5 days whose peak (which is also the peak of the corresponding hydrograph) is greater than the average flow at the basin outlet. Recession curves with peak values smaller than basin average flow were not considered in order to exclude very low flow discharge observations which are typically associated with observational errors. This criterion, however, does not affect the results very much. The peak values of recession curves also were discarded as they are likely to be significantly influenced by surface flows.

$-dQ/dt - Q$ relationship for a basin can vary from event to event, which can be seen in Figure 1 that displays a set of three $-dQ/dt$ versus Q curves for Bull basin (489 sq km, Missouri). Though individual $-dQ/dt$ versus Q curves robustly exhibit power law relationships (in all three cases $R^2 > 0.98$, see Figure 1), they show significant deviations from one another in the log-log plot. It can be noticed that their k values are different from one another while their α values remain fairly constant. The recession curves of the basin were therefore analysed individually following Biswal and Marani (2010): α of the recession curves were computed individually, and the median of the distribution was considered as the representative value of α (α_o) for Bull basin, which was found to be 1.97 (with a standard error of ± 0.51). Arroyo Seco, Beaver and Sipsey Fork gave α_o values equal to 2.23, 2.11 and 1.91, respectively.

The variation of k from one event to another for the same basin demonstrates the dynamic behaviour of recession curves. It can be observed that the $-dQ/dt$ versus Q curves in Figure 1 follow an interesting pattern that $-dQ/dt$ versus Q curves with higher peak values occupy the right side in the panel. This means that $-dQ/dt$ versus Q curves with higher peaks would have lower k values. However, in order to quantify this pattern, it is necessary to analyse how k changes from event to event while α remains constant, as a slight change in α may result in significant variation of k . Hence, α for each recession curve of the basin was set at its α_o (median of the distribution of α), and its k value was computed. Biswal and Marani (submitted manuscript) found that k exhibits a power law relationship with the characteristic discharge Q_n , for any n (also see Figure 2):

$$k(Q_n) = k'_n Q_n^{-\lambda_n} \tag{2}$$

k'_n and λ_n are constant for a basin for any given n . It is noticed that the correlation (R^2) of the power law relation (Equation (2)) for Bull basin increases with n (Figure 2):

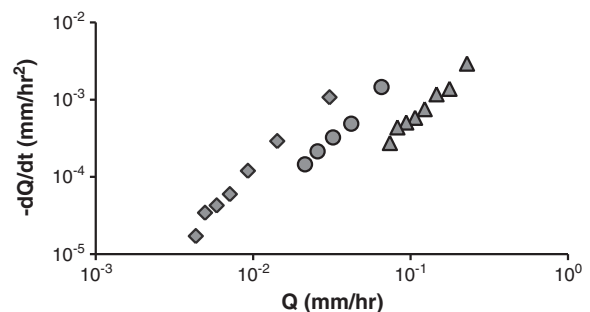


Figure 1. A set of three $-dQ/dt$ versus Q curves for Bull basin displaying deviation from one another

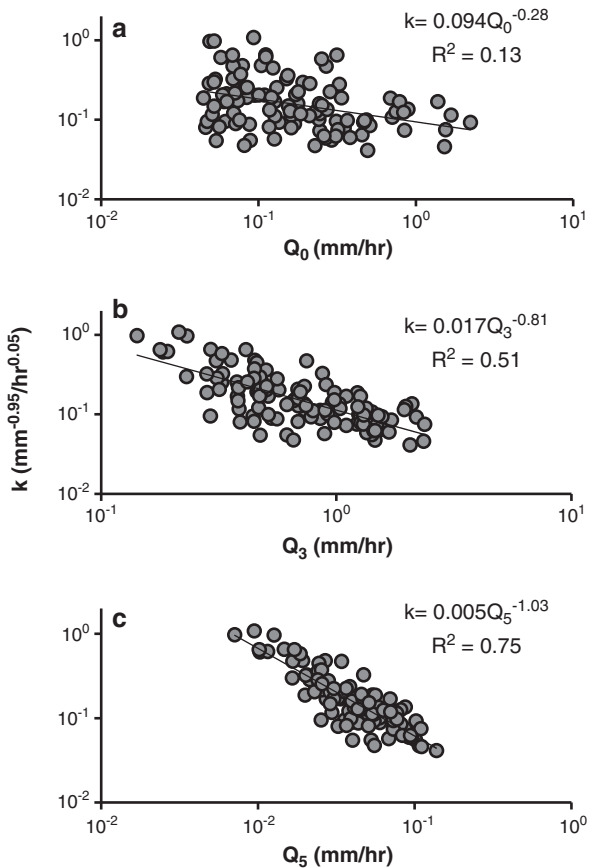


Figure 2. Power law relationship between k and Q_n , mean discharge observed in the n -th day after peak, for $n=0$ (a), $n=3$ (b) and $n=5$ (c)

$R^2=0.13$ for $n=0$, $=0.51$ for $n=3$ and $=0.75$ for $n=5$. Note that the value of λ_n also increases with n : 0.28 for $n=0$, 0.81 for $n=3$, and 1.03 for $n=5$. The analysis was repeated for the other study basins, and similar results were recorded.

The variation of k from event to event and its eventual display of the power law relationship with Q_n (Equation (2)) indicate that there is a distinct physio-climatological process governing it. The second observation that $k(Q_n)$ and Q_n show weak correlation for small values of n is interesting as well as counter intuitive, as one would expect Q_n , for large n , to be considerably influenced by observational errors, thereby decreasing the correlation. The low correlation between k and Q_n for small n essentially implies that they don't vary together during early recession flow periods. One possibility is that Q_n , for small values of n , will be a mixed flow of surface water and subsurface water, whereas Q_n at large n is expected to be composed of subsurface flow only (Biswal and Marani, submitted manuscript). Though it seems to be a plausible explanation, it invokes the question of why subsurface flow would not vary with surface flow. In fact, the relative composition of surface flow and subsurface flow is still an active field of research (e.g. Sophocleous, 2002; Ward *et al.*, 2012).

In this study, the causes for the power law relationship between k and Q_n and the low correlation between k and Q_n for small n are analysed through the network morphology-based recession flow model proposed by Biswal and Marani (2010).

THE GEOMORPHOLOGICAL RECESSON CURVE THEORY

A sizeable portion of streamflow comes from subsurface storage systems, even during storm periods (Beven, 1989; Dingman, 1994). Streamflow controlled by ground water storages is difficult to assess based on the fundamental equations of hydrodynamics, such as Darcy's law, as it is not possible to make any direct observation of subsurface storage systems. Also, single reservoir models (e.g. Brutsaert and Lopez, 1998; Kirchner, 2009) do not explain the dynamical behaviour of recession curves, as they usually assume a unique relationship between $-dQ/dt$ and Q for a basin. Single reservoir models assume a basin to be behaving as a single phreatic aquifer resting on a horizontal impermeable bed; thus, they ignore the effect of the spatial heterogeneity. In reality, the distribution of water and hydraulic conductivity in the subsurface zones of a basin can be highly uneven (Toth, 1962; Beven, 1989; Sophocleous, 2002). During recession periods, subsurface storage systems can evolve not only temporally but also spatially. Channel saturation, which is maintained by the stream feeding aquifers, can therefore vary both spatially and temporally. Typically, the saturated channelled network or the *active drainage network* (ADN) shrinks towards downstream direction during a recession event, i.e. the lower order streams will dry up before the higher order streams. It is widely acknowledged that flood responses in a basin are controlled by the dynamics of the saturated area (including the saturated parts of hillslopes). This is known as *variable source area* mechanism in the hydrologic literature (e.g. Hewlett and Hibbert, 1967; Dunne and Black, 1970). However, the role of the ADN in transporting baseflows was not investigated in earlier studies; for example, the Dupuit-Boussinesq aquifer model-based studies assume that the whole channel network remains active during a recession event. Perhaps, the dynamics of the ADN is one of the major causes of the dynamical relation between $-dQ/dt$ and Q . Biswal and Marani (2010) utilized the dynamics of ADN to model recession flows by considering channel network as the only entity that controls the recession flows.

It was assumed that Q at the outlet of a basin is a product of the length of ADN at time t , $G(t)$, and the flow generation per unit length of the ADN, $q(t)$:

$$Q(t) = q(t) \cdot G(t) \quad (3)$$

Differentiating both sides of Equation (3):

$$\frac{dQ}{dt} = \frac{dq}{dt} \cdot G + q \cdot \frac{dG}{dt} \quad (4)$$

dq/dt accounts for time rate of change in aquifer depletion, and dG/dt for the dynamics of the ADN. If the role of ADN dynamics is neglected, Equation (4) becomes a single aquifer model (e.g. Brutsaert and Nieber, 1977; Brutsaert and Lopez, 1998). In Biswal and

Marani (2010), however, the role of aquifer depletion was neglected; i.e. it was assumed that $dq/dt=0$, or q remains constant over time, for a recession event.

Also, it was assumed in Biswal and Marani (2010) that the speed of desaturation (c) at each channel head of the ADN is constant in both space and time, which quantitatively implies, $l=c(t-t_0)$, where l is defined as the distance of a channelled pixel from its farthest source (channel head) measured along the stream network and t_0 is the reference time. For this linear relationship, t and l can be easily interchanged whenever necessary. The expression for $-dQ/dt$ can thus be obtained as:

$$-\frac{dQ}{dt} = -q \cdot \frac{G(t)}{dt} = -q \cdot \frac{G(l)}{dl} \frac{dl}{dt} = qc \cdot N(l) \quad (5)$$

$G(l)$ is defined as the length of ADN at a distance greater than or equal to l and $N(l)$, the number of links exactly at a distance l in the ADN configuration, where l is the distance of an arbitrarily chosen point in the channel network from its furthest source. 30 m resolution digital elevation model data were obtained for the study basins from USGS (available at: <http://nationalmap.gov/viewer.html>). Drainage networks for the basins were obtained by imposing suitable flow accumulation threshold values (O'Callaghan and Mark, 1984). Figure 3 shows the ADN of Bull basin undergoing desaturation during a hypothetical recession event following the assumptions made in Biswal and Marani (2010). In the beginning of the recession event, i.e. at $t=0$ or $l=0$, the whole network is saturated and with time the ADN shrinks towards downstream direction (Figure 3a and b show ADN configurations for $l=0$ km and 2 km respectively). Using the expressions for Q and $-dQ/dt$ in Equation (1), we get

$$N(l) = \frac{kq^{\alpha-1}}{c} G(l)^\alpha \quad (6)$$

or

$$N(l) = \rho G(l)^\alpha \quad (7)$$

where

$$\rho = kq^{\alpha-1}/c \quad (8)$$

Typically, $N(l)$ versus $G(l)$ curve of a basin exhibits two scaling regimes: AB and BC, as shown in the case of Bull basin (Figure 3c). Only the scaling regime AB is perceived to be representing the observed $-dQ/dt$ versus Q curves, as it is not possible to obtain $-dQ/dt$ versus Q curves for very low flow values, due to short interval between rainfall periods and due to measurement errors (Biswal and Marani, 2010). Biswal and Marani (2010) using an extensive dataset showed that for most basins α obtained from basin network morphology, α_g , matches well with α_o , particularly when the basins are natural type. For Bull basin, the modelled power law exponent α_g (slope of the AB portion of its $N(l)$ versus $G(l)$ curve in log-log plot) is equal to 1.95, which is nearly

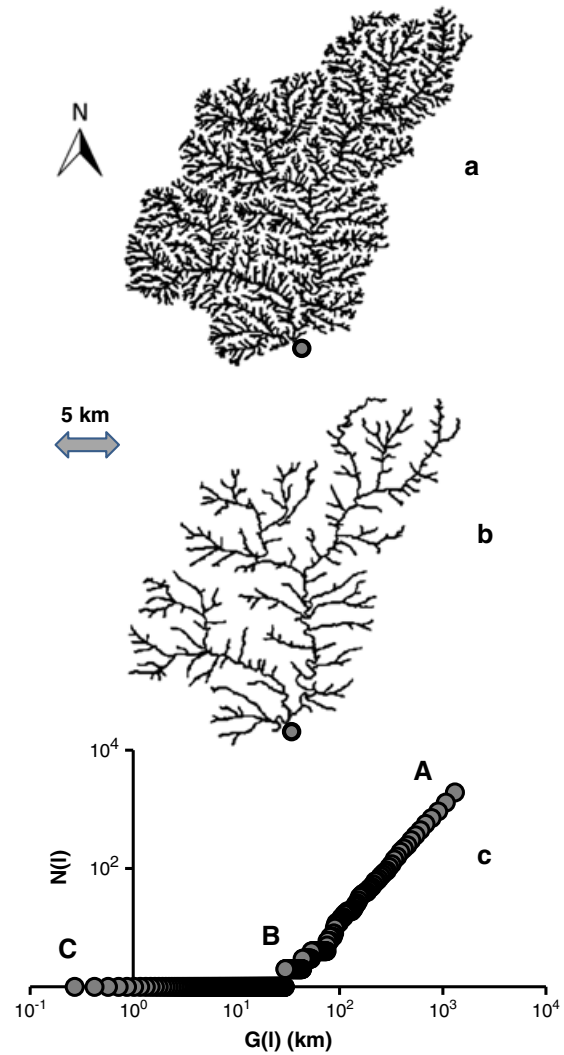


Figure 3. Evolution of the ADN of Bull basin during a hypothetical recession event following the assumptions of Biswal and Marani (2010): q and c remain constant both spatially and temporally throughout recession event. Drainage network for the basin was obtained by using a flow accumulation threshold of 100. a) In the beginning of the recession event (time $t=0$ or $l=0$), the whole network is saturated. b) ADN of the basin at $l=2$ km. c) Modelled recession curve $N(l)$, number of channel heads of ADN at a distance l , versus $G(l)$, total length of ADN at a distance greater than or equal to l

equal to 1.97, representative α obtained from the observed recession flows (α_o). Similarly, the values of α_g obtained for Arroyo Seco, Beaver, Sipsey Fork were found to be 2.25 (against $\alpha_o=2.23$), 1.95 (against $\alpha_o=2.11$) and 2.10 (against $\alpha_o=1.91$), respectively. This result ($\alpha_o \approx \alpha_g = \alpha$) is utilized in this study for further analysis.

CAUSES OF DYNAMICAL BEHAVIOUR OF RECESSION CURVES

Many factors can simultaneously influence $-dQ/dt$ versus Q curves. The power law relationship between $-dQ/dt$ and Q (Equation (1)) implies that both $-dQ/dt$ and Q decrease with time. As Q decreases slowly over time during recession events, it is likely that observational errors can significantly affect the computation of $-dQ/dt$, particularly during very low flow periods that occur

several days after (recession) peaks. Such observational errors may be causing the non-unique relationship between $-dQ/dt$ and Q , i.e. making $-dQ/dt$ versus Q curves different for different recession events (e.g. Rupp *et al.*, 2009). However, due to limited data here, the analysis is restricted to a few physical and climatological factors that cause the dynamical relationship between $-dQ/d$ and Q of a basin.

Individual recession curves of a real basin robustly exhibit power law form as in Equation (1), which indicates that the power law exponent α can be assumed to be constant throughout a recession event. A constant α for a recession event also means a constant value of k for it. If the obtained α for a recession event is equal to the α obtained from the basin geomorphology α_g , according to the geomorphological recession curve theory, both c and q can be assumed to be constant for the recession event. Conversely, if the assumption of constant c and q remains valid for all individual recession events of a basin, the obtained α will be equal to α_g in all cases, which is true for most of the real basins. Note that in Equation (6), the value of α does not depend on either q or c . Hence, the variation of q and c across recession events will not affect α , as long as they remain constant during individual recession events. That means α of a basin will be controlled only by its channel network morphology, not by any physio-climatic factor that would otherwise be represented by means of either q or c . It might be possible that both q and c vary during an individual recession event and alter the value of α , but it is assumed that such effects will not introduce any systematic bias. This seems to be supported by the fact that α_o is nearly equal to α_g for natural basins (Biswal and Marani, 2010). The notion is also strengthened by the observation that no correlation is observed between α_o and basin area or average streamflow (Biswal, 2011). Therefore, the deviation of α_o from α_g may be attributed mainly to observational errors.

In contrast, k is related to both c and q (see Equation (8)); thus, the variation of either c or q (or both) across recession events will ensure the variation of k . There is no observational data available here to analyse the effect of variability of c on k . The effect of q can, however, be assessed as there are some indirect indicators for q . In this study, two factors will be analysed: (i) variation in degree of aquifer recharge and (ii) spatial variation of rainfall.

Variation in degree of aquifer recharge

Stream feeding aquifers will be recharged during a rainfall event, and after the rainfall ceases, the aquifers will release water to their nearby streams. The amount of water released from an aquifer will depend on how much the aquifer was recharged during the rainfall event. Understandably, a rainfall event with higher effective rainfall (total rainfall minus losses such as evapotranspiration, i.e. amount of rainfall transforming into streamflow) will recharge the aquifer to a higher degree. A rainfall event with lower effective rainfall intensity lasting for a long time may also produce higher degree of aquifer recharge, if the rate of aquifer recharge is more than the rate at which the aquifers

are releasing water. Higher degree of aquifer recharge will produce higher rate of aquifer depletion (q) during a recession event, which also means higher value of Q_n (Equation (3)). According to the geomorphic recession flow model, the value of α is not dependent on the degree of aquifer recharge. This is supported by the observation in this study that no appreciable correlation exists between α and Q_n . For example, R^2 of the power law relationship between α and Q_n for Bull basin is less than 0.02 for all values of n (this also means there is no correlation between α and k). The value of k , however, is dependent on q . As the geomorphologic parameter ρ is constant for a basin, manipulation of Equation (8) yields the relationship: $k \propto c/q^{\alpha-1}$. For $\alpha > 1$, which is true for most of the real world basins (Biswal and Marani, 2010), k will be lower for higher q . Assuming that c remains constant across recession events in a basin, it can be found that higher q for a recession event will produce a recession hydrograph with higher peak (Q_0), or more generally, higher Q_n . This is verified by the fact that the exponent of the power law relationship between k and Q_n is negative for all values of n (Equation (2)).

Degree of recharge of aquifers, characterized by q , may also be affected by the history of aquifer recharges. Consider a stream feeding aquifer which is recharged during a rainfall event and later releasing water to its nearby stream during the recession event following it. If the aquifer is not drained completely during the recession event, then it is likely that the aquifer has contribution to streamflow in the future recession events. Thus, q for a recession event is likely to be higher if more water was stored in the aquifer in the past, i.e. if effective rainfall volume in the past was more. Past streamflow in a basin is assumed to be an indicator of water stored in the past within it for a time period more than 8 days. For each recession event, the average discharge from 10 to 2 days before the peak (of the recession hydrograph), i.e. past 8 days before the recession event (Q_{P8}), is calculated. Streamflow before 2 days of the peak was not considered to make sure that flows from the current rainfall event are not considered. Observations here show that a power law relationship exists between k and Q_{P8} :

$$k = k^* Q_{P8}^{-\delta_{P8}} \quad (9)$$

where k^* is a constant. The value of δ_{P8} is observed to be positive, which implies that k decreases with Q_{P8} . This again confirms that k decreases with the degree of aquifer recharge. Figure 4a shows k versus Q_{P8} curve for Bull basin with its δ_{P8} and R^2 are 0.35 and 0.42, respectively. Figure 5a, b and c show k versus Q_{P8} curves for Arroyo Seco, Beaver and Sipsy Fork basins, respectively. Stronger correlation between k and Q_{P8} would imply that past storage has more contribution to stream flow. Also, the correlation between past average discharge and k is expected to decrease with chosen (past) time span, as water stored in an aquifer may not be contributing to stream flow after a long time. To test this hypothesis, the relationship between k average past discharge for different

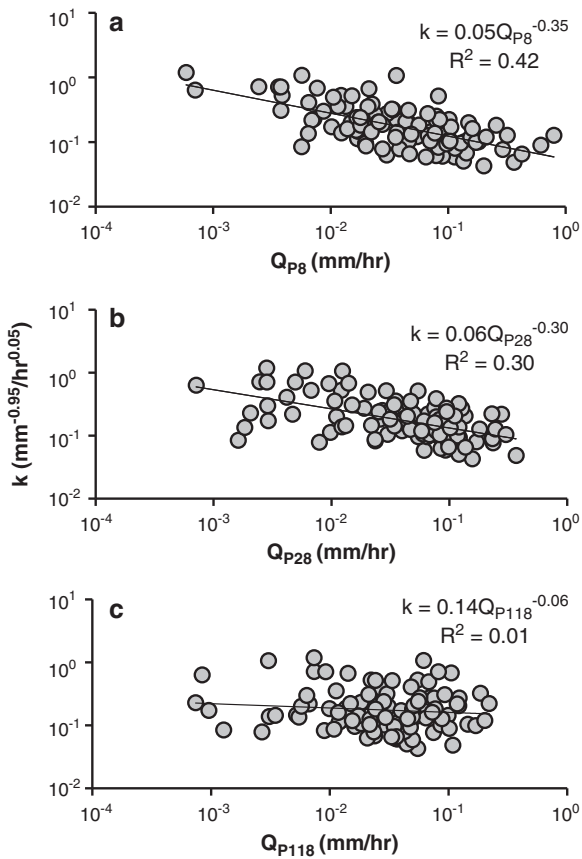


Figure 4. k of a recession event versus past average discharge for Bull basin for time spans: a) from 10 to 2 days (Q_{P8}), b) from 30 to 2 days (Q_{P28}) and c) from 120 to 2 days (Q_{P118})

time spans was analysed. Figure 4b and c show the relationship between k and past average discharge for Bull basin for time spans from 30 to 2 (Q_{P28}) days and 120 to 2 (Q_{P118}) days, respectively. The (R^2) correlation is observed to be decreasing with time span for the basin.

The correlation between past average discharge and k indicates the ability of a natural basin to hold water for a time period longer than its normal recession duration. This ability of a basin will be dependent on many factors such as topography, geology and distribution of hydraulic conductivity and flow paths (e.g. Tromp-van Meerveld and McDonnell, 2006; Tetzlaff *et al.*, 2009; Nieber and Sidle, 2010; Spence *et al.*, 2010). Equations (2) and (9) suggest that drainage from basin subsurface storage system of a real basin does not mimic that of a single reservoir following a unique storage-discharge relationship. It rather seems that basin storage-discharge relationship is strongly characterized by the dynamics of its ADN.

Spatial variation of rainfall

Most studies aiming to model hydrological responses assume spatially invariant rainfall input due to practical constraints. However, rainfall intensity hardly remains constant in real basins, even within a basin of size as small as 0.5 sq km (e.g. Goodrich *et al.*, 1995). Spatial variation of rainfall intensity can therefore affect spatial distribution of

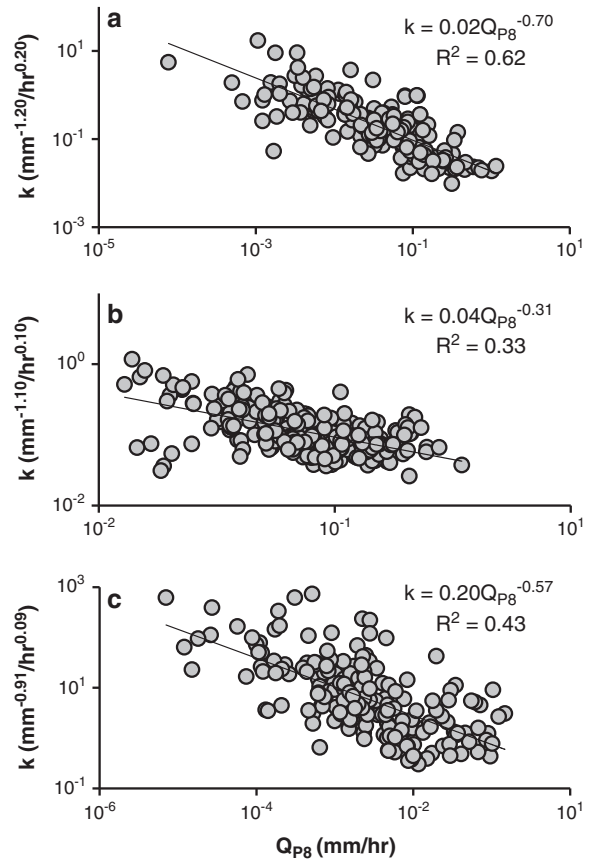


Figure 5. k of a recession event versus past average discharge from 10 to 2 days (Q_{P8}) for: a) Arroyo Seco, b) Beaver and c) Sipsey Fork

aquifer recharge in a basin. The impact is assessed here by blocking parts of stream networks from receiving rainfall. Figure 6a shows whole of the stream network of Bull basin receiving rainfall, while Figure 6b and c show parts of the stream network receiving rainfall. It can be noticed that due to spatial variation of rainfall, $N(l)$ versus $G(l)$ are different in each case. The value of α is 1.95 for (a), 1.90 for (b) and 2.15 for (c). Thus, in addition to observational errors, spatial rainfall variation can be considered as one of the factors affecting α . The variation of ρ from event to event is evident in Figure 6. Fixing α at 1.95 (as a slight change in α can result in a significant variation of ρ), it is found that ρ ($\text{km}^{-1.95}$) values are 0.0015, 0.0025 and 0.0021, respectively, for (a), (b) and (C). Similarly, G_0 can be affected by spatial rainfall variation. The values of G_0 for (a), (b) and (c) are 1673 km, 878 km and 903 km.

Interestingly, it is possible to get different recession curves with the same set of k and α . Integrating Equation (1):

$$-\int_{Q_0}^Q Q^{-\alpha} dQ = k \int_0^t dt \tag{10}$$

where Q_0 is discharge at time $t=0$. Equation (10) can be transformed into

$$Q = Q_0 [1 + kt(\alpha - 1)Q_0^{\alpha-1}]^{\frac{1}{1-\alpha}} \tag{11}$$

It can be noticed that the shape of recession curve (Q) is not only dependent on k and α but also on the initial discharge Q_0 , or peak discharge. That means, different curves will be obtained for same k and α if Q_0 does not remain same. However, when $1 \ll kt(\alpha - 1)Q_0^{\alpha-1}$, i.e. when t is large enough, Equation (11) becomes independent of Q_0 :

$$Q = [kt(\alpha - 1)]^{\frac{1}{1-\alpha}} \tag{12}$$

As can be seen in Equation (12), the shape of a recession curve depends only on k and α for large t , i.e. it becomes independent of Q_0 . Thus, k can be expected to have correlation with Q_n for large t , but not for small t . This is probably one of the reasons why low correlation is observed between k and Q_n when n is small, as recession curves with same k , Q_n can be different (see Figure 3). Figure 7a shows two recession curves for Bull basins that are different during early recession phase due to different values of Q_0 , but converging later on into a single curve, with its inset showing $-dQ/dt$ versus Q curves for the two events tending to overlap on one another.

The dependence of the shape of a recession curve on its peak can be seen through the geomorphological recession flow model. Integrating both sides of Equation (7), an expression similar to Equation (11) is obtained:

$$G(l) = G_0 [1 + \rho l(\alpha - 1)G_0^{\alpha-1}]^{\frac{1}{1-\alpha}} \tag{13}$$

It can be noticed that similar to Equation (11), the shape of the geomorphic recession curve, $G(l)$, is also dependent on the length of the ADN at time $t=0$, G_0 . Again, when $1 \ll \rho l(\alpha - 1)G_0^{\alpha-1}$, i.e. when l is large enough, Equation (13) becomes independent of G_0 :

$$G(l) = [\rho l(\alpha - 1)]^{\frac{1}{1-\alpha}} \tag{14}$$

The situation of same ρ and α but different G_0 can also be expected when initial aquifer recharge is spatially uneven, i.e. if rainfall input varies spatially. This phenomenon can be best viewed from Figure 3a and b where the initial ADN configurations for Bull basin during two recession events are different. In both cases, the $N(l)$ versus $G(l)$ curves will have same α and ρ (since the ADN configuration in Figure 3b is a result of desaturation of the ADN shown in Figure 3a following the constant q and constant c assumption, α and ρ will be same for both the cases), but will have different G_0 values due to their different initial ADN configurations (see Figure 7b). As can be seen from Figure 7b, both the curves converge for large l .

DISCUSSIONS AND CONCLUSIONS

Storage-discharge relationship or $-dQ/dt$ - Q relationship of a basin is not unique, which demonstrates the dynamic behaviour of recession flows. Individual $-dQ/dt$ versus Q curves of a basin exhibit power law relationship of type: $-dQ/dt = kQ^\alpha$. The coefficient of the power law relation k has a power law relationship with the mean discharge observed in the n -th day after the recession peak (Q_n): $k \propto Q_n^{-\lambda_n}$, with λ_n being positive for all values of n .

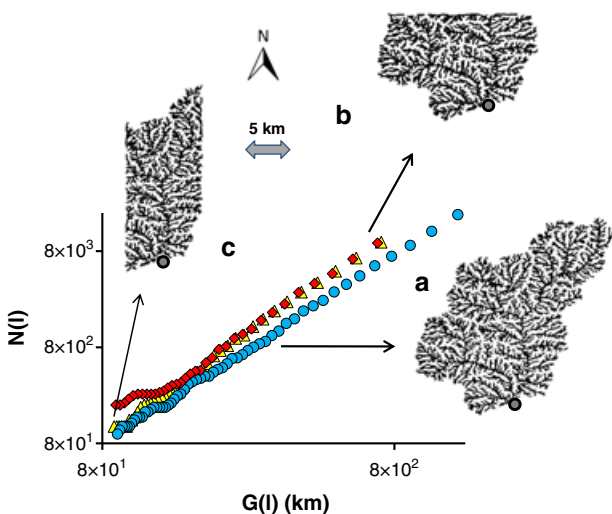


Figure 6. Parts of the stream network of Bull basin (obtained by using a flow accumulation threshold of 100) receiving rainfall during three hypothetical rainfall events and the corresponding $N(l)$ versus $G(l)$ curves. Note that the spatially and temporally constant q and c condition was applied to the parts of the stream network receiving rainfall in all three cases

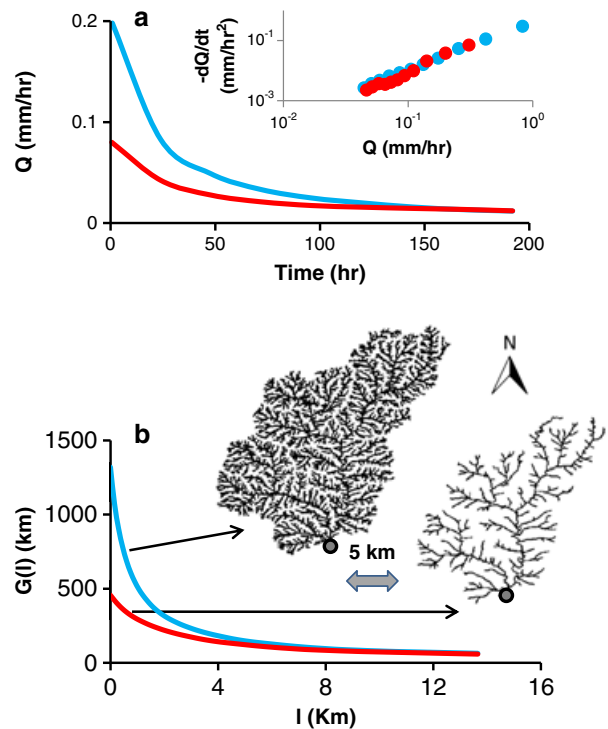


Figure 7. a) Two observed recession curves from Bull basin with different shapes for small t , but converge when t is large enough. The inset shows the $-dQ/dt$ versus Q curves for the two recession curves. b) $N(l)$ versus $G(l)$ curve obtained for two spatial rainfall patterns by using the geomorphological recession flow model. The two curves are different for small l , but converge when l is large enough. The channel network was extracted by using a flow accumulation threshold of 100

Interestingly, the correlation of the k - Q_n relationship (R^2) increases with n . These observations nurture the idea that the dynamical relationship between $-dQ/dt$ and Q of a basin is governed by a distinct physical processes, i.e. observational errors only cannot explain it.

The analysis for the dynamic behaviour was assessed in this study by using the geomorphological recession flow model, which assumes that the flow generation per unit channel length (q) as well as the speed of channel desaturation (c) during a recession period are constant both spatially and temporally. According to the geomorphological recession flow model $k \propto c/q^{\alpha-1}$, therefore, k will increase with c and decrease with q (note that for most real basins $\alpha > 1$). The dependence of k on q is supported by the observational evidence that k decreases with Q_n , which is supposed to increase with q .

Higher q may also reflect higher degree of historical aquifer recharge. This is possible because water stored in the aquifer during a rainfall event may not drain completely during the recession event following it, especially if the recession event does not last long enough (i.e. if another rainfall event appears too shortly), and will pass on to be part of streamflows in the next recession events. This was confirmed in this study by the observational evidence that k of a recession event exhibits a power law relationship with the mean discharge observed from 10 to 2 days before the peak of the recession event (Q_{PS}): $k \propto Q_{PS}^{-\delta}$, with δ being negative.

The relationship between k and past average discharge indicates the ability of basins to store water, which is arguably controlled by many factors such as soil properties, topography and subsurface flow pathways. With time, the contribution of stored water during a rainfall event to streamflow will decrease. This was confirmed by the observation here that the correlation between k and past average discharge decreases as time span (of past streamflow observation) increases.

Spatial variation of rainfall affects recession curves. This effect was assessed in this study by blocking parts of a stream network from receiving rainfall input and then simulating recession flow by using the geomorphological recession flow model. It was found that spatial variation of rainfall can alter both k and α of a recession event.

The shape of a recession curve not only depends on k and α but also on peak discharge (Q_0), when t is small. However, for large t , it becomes independent of Q_0 . By using the geomorphological recession flow model, it was shown in this study that spatial variation of rainfall can produce different recession curves ($N(l)$ versus $G(l)$) that display same α and ρ with different G_0 values for small l . However, when l is large enough, the curves become independent of G_0 . This dependence of the shape of a recession curve on its Q_0 explains why R^2 of the power law relationship between k and Q_n is small for small values of n .

Finally, the parallels between the behaviour of observed recession curves and the behaviour of recession curves obtained by the network morphology-based recession flow model demonstrate the ability of channel networks in controlling hydrological responses of natural basins.

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