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Multiobjective fuzzy optimization for sustainable groundwater management using particle swarm optimization and analytic element method

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Abstract:

Groundwater management involves conflicting objectives as maximization of discharge contradicts the criteria of minimum pumping cost and minimum piping cost. In addition, available data contains uncertainties such as market fluctuations, variations in water levels of wells and variations of ground water policies. A fuzzy model is to be evolved to tackle the uncertainties, and a multiobjective optimization is to be conducted to simultaneously satisfy the contradicting objectives. Towards this end, a multiobjective fuzzy optimization model is evolved. To get at the upper and lower bounds of the individual objectives, particle Swarm optimization (PSO) is adopted. The analytic element method (AEM) is employed to obtain the operating potentio metric head. In this study, a multiobjective fuzzy optimization model considering three conflicting objectives is developed using PSO and AEM methods for obtaining a sustainable groundwater management policy. The developed model is applied to a case study, and it is demonstrated that the compromise solution satisfies all the objectives with adequate levels of satisfaction. Sensitivity analysis is carried out by varying the parameters, and it is shown that the effect of any such variation is quite significant.

KEY WORDS multiobjective fuzzy optimization; groundwater management; analytic element method; particle swarm optimization

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INTRODUCTION

Fresh surface water resources are depleted day by day because of various competitive demands such as agricultural, municipal and industrial. This necessitated augmentation by use of groundwater or resulting to groundwater alone in some situations. Indiscriminate and sometimes unlimited pumping is deteriorating groundwater quality. The problem becomes more complex because of the multiobjective nature of the groundwater management. For example, maximum discharge objective to the region that facilitates maximum pumping may not satisfy the minimum pumping cost or piping cost criteria. For sustainable groundwater management in either developing or developed countries, these three objectives are required to be simultaneously satisfied (may not be equally) rather than mere maximization or minimization of any one of them (Duckstein *et al.*, 1994). This requires evolving compromise solutions in a multiobjective environment, which provide realistic and acceptable strategy to a policy maker for possible implementation (Loucks *et al.*, 1981; Deb, 2001; Raju and Nagesh Kumar, 2014).

In addition, the inherent uncertainty and imprecision in the available data such as uncertainty in cost of pumping due to increase/decrease in water level, variation in piping cost due to market fluctuations and change in groundwater policy from time to time makes groundwater management problems more complex, especially in the face of unexpected and sustained extremes such as droughts where groundwater pumping has to augment the meagre rainfall. In such a situation, fuzzy approach can be applied to handle the vagueness in objectives in real world situation (Chang *et al.*, 1997).

Most of the groundwater management problems were solved using the simulation-optimization approach. In this, numerical models are used to simulate the groundwater flow, and its output is used by optimization model for getting various parameters such as groundwater head and cost that can be used as the basis for effective

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decision making required for formulating a fuzzy model (Gaur *et al*., 2013). Brief but relevant literature review is presented in the succeeding text:

Bogardi *et al*. (1991) analysed groundwater management problems using interactive multiobjective decision-making method. The objectives of the study were maximization of total yield, minimization of maximum compression at the selected wells and minimization of total pumping cost. A finite-difference method-based flow model was used to generate the response matrix. Cieniawski *et al*. (1995) used genetic algorithms to solve multiobjective groundwater management problem for selecting a system for monitoring wells. McPhee and Yeh (2004) carried out a study of multiobjective optimization and demonstrated the use of groundwater simulation and optimization for solving the groundwater management problem.

Numerous authors employed multiobjective fuzzy optimization (MOFO) for various studies because of its (a) limited sensitivity analysis requirement and (b) less computational burden as compared with other multiobjective optimization techniques such as constraint method (Deb, 2001). In addition, ability to relate transitions as soft boundaries rather than hard boundaries makes the fuzzy approach more promising. Yang and Yu (2006) considered fuzzy multiobjective problem comprising three objectives, namely, reducing saltwater demand, reducing freshwater demand and increasing the total fisheries gross profit. The model was coupled with a global optimization algorithm to find suitable aquaculture scenarios for Tachen Village, Changhua County, Taiwan. It was concluded that analytical results can be used for revising the aquaculture structure. Kentel and Aral (2007) used fuzzy logic based multiobjective decision making approach for groundwater management. Conventional simulation-optimization model was used to optimize additional groundwater withdrawal at multiple demand locations in a coastal aquifer. The methodology was applied to a hypothetical case consisting of six groundwater demand locations in Savannah. Ordered weighted averaging operator was used to calculate overall performance based on the model outcome. Deep *et al*. (2009) developed fuzzy interactive method for efficient management of multipurpose multireservoir problems and applied to a case study. Two objectives, namely, irrigation and hydropower generation, were considered in fuzzy environment. These objectives were combined into a single objective using the product operator and genetic algorithms. It was concluded that the interactive approach was found to be satisfactory. Choudhari and Raj (2009) applied fuzzy linear programming (with linear membership function) to a case study of four reservoir system, Maharashtra, India. It was concluded that the reservoir operation policies evolved could tackle the complexities associated with the problem. Raju and Nagesh (2014) explored fuzzy multiobjective programming to a case study of Sri Ram Sagar Project with three objectives, net benefits, agricultural production and labour employment. Linear membership functions based on linear programming were considered for the three objective functions. The observation was the decrease of net benefits, agricultural production and labour employment by 2.38%, 10.26% and 7.22% as compared with ideal values. Mirajkar and Patel (2013) applied MOFP (with linear membership function) approach to a case study of Ukai irrigation project Gujarat, India. The model was solved for four situations of 90%, 85%, and 75% and 60% exceedance probability. It was concluded that the inflow corresponding to 75% exceedance probability was marginally sufficient to meet the requirements of the study area.

However, no study considered simultaneously nonlinear/linear fuzzy membership functions in the coupled analytic element method (AEM) and particle swarm optimization (PSO) in ground water planning environment. Keeping this aspect, MOFO methodology is explored in the present study to address groundwater management problem for a case study in France. Coupled simulation-optimization modelling involving AEM and PSO are employed to obtain the required inputs.

To the authors' knowledge, the present study is the first of its kind where MOFO is developed for a groundwater management problem in combination with PSO and AEM.

METHODOLOGY AND MATHEMATICAL MODELLING

Multiobjective fuzzy optimization model is employed in the present analysis. The three objectives considered are maximum discharge, minimum pumping cost and minimum piping cost. Initially, the three objectives were solved individually by taking only one of the objectives at a time. Using the results from individual objectives, the corresponding values of the two remaining objectives for each solution were derived. Following the process, a best and a worst value (maximum and minimum value) for each objective was calculated and correspondingly fuzzy membership functions were formulated. Further, the MOFO model was formulated, and finally, the compromise solution along with degree of satisfaction was determined.

Mathematically, a typical groundwater management problem can be defined using three main components, namely decision variables, objective function and constraints (Ahlfeld and Mulligan, 2000). The description of the three objectives functions, namely maximum discharge, minimum pumping cost and minimum piping cost with constraints, are as follows:

Maximization of discharge

Determination of maximum amount of water that can be withdrawn from the aquifer through given number of wells is

$$f = Max\left\{\sum_{i=1}^{N_w} Qi - a_1P(h) - a_2P(Q)\right\} \quad (1)$$

where Q is discharge (m^3/s), h is head (m), $P(Q)$ and $P(h)$ are penalty terms, α_1 and α_2 are weighting factors and N_w is total number of wells.

Minimization of pumping cost

The total cost of new system of pumping wells consists of the cost of well installation and cost of pumping. The major factors that influence the pumping cost depend on the volume of water to be pumped, density of the water, hydraulic head, efficiency of the pump and energy cost (Moradi *et al*., 2003; Sharma and Swamee, 2006). The total cost of pumping consists of the cost of pump units and the capitalized electricity cost (pumping cost) including the annual repair and maintenance cost and can be expressed (Swamee and Sharma, 1990; Swamee, 1996) for a single well as

$$C_{pum} = Min\left\{\sum_{i=1}^{N_w}\left(k_P\frac{\gamma Q_iH_i}{\eta} + \frac{8.76R_E\gamma Q_iH_ir_T}{\eta}\right) + a_1P(h) + a_2P(Q)\right\} \quad (2)$$

where γ = density of the fluid (N/m^3); H = pumping head (m), which is equal to the head from water table in aquifer to the height of storage tank including head losses in pipes; η = combined efficiency of the pump and the prime mover; R_E = the cost of the electricity per kilowatt hour (€/kwh); r = the rate of interest expressed as Euros per Euros per year (€/€/year); T = life of project (year); and k_P = cost of a pump per unit kilowatt hour (€), which can be obtained by interpolating values from a curve between cost and pump capacity. For long life of project ($T \to \infty$) Equation (2) gives $r_T = 1/r$. The pump parameters have been ascertained from market surveys.

Minimization of piping cost

Total cost of new pipe system consists of the cost of piping network to carry the water from pumping wells to storage tank. The piping cost depends on the location of new wells, the cost of the earthwork, the cost of pipes, jointing separate pipe sections by welding, applying an outer insulation, lowering the pipeline into a trench and filling. In this study, the piping length was considered from the wells to a reference location only, and all the pipes were considered as of the same diameter and material. The reference location consists of a water storage tank where water from all the wells will be stored and subsequently transported for water supply.

$$C_{pip} = Min\left\{\sum_{i=1}^{N_w}(A_2L_i) + a_1P(h) + a_2P(Q)\right\} \quad (3)$$

A_2 = cost of piping (€/m), L = total length of pipes.

The constraints incorporated into the model are as follows

$$Q_{i,\min} < Q_i < Q_{i,\max} \quad (3a)$$

$$\sum_{i=1}^{N_w} Q_i > Q_{total} \quad (3b)$$

$$h_i > h_{i,\min} \quad (3c)$$

$$(x_i, y_i) \neq A_i \quad (3d)$$

$$\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \geq S_{w,\min} \quad (3e)$$

$$P(h) = \begin{cases} h_{i,\min} - h_i & \text{if} \quad h_i < h_{i,\min} \\ 0 & \text{if} \quad h_i \geq h_{i,\min} \end{cases} \quad (3f)$$

$$P(Q) = \begin{cases} Q_{tot} - \sum Q_i & \text{if} \quad \sum Q_i < Q_{tot} \\ 0 & \text{if} \quad \sum Q_i \geq Q_{tot} \end{cases} \quad (3g)$$

where $Q_{i,min}$ and $Q_{i,max}$ are the minimum and maximum discharge limits for ith well; $h_{i,min}$ = minimum allowable head of groundwater at ith well; $S_{w,min}$ = minimum distance between any pair of wells; and x_j and y_j are coordinates of remaining well, that is, $i \neq j$.

DESCRIPTION OF AEM-PSO MODEL

Groundwater management problems are solved using the simulation-optimization approach. In this approach, the coupling of a groundwater flow model with an optimization model is used to find out the best management practices to address the groundwater management problems. The groundwater flow model is used to simulate the flow and to check the constraints of the problem that are based on state variable, namely groundwater head. The objective function is evaluated using an optimization model, which, in turn, utilizes the flow model to satisfy the constraints.

Thus, the coupled simulation-optimization modelling is an iterative process involving considerable computing. Therefore, selecting the appropriate and efficient computational flow model is the key factor. In this study, an AEM for groundwater flow was adopted (Gaur *et al*., 2011a, b). Majumder and Eldho (2013) mentioned the advantages of AEM over grid methods. They mentioned that the computational effort in AEM depends on the number of features and their discretization level but not on the spatial extent of the domain, thus making it possible to model the main features of large geographic areas at high resolution without excessive computation time. Olsthoorn (1999) performed comparative analysis of AEM and finite difference method (FDM) based models and concluded that the AEM model was more efficient than the FDM based model. Similar views are expressed by Strack (2003); Matott *et al*. (2006); Bandilla *et al*. (2007). In this study, the PSO was used as the optimization model, which was coupled with the AEM for getting at the head. Brief descriptions of AEM and PSO are given in the succeeding text.

Analytic element method

The AEM uses superposition of analytic solutions to basic flow features called 'analytic elements.' (Strack, 1989; Haitjema, 1995). AEM is based on potential theory where the discharge potential of a given aquifer is determined by superimposing the contribution from individual analytic elements that correspond to particular hydraulic feature (e.g. wells, rivers and lakes). The analytic elements are line sinks and point sinks. Surface waters are often represented by strings of line sinks and wells are represented by point sinks. Each type of geohydrological feature can be simulated by corresponding analytic element, for example, extraction wells by point sinks, rivers by line sinks element, infiltration areas by strings of line sinks, inhomogeneity by line-doublet and recharge by area-sink. The (potentiometric) head and groundwater flow in the aquifer is then obtained by adding the contributions of all these analytic elements.

In the AEM, groundwater flow is often expressed in terms of a complex potential Ω (m^3/s) as

$$\Omega = \Phi + i\psi \tag{4}$$

where discharge potential Φ (m^3/s) and the stream function Ψ (m^3/s) fulfil the Cauchy–Riemann condition, and therefore, Φ and Ψ may be represented as the real and the imaginary parts of an analytic function $\Omega = \Omega(z)$ of the complex variable $z = x + iy$, defined in the flow domain. Finally, the potential is converted into head.

The mathematical details of the AEM are found in Strack (1989) and Haitjema (1995). The MATLAB (MathWorks Inc. 2001) platform was used to implement the AEM.

Particle swarm optimization

The PSO is an efficient method for solving large nonlinear, complex global optimization problems and, in some cases, performs more efficiently compared with other evolutionary computation techniques (Kennedy *et al*., 2001). Analogously, it can be compared with birds searching for food, which consider two factors to achieve their goal: their own previous best experience (i.e. *pbest*) and the best experience amongst all other members (*gbest*). This is also similar to human behaviour in decision making when people consider their own best past experience and the best experience of other people around them. The working steps of the PSO method for the solution of any optimization problem are as follows:

1. Initialize a population (array) of particles with random positions and velocities for the dimensions in the problem space. Let $X_i(t)$ denote the position of particle i in an n-dimensional vector, where n is the number of optimization variables (dimension of search space) and t denotes the time step (generation). A particle can be described as $X_i(t) = (x_{i1}, x_{i2}, \ldots, x_{in})$. Each particle has an n-dimensional vector for velocity, and can be described as $V_i(t) = (v_{i1}, v_{i2}, \ldots, v_{in})$, where $i = 1, 2, \ldots, K$ and K is the size of the swarm. Decision variables in PSO are accounted by the dimension of each particle, whereas the velocity and the position of the particle is calculated by Equations (5) and (6), respectively

$$v_{ij}^t = \chi\left[\omega v_{ij}^{t-1} + c_1 r_1\left(P_{ij}^{t-1} - x_{ij}^{t-1}\right) + c_2 r_2\left(G_j^{t-1} - x_{ij}^{t-1}\right)\right] \tag{5}$$

$$x_{ij}^t = x_{ij}^{t-1} + v_{ij}^t \tag{6}$$

where c_1 and c_2 are acceleration constants, and r_1 and r_2 are random real numbers between 0 to 1. P and G^t denotes the *pbest* and *gbest* values of particles. Thus, the particle flies through potential solutions towards P and G^t in a navigated way whilst still exploring new areas by the stochastic mechanism to escape from local optima. ω is called inertia weight, which is used to control the impact of the previous history of velocities on the current one. χ is the constriction coefficient, which is used to restrain velocity.

2. Determine the fitness value of each particle by a fitness function.
3. Compare each particle's fitness evaluation with the particle's *pbest*. If the current value is better than *pbest*, then set the *pbest* value as equal to the current value and the *pbest* location equal to the current location in d-dimensional space.
4. Compare the fitness evaluation with the population's overall previous best. If the current value is better than *gbest*, then reset *gbest* to the current particle's array index and value.
5. Change the velocity and position of the particle that depends on total number of swarms, acceleration constants, constriction coefficient and random numbers

(between 0 to 1).

6. Repeat the process until a sufficiently good fitness or maximum number of iterations (generations) are reached.

Because there is no actual mechanism for controlling the velocity of a particle, it is necessary to control the maximum travel distance in each iteration to avoid the particle flying past good solutions. Also, after updating the positions, it must be checked that no particle violates the boundaries of the search space. If a particle has violated the boundaries, it will be set at the boundary of the search space. The PSO model was developed on the MATLAB platform (MathWorks Inc. 2001).

MULTIOBJECTIVE FUZZY OPTIMIZATION APPROACH

In the present study it is assumed that objectives are imprecise and uncertain, and so can be represented by fuzzy sets in the form of membership functions (Raju and Nagesh Kumar, 2014). Numerous membership functions are available such as nonlinear/linear, hyperbolic, exponential, trapezoidal, triangular, spike, gaussian, cosine, sigmoid and increasing concave (Shinghal, 2013). All these membership functions handle the impreciseness in objectives or constraints or both effectively as the case may be. These membership functions are based on the lower and upper bounds of each objective function and parameters relevant to each of them. In the present study, nonlinear/linear membership function is employed to explore their applicability for the present planning problem (Morankar *et al.*, 2013) and shown in Figure 1(a to c) (Ross, 1995; Raju and Nagesh Kumar, 2014). Decision space in fuzzy environment is based on objectives and constraints (Raju and Nagesh, 2014). By introducing degree of membership,

$$\mu_D = (\mu_G \cap \mu_C) \tag{7}$$

where μ_D, μ_G, μ_C represent degree of membership functions for decision space, objective function and constraints. With more number of fuzzy objective function(s) and constraint(s), the equation can be expanded to

$$\mu_D(X) = [\mu_{G1}(X) \cap \mu_{G2}(X) \cap ....\cap \mu_{Gn}(X) \cap \mu_{c1}(X) \cap \mu_{c2}(X) \cap ....\cap \mu_{cm}(X)] \quad or$$
$$\mu_D(X) = Min[\mu_{G1}(X), \mu_{G2}(X), \; ...., \mu_{Gn}(X), \mu_{c1}(X), \mu_{c2}(X), \; ...., \mu_{cm}(X)] \tag{8}$$

Optimum solution $\mu_D^*(X) = Max[(\mu_D(X)]$ or

$$\mu_D^*(X) = Max[Min(\mu_{G1}(X), \mu_{G2}(X), \; ...., \mu_{Gn}(X), \mu_{c1}(X), \mu_{c2}(X), \; ...., \mu_{cm}(X))] \tag{9}$$

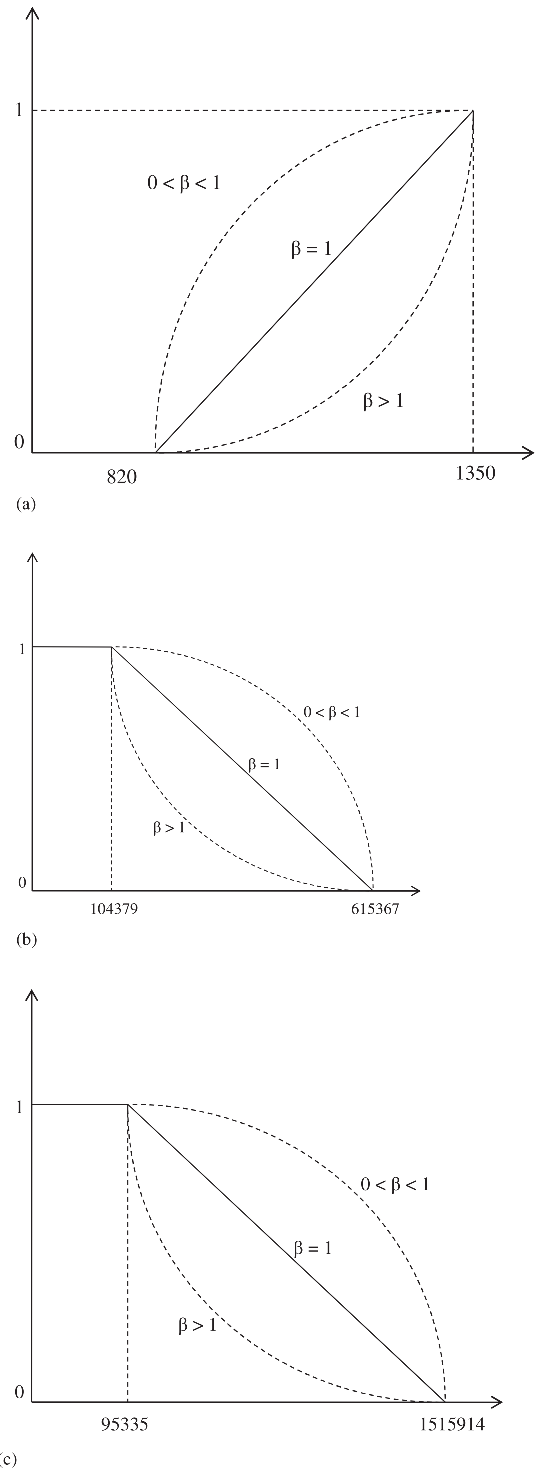


Figure 1. (a) Nonlinear/linear membership function for discharge (maximization). (b) Nonlinear/linear membership function for pumping cost (minimization). (c) Nonlinear/linear membership function for piping cost (minimization)

Nonlinear/linear membership functions $\mu_Z(X)$ for any objective *Z*/constraint (either maximization or minimization) can be expressed as

For maximization (such as benefits)

$$\mu_Z(X) = 0 \text{ for } Z \le Z_L$$
$$\mu_Z(X) = \left[\frac{Z - Z_L}{Z_U - Z_L}\right]^\beta \text{ for } Z_L \le Z \le Z_U$$
$$\mu_Z(X) = 1 \text{ for } Z \ge Z_U$$

For minimization (such as costs)

$$\mu_Z(X) = 1 \text{ for } Z \le Z_L$$
$$\mu_Z(X) = \left[\frac{Z_U - Z}{Z_U - Z_L}\right]^\beta \text{ for } Z_L \le Z \le Z_U \quad (10)$$
$$\mu_z(X) = 0 \text{ for } Z \ge Z_U$$

$\mu_Z(X)$ reflects the degree of achievement for the objective/constraint. Highest and lowest acceptable levels of the objective, obtained with individual optimization, are denoted by $Z_{U,}$ Z_L with β representing the shape of the desired membership function. Assigning value of 1 to β leads to linear membership function whereas any other value leads to nonlinear membership function (Sasikumar and Mujumdar, 1998; Raju and Nagesh Kumar, 2014). Introducing a new variable, λ, the problem can be expressed as

$$f(x) = \text{Max } (\lambda) \quad (11)$$

subject to

$$\left[\mu_{G_J}(X)\right]^\beta \ge \lambda \quad \text{for each fuzzy objective function } j = 1, 2, \ldots.., n \quad (11a)$$

$$\left[\mu_{C_i}(X)\right]^\beta \ge \lambda \quad \text{for each fuzzy constraint } i = 1, 2, \ldots, m \quad (11b)$$

$$0 \le \lambda \le 1 \quad (11c)$$

and all other existing constraints (Equation (3a) to (3g)) and bounds.

STUDY AREA

The present study is carried out for the town of Thiers, which is one of the major towns in the Loire region, France. The study area lies between 45°54^0 N to 46°N latitude and 3°25^{0}E to 3°29^{0}10^{00}E longitude. The Dore river catchment, which is situated in the eastern part of the Massif-Central in France (Figure 2), was considered to establish the new pumping wells to fulfil the water demand of Thiers. The study area consists of two rivers, that is, Allier and Dore, where the Dore River is an important tributary of the Allier river. The low-flow period in the river occurs in summer, that is, June–August but can extend up to November. The major part of the area is covered by fluvial quaternary sediments underlain by marl and clay. The quaternary alluvium is composed of gravel, sand and pebbles with silt. The impervious substratum is composed of clay and sand (oligocene period). The hydraulic conductivity in the domain varies from 1×10^{-3} to 3×10^{-3} m/s, whereas the thickness of the aquifer varies from 12 to 15 m (Bertin *et al*., 2009). The elevation of the bottom impervious layer of aquifer varies from 254 to 258 m from mean sea level. The location of different hydrological features and other required data were extracted from the geological maps provided by the Bureau de Recherché Géologiques et Minières (Bertin *et al*., 2009). A total of 12 piezometric measurements, which are shown in Figure 2, were considered to calibrate the groundwater flow model. Results of single-objective optimization and MOFO perspective are discussed in detail in the next section.

RESULTS AND DISCUSSION

Single-objective optimization

Initially, the developed model was applied one at a time on three objectives related to maximum discharge, minimum pumping cost and minimum piping cost, which are denoted as QMAX, PUMMIN and PIPMIN, respectively. Three objective functions are optimized individually as single-objective optimization problems (subject to constraints 3a to 3g) to determine the maximum and minimum values that can be possible for each objective.

In the cost function, $A_2 = 140$ €/m was obtained from experienced field experts. The values of other parameters were selected as $R_E = 0.08$ €/kwh, $\gamma = 9810$ N/m^3, $\eta = 80\%$, $r = 6\%$ €/€/year and $T = 25$ years. The location of storage tank was fixed after consulting the local authority having coordinates $X = 687\,000$ and $Y = 218\,000$ for computing the pipe length. The problem constraints were finalized with the help of water authority officials and stakeholders. Constraint 3a prescribes the maximum and minimum discharge limit of a single well. On the basis of aquifer properties and availability of pumps, the discharge limits were selected at 100 m^3/h $< Q_i <$ 270 m^3/h. Constraint 3b defines the minimum water discharge from all wells, which is taken as $Q_{tot} >$ 820 m^3/h as per the water demand of the city. Constraint 3c was to limit the drawdown of groundwater within permissible limit defined by stakeholders, thus $h_i >$ 262 m. Constraint 3d was incorporated by creating buffer zones of 145 m around the river, and then the penalty was incorporated if the coordinates of the wells came within the buffer zones. Constraint 3e accounts for the minimum distance of the wells from the river to ensure the minimum retention time for the

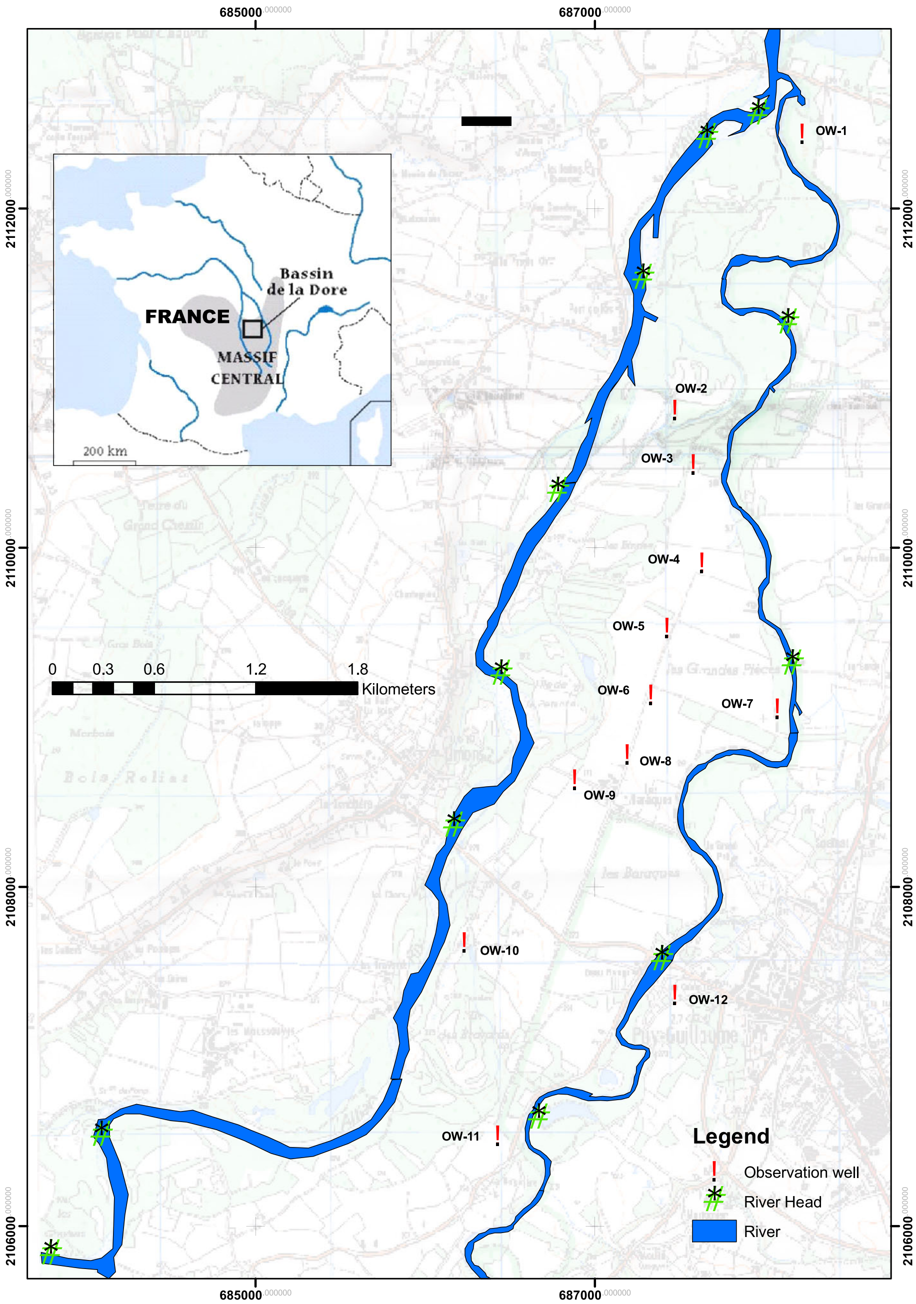


Figure 2. Map of study area

groundwater in the aquifer and to avoid the influence of pumping on the water level in the river.

The AEM model was developed with given line sinks and wells with constant hydraulic conductivity in the whole domain. The rivers considered in the study area were represented by strings of head-specified line sinks (39 in total). The water level in the river is monitored at 11 different locations (Figure 2) and was used to specify the heads at the centres of the line sinks that represent the river. The discharge wells were represented by well elements (i.e. point-sinks). In the model, a constant elevation for the bottom layer was taken as 257 m. The model was run in the steady state condition for the low-flow period, that is, June 2007. This period was found to be suitable for the steady state model run as the river condition was found to be almost stable along with a static groundwater head. The computed heads were compared with observed heads at 12 different locations in the domain to calibrate the model with real field conditions (Gaur *et al.*, 2011a, b, 2013). Figure 3 shows the graph of computed (AEM model) and observed values of groundwater head. The model was found to be well

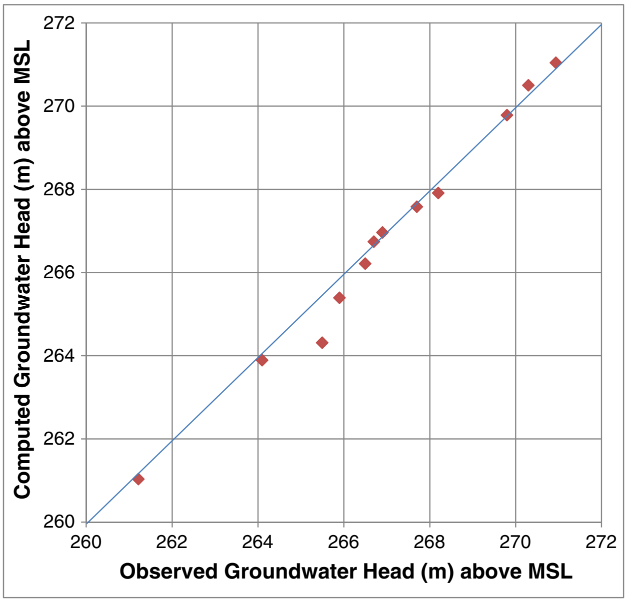


Figure 3. Calibration graph between observed and computed groundwater head

calibrated, as calculated heads and observed heads were analyzed with a 95% confidence level. In the calibration process, a hydraulic conductivity value was adjusted systematically, and the model output was compared with observed values. The results of the model showed that changing the hydraulic conductivity values up to 25% did not affect the groundwater head more than 1 m. In addition, in the calibration process, part of the river outside the area of interest was included, and its effect on groundwater head was examined. This process helps to establish the effect of far field features on the area of interest. The model was not found to be very sensitive for the far features as the area of interest is located between the two neighbouring rivers.

The PSO model was run for sets of 2, 3, 4, 5 and 6 wells, and the overall system cost (pumping and piping cost) for the given set of wells was computed. The model was run for different sets of PSO particles, and 25 particles were found as being efficient for the solution. The PSO model was considered converged when the value of the objective function did not change for 50 iterations. PSO parameters, that is, inertia weight, was taken as 0.8 to 0.4 and two acceleration constants were taken as 2. Ten runs were performed for each set of wells. A minimum value out of 10 runs was considered as an optimal solution. Set of 2 and 3 wells was not found feasible to satisfy the water demand of the city, that is, 820 m^3/h because of the dependence of high pumping rates of an individual well. Decision variables for the set of 4, 5 and 6 wells were taken at 12, 15 and 18, respectively, which consist of the discharge and *X* and *Y* coordinates of the wells. The results from different sets of wells were compared, and the set of 5 wells was identified as an optimal number of wells. Further, the single-objective optimization and MOFO were carried out for the set of 5 wells.

Salient results including discharge, pumping cost and piping cost are presented in Table I. The notations '*' and '**' represent the upper and lower bounds (maximum and minimum values) for each objective. Summarized points as observed from Table I are the following:

1. In QMAX case, discharge for wells 1 to 5 is 270 m^3/h (totaling to 1350 m^3/h), which is the maximum discharge limit for each well; in the case of PUMMIN, these are 120.1, 144.1, 176.3, 176.4, 203.1 m^3/h (totaling to 820 m^3/h); and 120.0, 155.5, 171.8, 172.3, 200.4, respectively for the case of PIPMIN (totaling to 820 m^3/h).
2. In QMAX case, head values (respectively for well 1 to well 5) are 270.1, 270.0, 269.2, 268.7, 267.6 m; In the case of PUMMIN, these are 270.8, 270.7, 269.5, 270.2, 270.9, respectively, for wells 1 to 5. These are 262.0, 262.7, 262.5, 262.6 and 262.8, respectively, for the case of PIPMIN.
3. It is observed that head values in QMAX case is close to PUMMIN as in both cases the wells are located in southern part of the aquifer, which has higher water head in comparison with other parts of the aquifer.
4. Piping length in QMAX case is 10514 m followed by 10828 m in the case of PUMMIN and 681 m in the case of PIPMIN. Piping cost per metre length is adopted as same for all three cases, that is, €140.
5. It is observed from Table I that discharges in QMAX case is 1.64 times more than in PUMMIN and PIPMIN cases. Pumping cost in PUMMIN case is 0.34 times of QMAX case and 0.169 times of PIPMIN case. Piping cost in QMAX case is 0.698 and 11.1 times of PUMMIN and PIPMIN cases, respectively.
6. It is noticed that all three individual objectives have satisfied the minimum requirements (3a to 3g) such as discharge, drawdown limit and other related values whilst optimizing (maximizing or minimizing) the individual objectives.
7. It is observed from Table I that the three groundwater management objectives are conflicting in all aspects as evident from different discharges, heads, pumping and piping cost values obtained for each objective.

Conflicting nature of objectives necessitate developing tradeoff relationships for selecting a compromise groundwater management plan, which is expected to provide realistic and acceptable strategy that can be implemented by a policy maker with much ease. MOFO is found to be suitable to analyze the conflicting situation because of its ability to (1) incorporate any number of objectives with

Table I. Salient results of the ground water management with various optimization models

Salient features	Single-objective optimization			Multiobjective fuzzy optimization		
	Maximum discharge QMAX	Minimum pumping cost PUMMIN	Minimum piping cost PIPMIN	$\beta_1=1.0$ $\beta_2=1.0$ $\beta_3=1.0$	$\beta_1=0.5$ $\beta_2=0.5$ $\beta_3=0.5$	$\beta_1=2$ $\beta_2=2$ $\beta_3=2$
Well 1: discharge (m^3/h)	270.0	120.1	120.0	170.2	196.9	150.0
Well 2: discharge (m^3/h)	270.0	144.1	155.5	207.8	265.8	189.6
Well 3: discharge (m^3/h)	270.0	176.3	171.8	232.1	230.5	255.4
Well 4: discharge (m^3/h)	270.0	176.4	172.3	254.2	177.2	266.4
Well 5: discharge (m^3/h)	270.0	203.1	200.4	268.8	262.2	268.1
Well 1: head (m)	270.1	270.8	262.0	266.3	265.2	268.7
Well 2: head (m)	270.0	270.7	262.7	266.9	270.3	266.3
Well 3: head (m)	269.2	269.5	262.5	267.3	268.2	267.8
Well 4: head (m)	268.7	270.2	262.6	270.3	266.3	269.9
Well 5: head (m)	267.6	270.9	262.8	270.4	270.8	269.2
Well 1: pumping cost (€)	40 777	11 455	95 386	76 980	105 647	39 585
Well 2: pumping cost (€)	42 924	14 890	114 952	84 075	34 650	86 357
Well 3: pumping cost (€)	60 093	35 034	129 733	85 974	68 708	85 875
Well 4: pumping cost (€)	70 824	25 239	128 741	32 935	80 146	45 528
Well 5: pumping cost (€)	94 432	17 758	146 552	33 759	25 010	59 458
Piping length (m)	10 514	10 828	681	4832	4840	4900
Total discharge (m^3/h)	1350**	820*	820*	1133.1	1132.6	1129.5
Total pumping Cost (€)	309 054	104 378*	615 368**	313 724	314 161	316 804
Total piping cost (€)	1 058 689	1 515 914**	95 335*	676 487	677 544	686 000
Total piping cost /m (€)	140.00	140.00	140.00	140.00	140.00	140.00
λ				0.59	0.76	0.34

**Maximum/
*Minimum

ease and without much computational burden (as each additional objective will be transformed into additional constraint) and (2) conversion of multiobjective problem as single objective with much ease (Raju and Nagesh Kumar, 2014). Details of MOFO model formulation and corresponding results are presented in the next section.

Multiobjective optimization

A MOFO model is applied to the present case study of Dore river catchment, France and formulation of MOFO model for the management problem with three objectives is

$$f_x = Max\{(\lambda\} - a_1 P(Z_Q) - a_2 P\left(Z_{pummin}\right) - a_3 P\left(Z_{pipmin}\right) \quad (12)$$

where $P(Z_Q)$, $P(Z_{pummin})$, $P(Z_{pipmin})$ are penalty terms, which vary linearly with the magnitude of constraint violation and α_1, α_2, α_3 are weighting factors, which can be selected according to the problem. Model is subjected to the following constraints, namely,

Constraint related to discharge (transformed objective function 1)

$$\left[\mu_{z1}(X)\right]^{\beta_1} \geq \lambda \ or \ \left[\frac{Z_Q - 820}{1350 - 820}\right]^{\beta_1} \quad (13)$$

$$P(Z_Q) = \begin{cases} \left[\mu_{z1}(X)\right]^{\beta_1} - \lambda & if \ \left[\mu_{z1}(X)\right]^{\beta_1} < \lambda \\ 0 & if \ \left[\mu_{z1}(X)\right]^{\beta_1} \geq \lambda \end{cases} \quad (14)$$

Z_Q = Objective function equation related to discharge (shape of membership function is presented in Figure 1(a))

Constraint related to pumping cost (transformed objective function 2)

$$\left[\mu_{z2}(X)\right]^{\beta_2} \geq \lambda \text{ or } \left[\frac{615368 - Z_{Pumping\text{cost}}}{615368 - 104378}\right]^{\beta_2} \geq \lambda \quad (15)$$

$$P\left(Z_{\text{pummin}}\right) = \begin{cases} \left[\mu_{z1}(X)\right]^{\beta_1} - \lambda & if \ \left[\mu_{z2}(X)\right]^{\beta_2} < \lambda \\ 0 & if \ \left[\mu_{z2}(X)\right]^{\beta_2} \geq \lambda \end{cases} \quad (16)$$

Z_{pummin} = Objective function equation related to pumping cost (shape of membership function is presented in Figure 1(b))

Constraint related to piping cost (transformed objective function 3)

$$\left[\mu_{z3}(X)\right]^{\beta_3} \geq \lambda \text{ or } \left[\frac{1515914 - Z_{piping\text{cost}}}{1515914 - 95335}\right]^{\beta_3} \quad (17)$$

$$P\left(Z_{\text{pipmin}}\right) = \begin{cases} \left[\mu_{z1}(X)\right]^{\beta_1} - \lambda & \text{if } \left[\mu_{z3}(X)\right]^{\beta_3} < \lambda \\ 0 & \text{if } \left[\mu_{z3}(X)\right]^{\beta_3} \geq \lambda \end{cases} \quad (18)$$

Z_{pipmin} = Objective function equation related to piping cost (shape of membership function is presented in Figure 1(c))

$$0 \leq \lambda \leq 1 \quad (19)$$

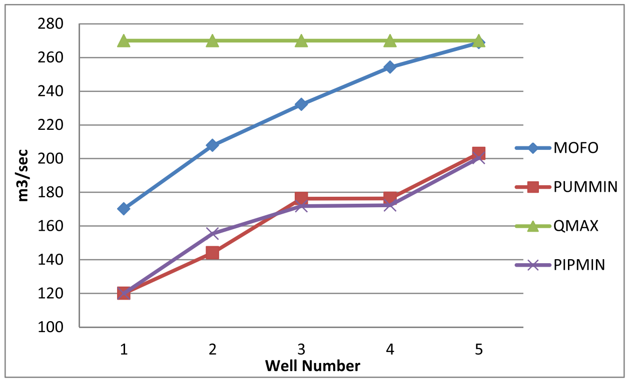
and existing constraint set and bounds (Equations (3a) to (3g)) in the management model. $\mu_{z1}, \mu_{z2}, \mu_{z3}$ are membership functions for maximum discharge, minimum pumping cost and minimum piping cost, respectively.

Results of MOFO with $\beta_1 = \beta_2 = \beta_3 = 1.0$ are presented in Table I and Figure 4 (a to c). Summarized points as observed from Table I with reference to MOFO ($\beta_1 = \beta_2 = \beta_3 = 1.0$) are

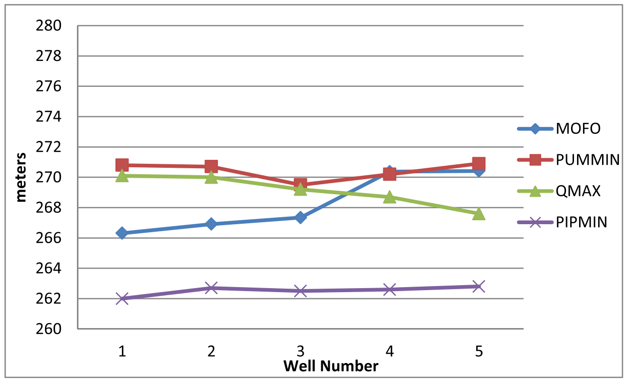
1. It is observed from MOFO that solution gives discharges for wells 1 to 5 as 170.2, 207.8, 232.1, 254.2, 268.8 (totalling to 1133.1 m^3/h). These values are less than for QMAX and more than PIPMIN and PUMMIN cases.
2. Head values (respectively for well 1 to well 5) are 266.3, 266.9, 267.3, 270.3 and 270.4 m. Values obtained from MOFO solution are more than PIPMIN and less than QMAX and PUMMIN.
3. Pumping cost in MOFO solution (€313 724) is more than QMAX (€309 054) and PUMMIN (€104 378) and less than PIPMIN (€615 368).
4. Piping cost in MOFO solution (€676 487) is less than QMAX (€1 058 689) and PUMMIN (€1 515 914) and more than PIPMIN (€95 335).
5. Piping cost per metre length is same as single-objective case, that is, €140.
6. It is observed that discharge has decreased by 216.9 m^3/h, pumping cost increased by €209 346, and piping cost increased by €581 152 as compared with the ideal values obtained with single objective analysis with degree of satisfaction λ value of 0.59.

Sensitivity analysis

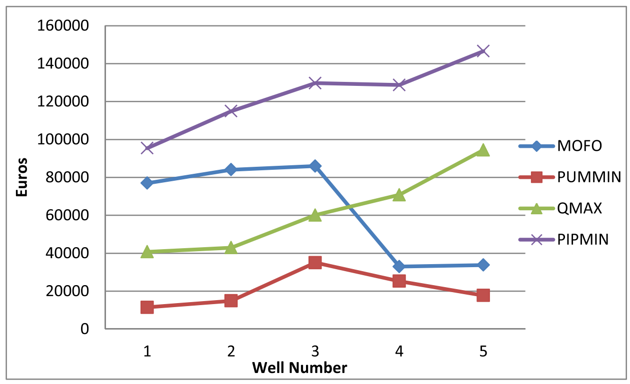
Two scenarios are investigated as part of sensitivity analysis: effect of exponents $\beta_1 = \beta_2 = \beta_3 = 0.5$ (termed as first scenario) and $\beta_1 = \beta_2 = \beta_3 = 2.0$ (termed as second scenario). It is observed from Table I that with reference to scenario 2, discharge, pumping cost and piping cost are 1129.5 m3/h, €316 804, €686 000, respectively, with λ value of 0.34, whereas for scenario, 1 these are 1132.6 m3/h, €314 161, €677 544 with λ value of 0.76. Results shows that total discharge has decreased by 220.5 and 217.4 m^3/h as compared to QMAX and pumping cost increased by €212 426 and €209 783 as compared to



(a)



(b)



(c)

Figure 4. (a) Values of discharge for QMAX, PUMMIN, PIPMIN, and multiobjective fuzzy optimization (MOFO) cases. (b) Values of head for QMAX, PUMMIN, PIPMIN, and MOFO cases. (c). Values of pumping cost for QMAX, PUMMIN, PIPMIN, and MOFO Cases

PUMMIN in scenarios 2 and 1, respectively. Results show that effects of exponents are having significant impact on the outcome.

Effort was made to see the effect of fixing degree of satisfaction of 0.5 on the discharge, pumping and piping cost for scenarios $\beta_1 = \beta_2 = \beta_3 = 0.5$, $\beta_1 = \beta_2 = \beta_3 = 2.0$ and $\beta_1 = \beta_2 = \beta_3 = 1.0$. It is observed that for $\beta_1 = \beta_2 = \beta_3 = 1.0$ scenario, discharge, piping and pumping costs are 1085 m^3/h, €805 624.5, €359 873, respectively. These

Table II. Salient results of the ground water management with sensitivity analysis

Salient features	Multiobjective fuzzy optimization				
	Case 1	Case 2	Case 3	Case 4	Case 5
Well 1: discharge (m^3/h)	267.5	170.2	238.1	243.4	223
Well 2: discharge (m^3/h)	191.9	207.8	229.5	171.6	216.2
Well 3: discharge (m^3/h)	161.2	232.1	268.1	264.6	268
Well 4: discharge (m^3/h)	249.3	254.2	207.4	235	234.2
Well 5: discharge (m^3/h)	232.2	268.8	207.1	241.3	220
Well 1: head (m)	270.9	271.9	266.3	267.6	267.2
Well 2: head (m)	266.7	267.8	266.9	268	266.5
Well 3: head (m)	266.2	268.5	267.3	270.5	267.2
Well 4: head (m)	271.8	265.9	270.3	265.3	266.3
Well 5: head (m)	267.2	267.0	270.4	266.6	270
Well 1: pumping cost (€)	24 028	76 980	74 948	85 129	85 085
Well 2: pumping cost (€)	80 693	84 075	112 010	54 561	94 520
Well 3: pumping cost (€)	74 703	85 974	5967	31 549	102 255
Well 4: pumping cost (€)	4161	32 935	60 668	125 155	106 113
Well 5: pumping cost (€)	88 226	33 759	107 827	103 576	34 975
Minimum pumping cost	104 379	104 379	114 816.9	163 111	220 371
Maximum pumping cost	461 525.25	615 367	769 209	810 910	810 910
Total discharge (m^3/h)	1102.1	1133.1	1150.2	1155	1161
Total pumping cost (€)	271 812	313 724	361 421	399 971	422 948
Piping length (m)	5427	4832	4502	4390	4214
Total piping cost (€)	759 714	676 487	630 315	614 635	590 030
λ	0.53	0.59	0.62	0.63	0.65

values are changed by $-48.1\,m^3$/h, €129 137.5, €46 149 as compared with $\beta_1=\beta_2=\beta_3=1.0$ with degree of satisfaction value of 0.59. In case of $\beta_1=\beta_2=\beta_3=0.5$, discharge, piping and pumping costs are $952.5\,m^3$/h, €1 160 769, €487 620.5 and changed by $-180.1\,m^3$/h, €483225, €173459.5 as compared with $\beta_1=\beta_2=\beta_3=0.5$ with degree of satisfaction value of 0.76. In case of $\beta_1=\beta_2=\beta_3=2.0$, discharge, piping and pumping costs are $1194.76\,m^3$/h, €511 413, €254 043.5 and changed by $65.26\,m^3$/h, €−174587, €62760.5 as compared with $\beta_1=\beta_2=\beta_3=2.0$ with degree of satisfaction value of 0.34.

Sensitivity analysis is also performed by varying the minimum and maximum limits of pumping cost/piping cost. It is observed from Table II that by increasing lower limit or upper limit of pumping cost, MOFO solution increases the pumping cost. It shows that increased pumping cost limit pushes wells towards storage location where more drawdown in aquifer occurred, which helps to reduce piping cost as well. At the same time, increased limit in pumping cost also helps to extract more water from aquifer.

Sensitivity analysis is also performed by varying the upper limit of piping cost, and it is observed that (1) reduced piping cost limit increases the value of optimized pumping cost with reduced total discharge limit; and (2) decreased piping cost limit pushes the wells near storage tank where pumping cost increases because of more drawdown in aquifer. Because of this reason, total discharge also gets reduced.

It is observed from sensitivity analysis that variations of exponents, upper and lower limits of pumping and piping cost have significant impact on the MOFO solution and degree of satisfaction, which clearly indicates that careful selection of exponents and other parameters are very much essential to make study more meaningful and practical.

SUMMARY AND CONCLUSIONS

The present paper discussed the application of MOFO in groundwater management context for the chosen case study with three conflicting objectives along with relevant sensitivity analysis. To the authors' knowledge, present study is the first of its kind where MOFO is applied to groundwater management problem in conjunction with coupled AEM and PSO. MOFO methodology is found to be advantageous as compared with single-objective management problems, as it can incorporate any number of objectives simultaneously with ease and without much computational burden. The additional and potential advantage is its ability to consider the uncertainty aspects that are very common in the groundwater planning in the form of nonlinear/linear membership functions. The following conclusions may be drawn from the study:

1. Analyses of the results indicate that for $\beta_1=\beta_2=\beta_3=1.0$, that discharge has decreased by $216.9\,m^3$/h, pumping cost

increased by €209 346, and piping cost increased by €581 152 as compared with the ideal values obtained with single-objective analysis with degree of satisfaction $\lambda = 0.59$. It is observed that for $\beta_1 = \beta_2 = \beta_3 = 2.0$, discharge, pumping cost and piping cost are 1129.5 m^3/h, €316 804 and €686000, respectively, with $\lambda = 0.34$, whereas for $\beta_1 = \beta_2 = \beta_3 = 0.5$, these are 1132.6 m^3/h, €314 161 and €677 544 with $\lambda = 0.76$. It can be inferred that as β values are increasing, λ values are decreasing significantly.

2. Analysis of results indicates that all the results obtained through MOFO are compromising solutions for the three conflicting objectives and the solution of MOFO always lies within the values obtained by QMAX, PUMMIN and PIPMIN.
3. Sensitivity analysis shows that variations in minimum, maximum limits of pumping cost influence the MOFO solution significantly. It shows that MOFO solution tries to accommodate the varying limits of pumping or piping cost by changing the optimal values of total discharge, pumping and piping. Sensitivity analysis shows that the developed model can be used by decision makers as they can understand the effect of different financial limits of project in pumping and piping cost.
4. It is observed from sensitivity analysis that effects of exponents, upper and lower limits of pumping and piping cost have significant impact on the MOFO solution and degree of satisfaction, which clearly indicates that careful selection of exponents and other parameters are very much essential to make study more meaningful and practical.

The present study in single and multiobjective environment provided a complete perspective on piping, pumping, discharge and impact of constraints on them. This will help the water resources planners and managers to understand the quantity of water that can be pumped from wells to augment the surface water resources as part of conjunctive use policy. In addition, farmers or other stakeholders will also be in a position to know the quantity of water they may receive for the respective purposes such as drinking/irrigation. This may lead to sustainable, scientific and replicable planning, which further enhances the efficiency of the outcome. This is mainly because of the employability and potentiality of mathematical models such as AEM, PSO and multiobjective perspective in fuzzy environment. . Further studies can be explored with various other type membership functions such as exponential and hyperbolic membership functions (Morankar *et al*., 2013).

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