J. Astrophys. Astr. (1994) **15,** 143–156

Seismology of the Solar Convection Zone

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Received 1993 October 5; accepted 1994 April 15

Abstract. An attempt is made to infer the structure of the solar convection zone from observed p-mode frequencies of solar oscillations. The differential asymptotic inversion technique is used to find the sound speed in the solar envelope. It is found that envelope models which use the Canuto-Mazzitelli (CM) formulation for calculating the convective flux give significantly better agreement with observations than models constructed using the mixing length formalism. This inference can be drawn from both the scaled frequency differences and the sound speed difference. The sound speed in the CM envelope model is within 0.2% of that in the Sun except in the region with $r > 0.99R_\odot$. The envelope models are extended below the convection zone, to find some evidence for the gravitational settling of helium beneath the base of the convection zone. It turns out that for models with a steep composition gradient below the convection zone, the convection zone depth has to be increased by about 6 Mm in order to get agreement with helioseismic observations.

Key words: Sun: oscillations—convection.

1. Introduction

Helioseismology has provided us with a tool to probe the interior of the Sun using the observed frequencies of solar oscillations. Although a lot of effort has been directed towards studying the deeper layers of the Sun, very little attention appears to have been paid to the envelope region below the solar surface. The structure of the surface layers is predominantly determined by convective heat transport. Although, there is no accepted theory of stellar convection, most stellar models are traditionally constructed using the mixing length theory (MLT), which is based on essentially *ad hoc* simplifying assumptions regarding turbulent convection. Recently, Canuto & Mazzitelli (1991) have proposed an alternative prescription of stellar convection based on a more detailed study of turbulence, which considers the full spectrum of turbulent eddies. In this paper we would like to examine if the available helioseismic data can distinguish between the MLT models and models constructed using the Canuto-Mazzitelli (CM) formulation. By comparing the frequencies of different solar models constructed using MLT and CM prescriptions with the observed frequencies, Paterno *et al.* (1993) conclude that the model constructed with the CM formulation gives better agreement with observations. However, in that work, they compared models obtained using stellar evolution codes, and as such, the results are liable to be affected by uncertainties in the theory of stellar evolution. In order to avoid this uncertainty, we use solar envelope models to study the difference between the CM and MLT formalisms. Further, we have used inversion techniques to identify the differences.

For most part of the solar convection zone, the temperature gradient is very close to the adiabatic gradient. Below the ionization zones of hydrogen and helium the adiabatic index is known reliably, and as a result, the uncertainties in the structure of this region are small. Beneath the convection zone, the temperature gradient depends on the opacity which is uncertain to some extent. Apart from the opacity there is a possibility of a composition gradient due to the gravitational settling of helium and metals (Cox, Guzik & Kidman 1989; Bahcall & Pinsonneault 1992; Christensen-Dalsgaard, Proffitt & Thompson 1993). The hope is that if we can reliably determine the structure of the solar convection zone using helioseismic techniques, it may be possible to infer uncertainties in opacity as well as in the composition gradient below the convection zone. Of course, it may not be possible to separate out the effect of composition gradient from that of the uncertainties in opacity and the depth of the convection zone.

In this work we use the differential asymptotic technique of sound speed inversion (Christensen-Dalsgaard, Gough & Thompson 1989) to determine the relative difference in the sound speed between the Sun and solar envelope models. From the sound speed difference we should be able to determine which model is closer to the Sun. We use envelope models in our analysis because a) they are relatively easy to construct, b) they are less affected by uncertainties in the theory of stellar evolution, c) it is possible to construct models with essentially arbitrary helium abundance and depth of the convection zone. In any case, the helium abundance and the depth of convection zone in the Sun can be independently determined from helioseismology. Although the depth of convection zone is known very reliably (Christensen-Dalsgaard, Gough & Thompson 1991), there is some uncertainty in the helium abundance (Vorontsov, Baturin & Pamyatnykh 1992; Kosovichev *et al.* 1992; Antia & Basu 1994 and references therein). Once the parameters of the envelope model are fixed, we can extend the model inwards towards the center. However, the composition gradient in the interior cannot be determined without constructing an evolutionary model, and hence the inward extension of the envelope model is uncertain. Nevertheless, it may be possible to distinguish between various models of helium diffusion using extended envelope models. It may be argued that the envelope model may not represent the Sun, since the boundary condition at the center may not be satisfied when the models are extended to the center. However, a small discrepancy in the central boundary condition can be attributed to uncertainties in the opacities, nuclear reaction rates and the composition profile.

2. The inversion technique

We have constructed several solar envelope models using different prescriptions for calculating the convective flux. Basically the envelope models depend on two parameters: the helium abundance Y, and the mixing length parameter, α. In conformity with the usual practice, in the models using MLT, we use the mixing length $L = \alpha H_p$, while for models with the CM formulation, we use $L = \alpha + z$, where z is the depth measured from the radius where the optical depth, $\tau = 1$. In the mixing length formalism, the convective flux is given by

$$F_C^{\rm MLT} = \frac{8\sigma T^4(\nabla - \nabla_{\rm ad})}{3\kappa\rho H_p} a_0 \Sigma^{-1}[(1+\Sigma)^{1/2} - 1]^3, \quad (1)$$

where a_0 = 9/4, and Σ is a quantity which depends on the superadiabatic gradient and is defined by equation (5) of Canuto & Mazzitelli (1991). In the CM formulation, the corresponding convective flux is expressed as

$$F_C^{\rm CM} = \frac{16\sigma T^4(\nabla - \nabla_{\rm ad})}{3\kappa\rho H_p} a_1 \Sigma^m[(1+a_2\Sigma)^n - 1]^p, \quad (2)$$

where, the coefficients a_1 = 24·868, a_2 = 0·097666, m = 0·14972, n = 0·18931, and p = 1·8503. This expression has been obtained by fitting the results of detailed numerical calculations of the spectrum of turbulent eddies and includes the contribution from eddies of all sizes. The main difference between the two formalisms occurs at the subsurface layer near the top of the convection zone where the CM formulation gives a steeper temperature gradient leading to the density inversion in a thin layer in that region. In the deeper layers, the superadiabatic gradient in CM models is much less than that in MLT models, however since the degree of super-adiabaticity is very small in both models, the temperature gradient is effectively adiabatic. Consequently, there is no measurable difference between the two models in the deeper layers of the convection zone.

It should be stressed that both sets of models are constructed using the same procedure, except for the expressions for convective flux and mixing length. The parameter α is adjusted to give a convection zone depth d_{cz} , of approximately 200 Mm. The helium abundance in the convection zone is taken to be between 0·24 and 0·26 which is the estimated value of Y from helioseismic data (Antia & Basu 1994). All the models have a uniform metal abundance of Z = 0·02. Below the convection zone we have used different composition profiles as given by various models of gravitational settling of helium. We have however, not included the gravitational settling of metals. In particular, we have used the composition profile as given by Bahcall &Pinsonneault (1992) (henceforth BP) and the models TD1 and TD2 using turbulent mixing from Christensen-Dalsgaard, Proffitt & Thompson (1993) (henceforth CDPT). The BP model has a sharp composition gradient just below the convection zone, while in the model TD2 which includes turbulent mixing, the composition profile is smooth.

All the envelope models use the MHD equation of state (Hummer & Mihalas 1988; Mihalas, Däppen & Hummer 1988; Däppen *et al.* 1988) since this equation of state is found to be close to that of the solar material (Vorontsov, Baturin & Pamyatnykh 1992; Antia & Basu 1994). Most models use OPAL opacities (Rogers & Iglesias 1992), however, we have also constructed models with opacities from Cox & Tabor (1976) to test the effect of uncertainties arising from opacity values. All the models used in this study extend up to a depth of 500 Mm, and their properties are summarized in table 1.

We use the differential asymptotic method for sound speed inversion (Christensen-Dalsgaard, Gough & Thompson 1989) to find the relative difference in sound speed between the Sun and envelope models. In this method, the difference between the frequencies of a solar model and the observed solar frequencies provides the basic

Table 1. Properties of solar envelope models.

Model	Y	d_{cz} (Mm)	Convection theory	Composition profile	Opacity
M1	0.24	200	MLT	Uniform	OPAL
M2	0.26	200	MLT	Uniform	OPAL
M3	0.24	200	CM	Uniform	OPAL
M4	0.26	200	CM	Uniform	OPAL
M5	0.25	200	CM	Uniform	OPAL
M6	0.25	198	CM	BP	OPAL
M7	0.25	200	CM	TD2	OPAL
M8	0.25	198	CM	TD2	OPAL
M9	0.25	200	CM	TD1	OPAL
M10	0.25	204	CM	BP	OPAL
M11	0.25	200	CM	Uniform	CT

input to find the corresponding sound speed difference between the solar model and the Sun. Following Christensen-Dalsgaard, Gough & Thompson (1989) we express the frequency difference in the form

$$S(w)\frac{\delta\omega}{\omega} = S(w)\frac{\omega_0 - \omega}{\omega_0} = H_1(w) + H_2(\omega), \tag{3}$$

where

$$S(w) = \int_{r_t}^{R_\odot} \left(1 - \frac{c_0^2}{w^2 r^2}\right)^{-1/2} \frac{\mathrm{d}r}{c_0}, \tag{4}$$

and

$$H_1(w) = \int_{r_t}^{R_\odot} \left(1 - \frac{a^2}{w^2}\right)^{-1/2} \frac{c_0 - c}{c_0} \frac{1}{a} \frac{\mathrm{d}r}{r}. \tag{5}$$

Here $r_t = c_0/w$ is the lower turning point, c_0 is the sound speed in the reference model and c is the sound speed in the Sun, $a = c_0/r$, ω_0 is the frequency of p-mode in the reference model, while ω is the observed frequency for the same mode and $w = \omega/(l + 1/2)$. Thus, the frequency difference depends asymptotically on the interior sound speed difference through a function of w, and on differences in the surface layers through a function of ω. Using the known frequency differences between a large number of modes, we can obtain the functions $H_1(w)$ and $H_2(\omega)$ by a least squares solution of equation (3). For this purpose we expand $H_1(\omega)$ in terms of B-spline basis functions in log w and likewise expand $H_2(\omega)$ using B-splines in w. For $H_1(w)$ we use 25 knots uniformly spaced in log w, while for $H_2(\omega)$ we use 20 uniformly spaced knots in ω. We have done experiments to find that the final results are not sensitive to the number of knots used. Since in this work we are interested in envelope models, we consider only those modes for which $1.0 < \omega < 5.5$ mHz and $w < 0.25$ mHz. Further, we restrict the sample of modes to those listed in the tables of Libbrecht, Woodard & Kaufman (1990) and weigh each point according to the quoted standard deviation in the frequencies. Since the asymptotic relation cannot be expected to hold for the f-mode, we do not include the f-mode ($n = 0$) in the set of modes used for inversion. After applying these cut-offs we are left with about 2530 eigenmodes to calculate

$H_1(w)$ and $H_2(\omega)$. For obtaining the least squares solution of equation (3), we use singular value decomposition (SVD) (refer Antia 1991) which directly gives the coefficients of both sets of *B*-splines.

With the help of the function $H_1(w)$ determined as above, we can obtain the difference in sound speeds between the model and that in the Sun using the relation,

$$\frac{\delta c}{c} = \frac{c_0 - c}{c_0} = -\frac{2r}{\pi}\frac{\mathrm{d}a}{\mathrm{d}r}\int_{a_s}^{a}\frac{\frac{\mathrm{d}H_1}{\mathrm{d}w}w\,\mathrm{d}w}{(a^2 - w^2)^{1/2}}, \tag{6}$$

where $a_s = a(R_\odot)$. Note that, equation (6) is slightly different from the corresponding equation in Christensen-Dalsgaard, Gough & Thompson (1989), but it can be readily shown that the two expressions are equivalent.

3. Comparison of convection theories

We have computed the relative sound speed difference between the various models and the Sun using the technique outlined in the previous section. Fig. 1 shows the relative sound speed difference between models Ml , M2, M3, M4 and the Sun. Models Ml and M2 are constructed using the mixing length formulation while models M3

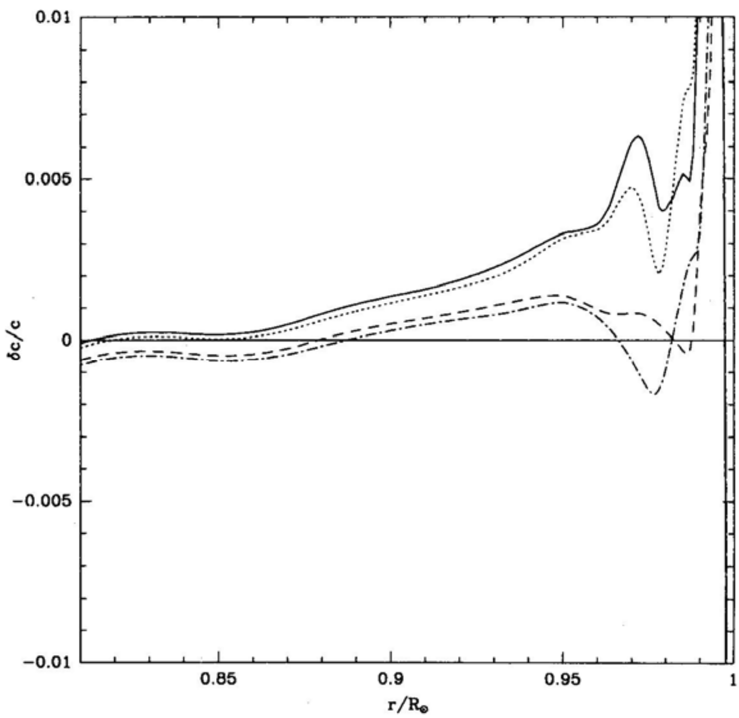

Figure 1. The relative sound speed difference between different envelope models and the Sun as a function of the fractional solar radius. The solid line represents model Ml, the dotted M2, dashed M3, and dot-dashed M4. Models Ml and M2 are MLT models and M3 and M4 are CM models.

and M4 use the CM prescription. We notice that for all the models the sound speed difference is fairly small (< 0.4%) in most of the convection zone. The main differences arise close to the surface (for $r > 0.95R_\odot$). It is clear that the sound speed in the CM models is distinctly closer to that in the Sun as compared to that in the MLT models. For $r > 0.985R_\odot$ there appears to be significant difference between the sound speed in the Sun and the models. However, in this region the applicability of the asymptotic inversion technique used in the present work is questionable.

In order to estimate the error due to the uncertainties in the observed frequencies, we have simulated 25 sets of frequencies where random errors with standard deviation quoted by the observers are added to the model frequencies. For each of the 25 sets of frequencies we have determined the relative sound speed difference. Using these 25 sets of simulations we can find the mean and variance of $\delta c/c$ at each radius. It is found that the error on the whole is very small, with the maximum variance being of the order of 0.01% for $r < 0.98R_\odot$. Thus we see that the difference between $\delta c/c$ obtained from CM and MLT models is more than the errors expected due to uncertainties in the frequencies.

Evidently, the sound speed very close to the solar surface cannot be reliably determined using the asymptotic technique. In this region, the basic assumption underlying the asymptotic theory will break down, since the vertical wavelength of the *p*-modes is not small as compared to the local scale heights. Apart from this,

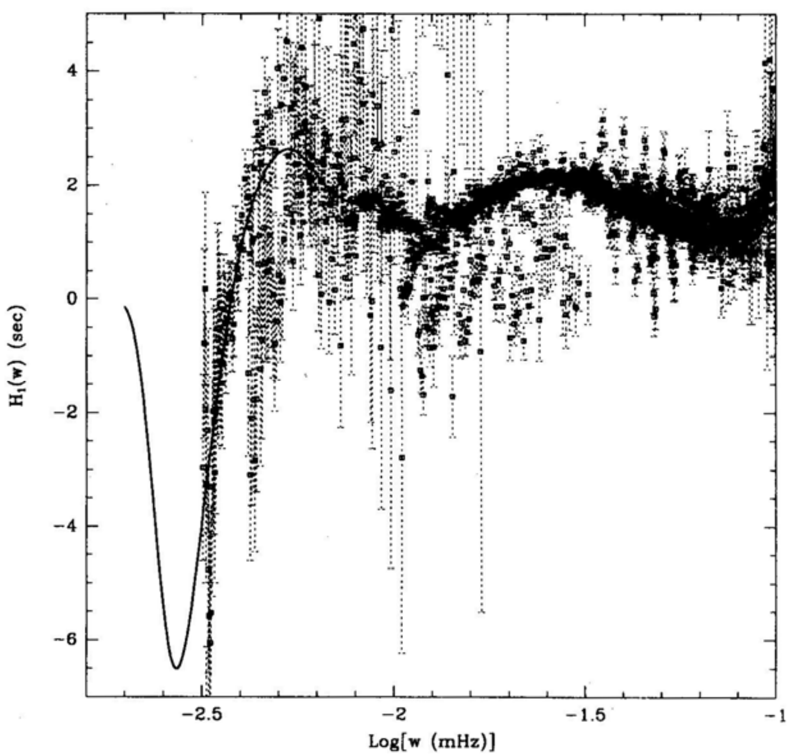

Figure 2. The function $H_1(w)$ for the Sun with model M5 as the reference model. The points mark $S(w)(\delta\omega/\omega) - H_2(\omega)$ against log w.

there is an additional difficulty because of the fact that there are no trapped modes with arbitrarily shallow turning points. For example, in the set of modes used in the present work, there is no mode with w < 0.0031 mHz, which corresponds to the lower turning point at a radius of $\approx 0.997R_\odot$. The function $H_1(w)$ for smaller value of w has then to be determined by extrapolation, which introduces uncertainties in the sound speed inversion close to the solar surface. Fig. 2 shows $H_1(w)$ for the Sun with respect to model M5. The points corresponding to the different modes are also marked in the figure. Each point shows ($\log w$, $S(w)(\delta\omega/\omega) - H_2(\omega)$) for the modes used in the present study. It is clear from the figure that the dip in the points towards the low w gets magnified in the function $H_1(w)$ because of the extrapolation. It is this dip in $H_1(w)$ that gives rise to the fairly large peak in $\delta c/c$ at r > 0.99R$_\odot$ for all models. Since it is not obvious how to extrapolate the function in this range, we have chosen the last knot for the *B*-spline basis functions at $\log[w(\mathrm{mHz})] = -2.5$, and the knot spacing is chosen such that $H_1(w) \to 0$ at the surface. In order to estimate the effect of the uncertainty in extrapolation, we have performed inversions by adding an arbitrary constant to $dH_1/d\log w$ for $\log[w(\mathrm{mHz})] < -2.5$ as in Christensen-Dalsgaard, Gough and Thompson (1989). Fig. 3 shows the result obtained by adding an extreme value of ± 50 sec. to $dH_1/d\log w$. The difference between these curves should give an upper limit to the error caused by extrapolation. Clearly, there may be a significant uncertainty in the relative sound speed difference for $r > 0.97R_\odot$.

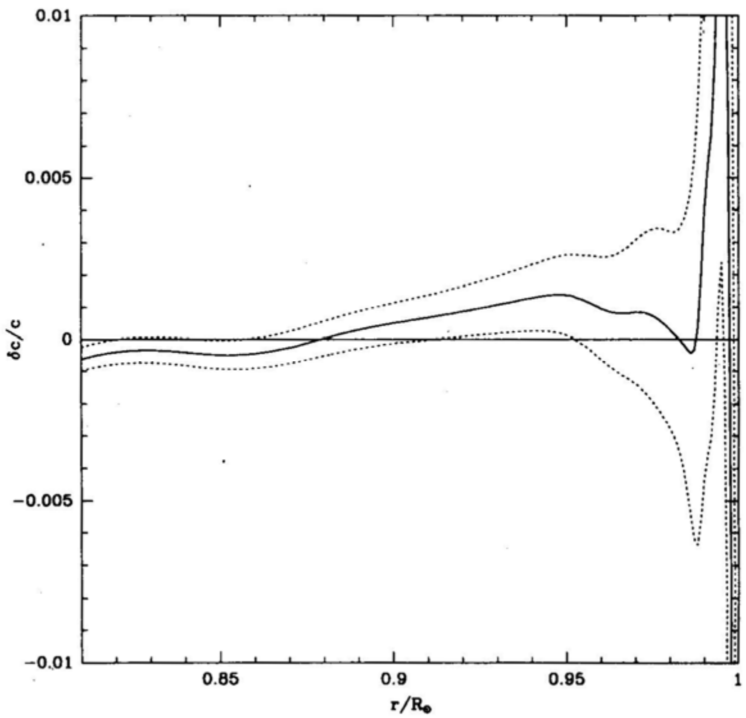

Figure 3. The effect of extrapolation of $H_1(w)$ at log w on the derived sound speed. The solid line is the relative sound speed difference between the model M3 and the Sun derived using the function $H_1(w)$ obtained directly from the least squares solution. The dotted lines are $\delta c/c$ obtained after adding a constant value of ± 50 sec. to $dH_1(w)/d\log w$ for $\log[w(\mathrm{mHz})] < -2.5$.

This technique can be tested by using the frequency differences between two known solar models to estimate the corresponding difference in the sound speeds. From such tests also, we find that the relative sound speed difference has significant errors for $r > 0.97R_\odot$. However, the extrapolated $H_1(w)$ for $\log[w(\mathrm{mHz})] < -2.5$ is similar for all the models considered here, and hence the uncertainty due to extrapolation may give similar errors in $\delta c/c$ relative to the Sun for each model. We therefore expect the difference between the CM and MLT models to be relatively unaffected by this uncertainty.

Another measure of the difference between the models and the Sun is provided by the function $H_1(w)$ which is a measure of the sound speed difference. The advantage of using the function $H_1(w)$ instead of $\delta c/c$ is that $H_1(w)$, for $\log[w(\mathrm{mHz})] > -2.5$, is obviously unaffected by the extrapolation at smaller values. The function $H_1(w)$ between models M1–M4 and the Sun is shown in Fig. **4.** We see that for the CM models, the systematic trend in $H_1(w)$ for $\log[w(\mathrm{mHz})] > -2.3$ is much less than that for the MLT models. It must be noted that $H_1(w)$ between similar models shows no systematic trend with w. The fact that CM models show a much flatter $H_1(w)$ indicates that they are probably closer to the Sun than the models constructed using MLT.

Apart from $H_1(w)$, the function $H_2(\omega)$ can also be used as a measure of the difference between models and the Sun. Since, $H_2(\omega)$ is supposed to reflect the differences in the surface layers, it may be better suited to see the difference between the two

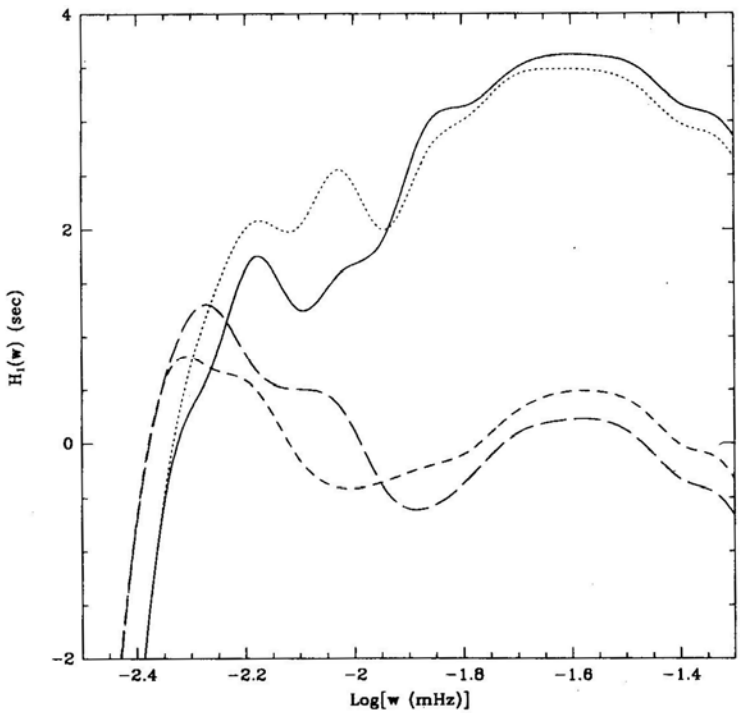

Figure 4. The function $H_1(w)$ between the Sun and different envelop models. The solid line represents model Ml, dotted line is for model M2, dashed line for model M3 and dot-dashed line for model M4.

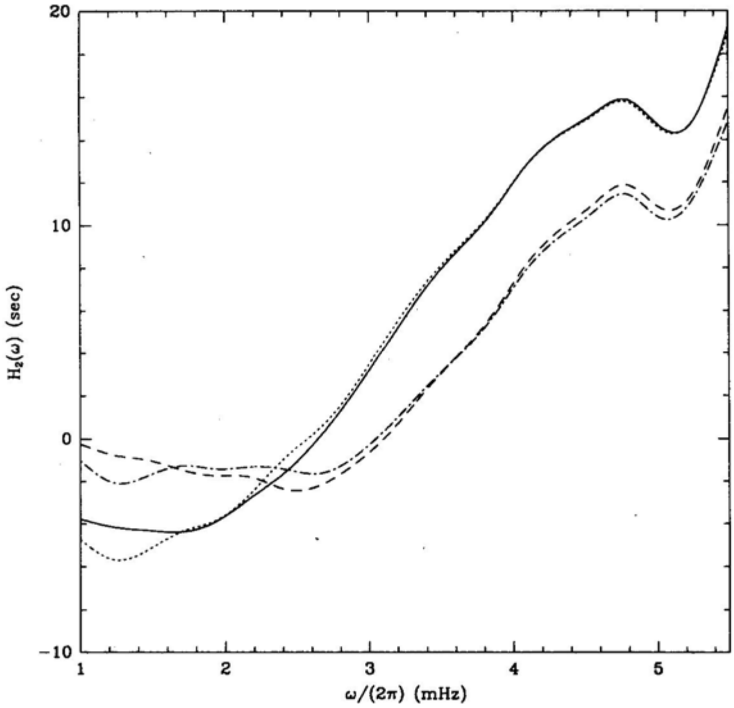

Figure 5. The function $H_2(\omega)$ between the Sun and different envelop models. The solid line represents model Ml, dotted line is for model M2, dashed line for model M3 and dot-dashed line for model M4.

formulations of stellar convection, which predominantly affects the surface layers. Fig. 5 shows $H_2(\omega)$ plotted for models M1–M4. We can see that the MLT models Ml and M2 have a much larger variation in $H_2(\omega)$ than do the CM models M3 and M4. In particular, $H_2(\omega)$ for the CM models is practically flat for $\omega < 3$ mHz. At higher frequencies the uncertainties in the treatment of atmosphere and the non-adiabatic effects may influence the frequencies of *p*-modes significantly.

With a view to provide an independent criterion to determine which of the two convection theories gives better results, we have considered the scaled frequency differences between the models and the Sun. We define the scaled frequency difference $\Delta_{n,l} = Q_{n,l}\delta\omega$, where $Q_{n,1} = E_{n,l} / \bar{E}_0(\omega_{n,l})$,

$$E_{n,l} = \frac{4\pi \int_0^{R_\odot} [|\xi_r(r)|^2 + l(l+1)|\xi_t(r)|^2]\rho r^2 \mathrm{d}r}{M_\odot [|\xi_r(R_\odot)|^2 + l(l+1)|\xi_t(R_\odot)|^2]}, \tag{7}$$

where, ξ_r and ξ_t are the radial and tangential components of the displacement for the given mode, ρ is the equilibrium value of the density; and $\bar{E}_0(\omega_{n,l})$ is the value of $E_{n,l}$ for $l = 0$ interpolated to the frequency $\omega_{n,1}$.

In Fig. 6 we have plotted the scaled frequency differences between the observed solar frequencies and those computed for models M2 and M4 as a function of *w*. It is clear that for $\log[w(\mathrm{mHz})] > -2.2$ the scaled frequency differences for the CM model are smaller by a factor of approximately 1.5 as compared to those for the

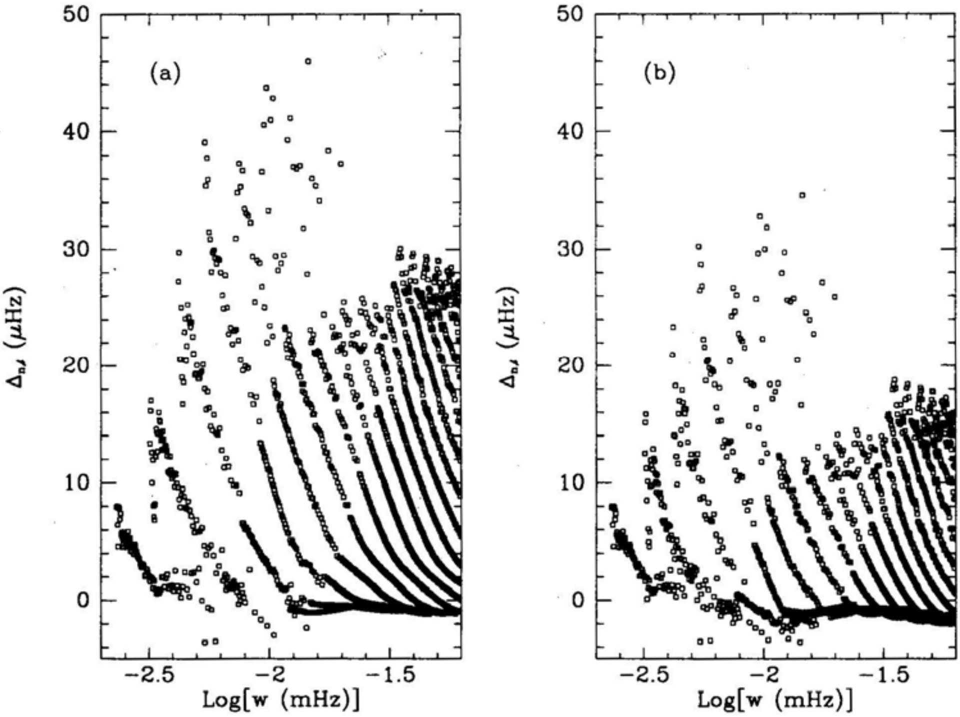

Figure 6. The scaled frequency difference between the Sun and (a) model M2, and (b) model M4. This figure also includes the frequency differences for the *f*-mode even though those are not used in inversion.

MLT model. These frequency differences are not very sensitive to small changes in helium abundance or the convection zone depth. Thus the scaled frequency differences should reflect the effect of the underlying convection theory: This difference can be attributed to the differences in $H_2(\omega)$ for the models.

Thus, on the basis of both the sound speed difference and the scaled frequency difference between the models and the Sun, we are led to the conclusion that the CM models give better agreement with observations than the MLT models.

4. Test for helium diffusion

Gravitational settling of helium and heavy elements introduces a composition gradient below the base of solar convection zone, which can be detected helioseismically. Unfortunately, the composition profile is not unique but depends on the adopted treatment of turbulence (Christensen-Dalsgaard, Proffitt & Thompson 1993). In the absence of turbulence, diffusion leads to a sharp increase in Y just below the base of the convection zone. Such a composition profile should give a clear signature in the sound speed profile, since the increase in the mean molecular weight will decrease the sound speed.

Guzik & Cox (1993) studied the effects of diffusion on solar oscillations and found that models with helium diffusion without turbulent mixing appear to give better agreement with the observed frequencies than either models without diffusion or models which have diffusion with turbulent mixing. They rely on simple comparison between the observed and calculated frequencies, hence it is difficult to separate the effect of diffusion from other uncertainties. Using evolutionary solar models CDPT found that the models using helium diffusion yield better agreement with helioseismic data. However, in their models the dominant contribution to the sound speed difference appears to be due to difference in the convection zone depth. The model without helium diffusion yields a rather shallow convection zone and hence the sound speed does not agree very well with observations. While it is true that models with helium diffusion give nearly correct convection zone depth, we cannot draw any definite conclusions from these results since the convection zone depth could depend on other uncertainties in the stellar evolution theory. In order to provide an independent test for the gravitational settling of helium, in this work, we have attempted to detect signatures of helium diffusion using envelope models, where the convection zone depth can be adjusted independently. Thus our work can supplement the conclusions of CDPT obtained using evolutionary models, which include the inner core.

Once the parameters of the envelope model are determined reliably, we can extend the model inwards assuming different profiles of δX, the change in chemical composition. The results are displayed in Fig. 7 which shows the relative difference

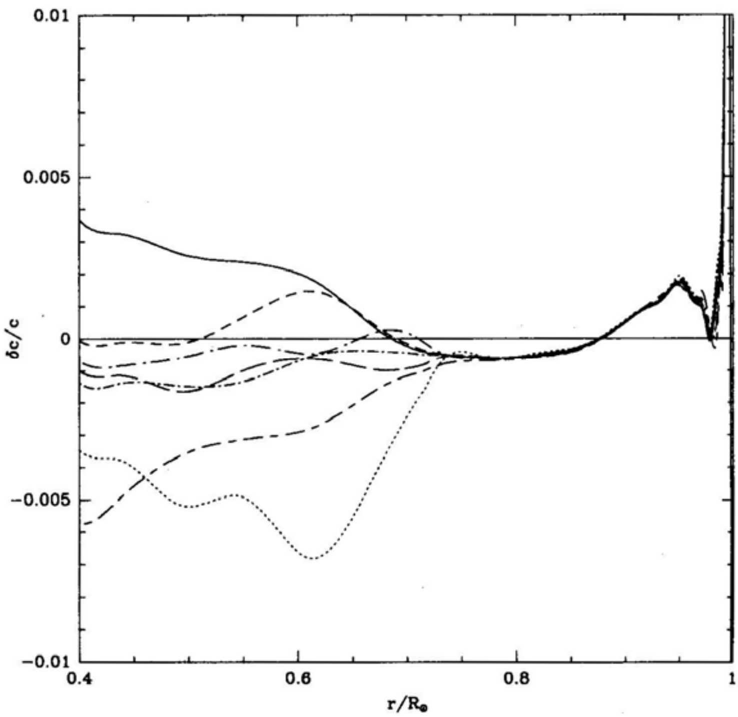

Figure 7. Relative sound speed difference between the Sun and models M5 (solid line), M6 (dotted line), M7 (short dashed line), M8 (long dashed line), M9 (dot short-dashed line), M10 (dot long-dashed line), and M11 (long dash-short dashed line).

in sound speed between the models and the Sun. If we consider the model M5 which has uniform composition, no sharp change is seen in $\delta c/c$ near the base of the convection zone. In this case $\delta c/c$ increases gradually below the convection zone. This increase could be attributed to either a lack of increase of helium abundance or an overestimation of opacity in these layers. Model M6, which uses the composition profile of BP, shows a sharp decline in $\delta c/c$ as compared to the Sun. From these results it is tempting to conclude that sharp changes in the composition profile of the form assumed in the BP model are unlikely to occur inside the Sun. However, uncertainties in opacities and the depth of the convection zone may mask the sharp change in $\delta c/c$ below the convection zone.

The model M7, which has the composition profile as given by CDPT (their model labelled TD2) shows a small hump in $\delta c/c$ near the base of the convection zone. This hump could be due to a small difference in the depth of the convection zone of the order of 2 Mm or due to a steep X gradient. In principle, uncertainty in the depth of the convection zone or overshoot layer could mask the changes due to composition profile. Alternatively, a sharp change in the composition profile could lead to some uncertainty in determination of the depth of the convection zone or the thickness of the overshoot layer. Using the oscillatory signal in the frequencies as a function of the radial order η arising from sharp changes in the temperature gradient below the base of the convection zone, Monteiro, Christensen-Dalsgaard & Thompson (1993); and Basu, Antia & Narasimha (1994) concluded that the observed solar frequencies are consistent with a no overshoot model. Further, an upper limit of $0.1H_p$ on the extent of overshoot was given. This result may be affected if there is a composition gradient, since that may also give rise to an abrupt change in the sound speed gradient.

In order to identify the origin of the hump in $\delta c/c$ for model M7, we have constructed models with slightly different convection zone depth and with small overshoot layer. For model M8 with a convection zone depth of 198 Mm, there is no hump, but $\delta c/c$ is systematically lower than that for model M7. Similarly, model M9 which has a convection zone depth of 200 Mm, but has the composition profile of the model TD1 of CDPT also gives a smooth $\delta c/c$. In this manner, a small increase in the depth of the convection zone appears to compensate for the somewhat sharp composition gradient in the model. In case of the composition profile given by the diffusion model of BP, the convection zone depth has to be increased to 204 Mm (model M10), in order to compensate for the change in sound speed due to the steep composition gradient. This depth is slightly larger than that estimated by Christensen-Dalsgaard, Gough, & Thompson (1991). The exact relative sound speed difference between this model and model M8, however, shows a small kink at the base of the convection zone which is not resolved by the inversion technique. Thus we can see that helium diffusion introduces an uncertainty in the depth of the convection zone as determined from the observed frequencies.

It is found that inclusion of overshoot below the convection zone does not change the sound speed substantially. It only gives rise to a small kink in $\delta c/c$ near the base of the convection zone. If the extent of overshoot in the models is less than the upper limit given by Basu, Antia & Narasimha (1994), we find that the kink in $\delta c/c$ is not noticeable in the inverted $\delta c/c$ profile. This result is consistent with the upper limit on the extent of overshoot.

In order to estimate the effect of uncertainties in opacity on $\delta c/c$, we have constructed one model (model M11) using Cox-Tabor opacities (Cox & Tabor 1976), the result of which is shown in Fig. 7. In this case, $\delta c/c$ decreases below the convection zone and since the inclusion of diffusion can only lead to a further reduction of $\delta c/c$, we may conclude that the opacity of solar material is higher than that given by Cox & Tabor (1976). This is consistent with the conclusion drawn by Christensen-Dalsgaard *et al.* (1985). On the other hand, for model M5 which uses the OPAL opacities, there is a gradual increase in $\delta c/c$ below the convection zone. As emphasized earlier this increase could be due to the absence of helium diffusion or due to an overestimate of the opacity.

From the sound speed profile of the envelope model with constant composition gradient, we can conclude that helium abundance increases below the solar convection zone, provided that the opacity of the solar material at the base of the convection zone is not less than that given by the OPAL tables. This result supports the hypothesis of gravitational settling of helium in the Sun. Nevertheless, considering the uncertainty in the depth of the convection zone, it is not possible to distinguish between different models of gravitational settling. Conversely, helium diffusion introduces an uncertainty in the depth of the convection zone of about 6 Mm.

5. Conclusions

We have attempted in the present study, to construct a solar envelope model which is close to the Sun as inferred from helioseismic observations. Apart from the usual mixing length formulation, we have tried the CM prescription for convective energy transport in the solar convection zone. Applying the differential inversion technique to the difference in frequencies of solar models and the observed frequencies, we find that the CM models appear to give a significantly better agreement with observations. This result follows from the relative difference in the sound speed $\delta c/c$, the functions $H_1(w)$ and $H_2(\omega)$, as well as from the scaled frequency differences between the models and the Sun. Thus, it appears that the CM formulation which gives a significantly steeper temperature gradient in the surface layers, is closer to reality as compared to the mixing length theory. The remaining difference in $\delta c/c$ around $r \approx 0.98R_\odot$could be due to uncertainties in the equation of state or due to the limitations of the asymptotic inversion technique.

By considering $\delta c/c$ just below the convection zone we can test different models with helium diffusion. The models with gravitational settling of helium which ignore turbulent diffusion, tend to give a composition profile with a sharp gradient just below the convection zone. Since $\delta c/c$ for model M5, which has a uniform composition, increases steadily below the convection zone, we surmise that the helium abundance should increase with depth. Uncertainties in the opacity will also affect the sound speed profile, but if OPAL tables do not overestimate the opacity of solar material below the convection zone, then indeed there is a need to invoke helium diffusion. Apart from the opacity, uncertainties in the depth of the convection zone will also affect the sound speed in this region. This uncertainty makes it difficult to choose between the different composition profiles that we have tested. If the envelope model has the composition profile given by BP, a convection zone depth of 204 Mm is required to obtain a sound speed close to that in the Sun. This depth is somewhat larger than the value obtained by Christensen-Dalsgaard, Gough & Thompson (1991). Models with the composition profile given by the model TD1 of CDPT require a convection zone depth of 200 Mm, while models with a smooth composition profile (TD2) need a depth of only 198 Mm.

A limitation of the present work is that we have used solar envelope models. If these models are extended to the center it is unlikely that the required boundary conditions on the mass and luminosity at the center will be satisfied. By adjusting the composition profile, opacities and nuclear reaction rates within reasonable limits, it may be possible to satisfy the boundary conditions. However, such calculations are unlikely to yield any information about the uncertainties in these quantities. On the other hand, it should be noted that we have considered inversion for sound speed only, which is essentially determined by the temperature profile. Thus even though the temperature profile in our models may be close to that in the Sun, the density may be somewhat different. In which case, the inward extension of the model may not be expected to yield a proper solar model. Thus it would be interesting to perform the inversion for density also to provide additional constraints on these envelope models. We plan to carry out the density inversion in future calculations.

Acknowledgement

We thank V. M. Canuto, I. Mazzitelli and L. Paterno for useful communications regarding the CM formulation for convective flux.

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