

# SOLAR CYCLE VARIATION IN SOLAR F-MODE FREQUENCIES AND RADIUS

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**Abstract.** Using data from the Global Oscillation Network Group (GONG) covering the period from 1995 to 1998, we study the change with solar activity in solar f-mode frequencies. The results are compared with similar changes detected from the Michelson Doppler Imager (MDI) data. We find variations in f-mode frequencies which are correlated with solar activity indices. If these changes are due to variation in solar radius then the implications are that the solar radius decreases by about 5 km from minimum to maximum activity.

**Keywords:** Sun: general – Sun: Oscillations

## 1. Introduction

The fundamental mode or f-mode of solar oscillations are believed to be surface gravity modes whose frequencies are essentially independent of the stratification in the solar interior. A large part of the difference between the observed f-mode frequencies at intermediate degree and those in a standard solar model has been interpreted as being caused by the incorrect radius of the solar models used (Schou et al. 1997; Antia 1998; Brown and Christensen-Dalsgaard 1998; Tripathy and Antia 1999). The frequencies of these modes can thus be used to measure the solar radius.

There have been many reports about possible variation of solar radius with time (Delache, Laclare and Sadsaoud 1985; Wittmann, Alge and Bianda 1993; Fiala, Dunham and Sofia 1994; Laclare et al. 1996; Noeel 1997). The reported change in measured angular semi-diameter varies from  $0.1''$  to  $1''$ , which implies a change of 70 to 700 km in

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radius. However, there is no agreement among observers about these variations. It would thus be interesting to look for corresponding variations in the f-mode frequencies. The reported variations in solar radius should change the f-mode frequencies by 0.1 to 1  $\mu$ Hz, which is much larger than the error estimates in these frequencies. Considering how small the estimated errors in f-mode frequencies are, it is possible to measure changes in the solar radius as small as a few kilometers over the solar cycle. Some variation in f-mode frequencies have been reported in the MDI data by Dziembowski et al. (1998) who find that the solar radius reached a minimum around the minimum activity period and was larger by about 5 km, 6 months before and after the minimum. If this variation is correlated with solar activity then we would expect much larger change between minimum and maximum activity.

In this work we look for possible variations in the solar radius using GONG data — which extend over longer time period than the MDI data used in previous studies. We also look for possible correlations between change in radius and solar activity. In Section 2 we outline the technique used in calculating the frequencies and examine possible systematic errors in computed frequencies. Section 3 gives the results while in Section 4 we summarize the conclusions from our study.

## 2. The technique

Since the f-modes have very low power they are barely visible in the spectra for modes of a given degree  $\ell$  and azimuthal order  $m$  that the GONG project fits to determine solar oscillation frequencies. As a result, it is difficult to determine the frequencies of these modes reliably from these spectra. To improve statistics we use the rotation-corrected,  $m$ -averaged, power spectra from GONG data (Pohl and Anderson 1998) to calculate the mean f-mode frequencies. Because summing over the  $2\ell+1$  spectra for a mode of degree  $\ell$  makes the peaks better defined, it is possible to fit them reliably. Each of these spectra was obtained from 3 GONG months (108 days) of data and has a frequency resolution of 0.107  $\mu$ Hz. These spectra extend up to  $\ell = 200$  and frequency  $\nu = 2083 \mu$ Hz. The f-mode peaks are clearly visible for  $\ell \gtrsim 100$ . We have also used two spectra covering only 1 GONG month, which gives a lower frequency resolution of 0.321  $\mu$ Hz. For these spectra the errors in computed frequency would be larger as compared to the 3 month spectra.

We fit each mode separately using a maximum likelihood technique (Anderson, Duvall and Jefferies 1991) with a symmetric Lorentzian profile for the peaks. The fit is performed over a region extending to

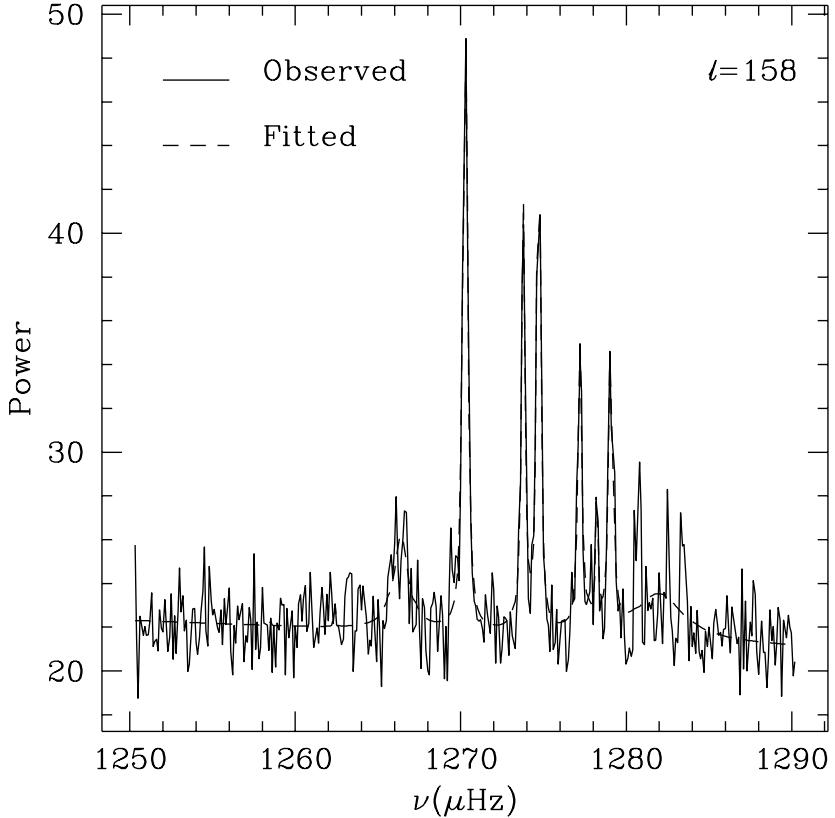


Figure 1. Fit to the  $\ell = 158$  f-mode spectra for months 33–35. Power is in arbitrary units.

about  $20 \mu\text{Hz}$  on either side of the peak and includes leaks from modes of degree  $\ell - 3$  to  $\ell + 3$ ,  $\ell$  being the degree of the target mode. It is found that peaks arising from the  $\ell + 1$  leak are split into two because of leaks from peaks of  $m \pm 1$ . Similarly, the  $\ell + 2$  leak is split into three because of leaks from peaks of  $m, m \pm 2$ . Although leaks from  $\ell - 1$  and  $\ell - 2$  are also split in a similar manner, the power in these peaks is much smaller. Thus we have explicitly fitted the  $m$ -leaks only from the  $\ell + 1$  and  $\ell + 2$  peaks. An example of the fit is shown in Fig. 1.

For  $\nu < 1170 \mu\text{Hz}$ , the modes are difficult to fit reliably since the width is smaller than the resolution limit of the spectra and the power is relatively low. As a result, in this work we only include higher frequency modes.

The use of  $m$ -averaged spectra may introduce some systematic errors in determining the frequencies. To check for these we compare the frequencies of p-modes fitted from  $m$ -averaged spectra with those obtained by the GONG project using the individual  $m$  spectra. The

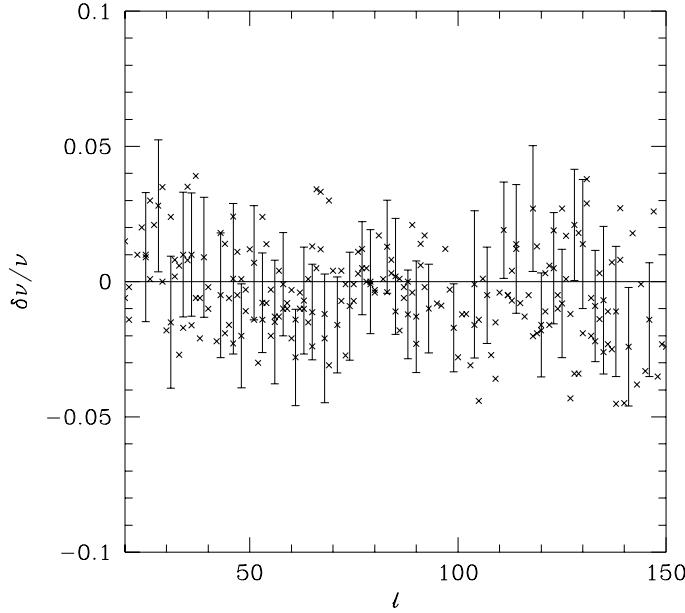


Figure 2. Difference in frequencies for months 33–35 between the values obtained from the  $m$ -averaged spectra and those computed from the individual  $m$ -spectra. For clarity only a few error-bars are shown.

GONG project has computed the mean frequency for each  $n, \ell$  multiplet by fitting the frequencies for individual values of  $m$  to polynomials in  $m$ . This mean frequency can be compared with the frequency computed from the  $m$ -averaged spectra. The results are shown in Fig. 2. It may be noted that f-mode has not been fitted in the individual  $m$  spectra and hence these are not included. It can be seen that the frequency difference is quite small being of the order of the estimated errors in the fitted frequencies. Systematic difference between these two sets of frequencies is  $\lesssim 10$  nHz.

Another possible source of systematic errors is the use of incorrect even-order splitting coefficients while constructing the  $m$ -averaged spectra. The GONG spectra were obtained by setting these coefficients to zero. A non-zero value of these even-order coefficients would introduce some systematic shift in frequencies. To estimate the effect of this we constructed some  $m$ -averaged spectra with either  $a_2 = -0.2$  nHz or  $a_4 = 0.1$  nHz. The resulting frequency shifts with  $a_2 = -0.2$  nHz are shown in Fig. 3. Similar shifts are found for non-zero  $a_4$ . Once again the frequency shifts due to non-zero  $a_2$  or  $a_4$  are quite small, being of the order of 10 nHz. Furthermore, the frequency shift for the f-mode is smaller than those for p-mode with the same  $\ell$ . Although these even coefficients are known to change with solar cycle, the variation is not

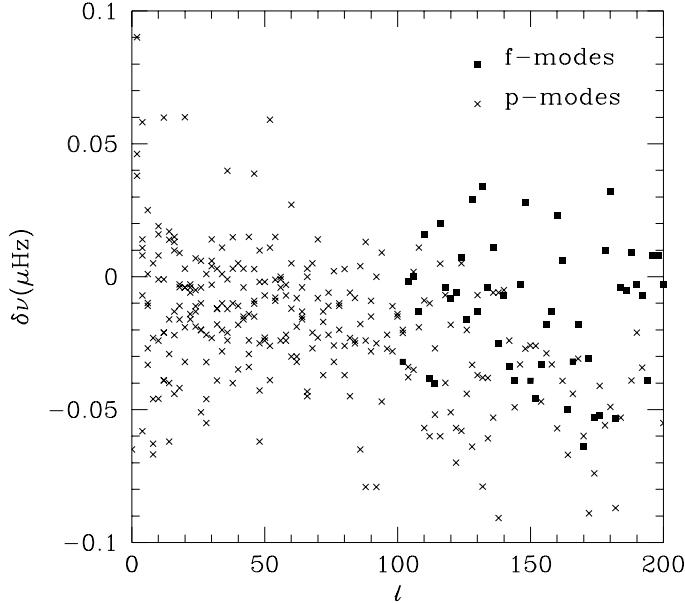


Figure 3. Shift in frequencies for months 21–23 arising due to non-zero  $a_2$

very large at the low frequencies which we are considering in this work. Thus the shifts shown in Fig. 3 can be considered to be conservative upper limits to errors that may be expected due to non-zero even coefficients. From Fig. 2 it can be seen that during a period where even coefficients are fairly large the difference in frequencies computed from individual  $m$ -spectra and the  $m$ -averaged spectra is quite small — much smaller than the differences seen in Fig. 3.

Considering possible systematic errors, it would be desirable to restrict ourselves to low frequency range where these errors are not significant. In the frequency range ( $\nu < 1440 \mu\text{Hz}$ ) considered in this work the systematic errors from these effects are expected to be about 10 nHz, and any time variation in these systematic effects would be smaller. Having established the reliability of frequencies from  $m$ -averaged spectra we calculate the frequencies from 7 different spectra during 1995 to 1998. These spectra are listed in Table 1. This table also lists the solar activity as measured by the mean daily sunspot number,  $R_I$  from the Solar Geophysical Data web page of the US National Geophysical Data Center (<http://www.ngdc.noaa.gov/stp/stp.html>).

Table I. List of GONG spectra used

GONG months	Starting date	Ending date	Mean daily sunspot no.
3 – 5	1995-07-18	1995-11-02	$14.5 \pm 1.3$
8 – 10	1996-01-14	1996-04-30	$6.0 \pm 0.6$
21 – 23	1997-04-26	1997-08-11	$14.7 \pm 1.1$
27 – 29	1997-11-28	1998-03-15	$39.5 \pm 1.7$
33 – 35	1998-07-02	1998-10-17	$79.5 \pm 2.5$
36	1998-10-18	1998-11-22	$62.4 \pm 4.1$
37	1998-11-23	1998-12-28	$82.6 \pm 4.1$

### 3. Results

We have fitted each of the 7 spectra listed in Table 1 to calculate the mean frequencies of the f modes. Fig. 4 shows the relative difference between the observed frequencies for Months 33–35 and Months 8–10. It can be seen that to a first approximation the relative frequency difference is independent of frequency and hence this frequency difference can be interpreted as arising from a change in the solar radius. Of course, suitable combination of other effects like magnetic field, convection etc. may also yield frequency differences which are independent of frequency but that will almost certainly require some fine-tuning of parameters, while change in solar radius would be a simpler explanation.

To estimate the possible change in radius we take an average over the relative frequency difference for all 56 f-modes that were successfully fitted in all spectra:

$$\langle \delta\nu \rangle = \frac{\sum_{\ell} \frac{\delta\nu_{\ell}}{\sigma_{\ell}^2}}{\sum_{\ell} \frac{1}{\sigma_{\ell}^2}} \quad (1)$$

The differences were taken with respect to calculated frequencies for a standard solar model, which is a static model, constructed with OPAL opacities (Iglesias & Rogers 1996) and low temperature opacities from Kurucz (1991). The OPAL equation of state (Rogers, Swenson & Iglesias 1996) was used to construct the model and convective flux was calculated using the the formulation of Canuto and Mazzitelli (1991). The radius of the model is 695.78 Mm (Antia 1998). The results are shown in Fig. 5, which also shows the mean daily sunspot number,  $R_I$ , for each of these periods. The change in frequencies appears to be correlated with the sunspot number.

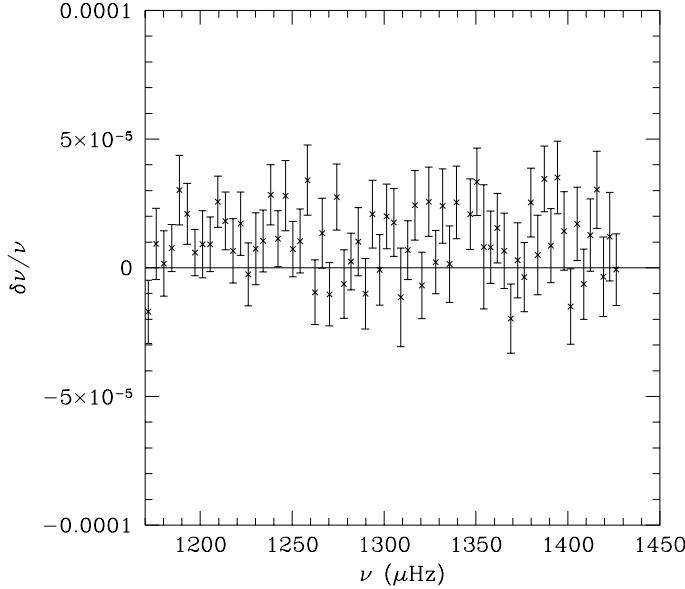


Figure 4. Relative frequency differences between f-mode frequencies from the spectra for months 33–35 and months 8–10.

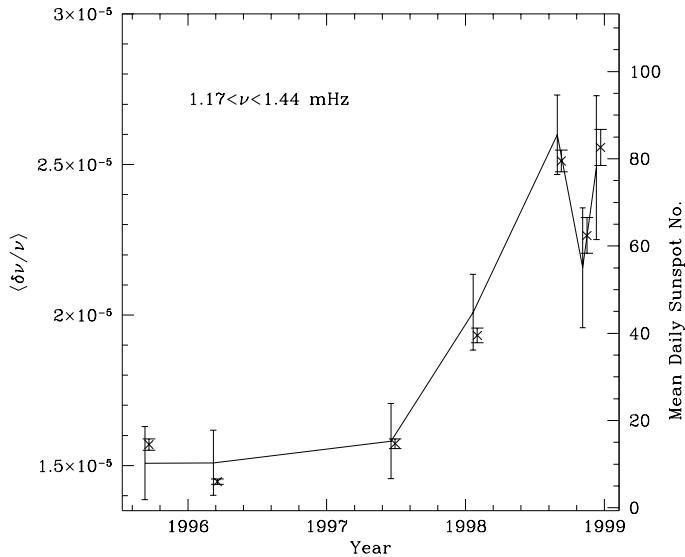


Figure 5. The averaged frequency difference for f-modes between the Sun and a standard solar model with radius  $R = 695.78 \text{ Mm}$  as a function of time is shown by the continuous line. The mean daily sunspot number  $R_I$  is also shown by points with scale shown on the right side. For clarity these points have been slightly shifted along the x-axis.

If the change in frequencies is entirely due to change in radius then the average frequency difference is related to change in radius (Antia 1998) by

$$\langle \frac{\delta\nu}{\nu} \rangle = -\frac{3}{2} \frac{\delta R}{R} \quad (2)$$

Thus a relative shift in frequencies by  $10^{-5}$  corresponds to a change in radius by  $0.67 \times 10^{-5} R_{\odot} \approx 4.7$  km. It may be argued that the change of frequency may be due to some other effect, e.g., magnetic field. Since the relative frequency difference between the frequencies at two different epochs is almost independent of frequency in the range of degrees considered here, a change in radius is the simplest explanation for the frequency shift. Of course, the magnetic field is directly correlated to activity indices, but the direct effect of magnetic field, is likely to yield frequency differences that will increase rapidly with frequency since the higher frequency modes are located closer to solar surface where these effects are likely to be more important (Campbell and Roberts 1989). It may be noted that a reduction in solar radius by 4.7 km in 2 years will generate about 100 times the solar luminosity through release of gravitational potential energy, if the contraction is homologous. This indicates that the change in radius is a superficial effect confined to outer layers where the f-modes are located.

However, the radius appears to decrease with increasing activity. This is not what is seen by Dziembowski et al. (1998). The magnitude of change found in GONG data during the period covered by their study is much smaller than what they have reported. The apparent discrepancy is most likely due to a different range of degrees used in their study. They have probably used modes up to  $\ell = 300$  and these high wavenumber modes are known to deviate significantly from the simple dispersion relation (Duvall, Kosovichev and Murawski 1998; Antia and Basu 1999). As a result, one can expect misleading results if average is taken over all modes, since it is possible that the deviation of the modes from the normal dispersion relation has some solar cycle dependence. A part of the deviation is known to be due to the use of symmetric profiles in calculating the f-mode frequencies (Antia and Basu 1999) and it is quite possible that the asymmetry changes with activity. Thus it would be better to confine our attention to modes with  $\ell < 200$ , where these effects are minimal. The results shown in Fig. 2 of Dziembowski et al. (1998) imply a decrease in frequency by  $10^{-5}$  between 1996.8 and 1997.2. If this change is correlated with sunspot number then we would expect a decrease by at least  $5 \times 10^{-5}$  between 1996 and 1998.6 in our results. Certainly no such shift is seen in our results. This shift is also much larger than the expected systematic errors in our results.

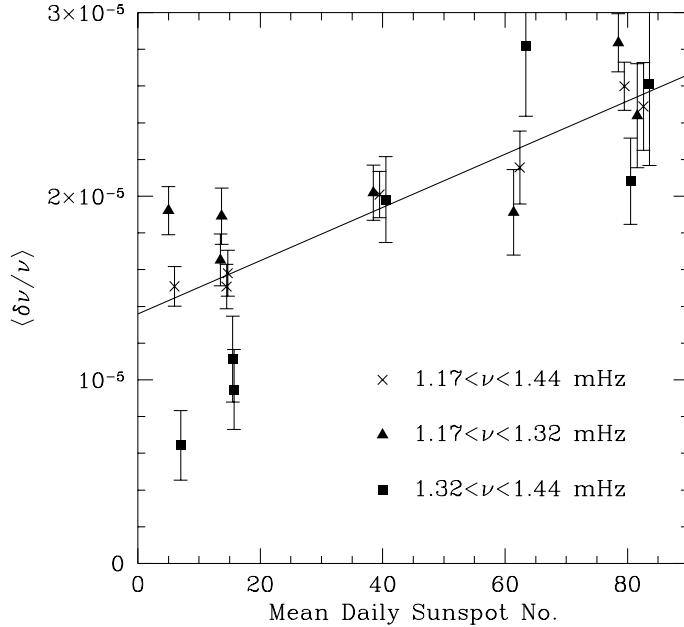


Figure 6. The averaged frequency difference for f-modes between the Sun and a standard solar model with radius  $R = 695.78$  Mm are plotted against the mean sunspot number during the period of observation. The average over three different frequency ranges as marked in the figure are compared with each other. The line marks the best linear fit for the full frequency range. The triangles and squares are slightly shifted along  $x$ -axis for clarity.

In order to check the stability of our results we have repeated the analysis by restricting the frequencies to  $1.17 < \nu < 1.32$  mHz and  $1.32 < \nu < 1.44$  mHz and the results are shown in Fig. 6. It is clear that the results are not particularly sensitive to the range of frequencies used. However, the correlation with solar activity is markedly weaker in the higher frequency range. From the relative frequency differences between the observed frequencies and those in a standard solar model it is known (Tripathy & Antia 1999) that there is a distinct tendency of relative frequency differences decreasing with frequency in this range. It is likely that this trend is due to some other effect like magnetic field which varies with solar cycle. In that case it will interfere with the signal due to change in radius. It is quite possible that at even higher frequencies used in MDI study, the correlation is further weakened because there is an even stronger decrease in the relative frequency differences.

#### 4. Conclusions

We have calculated the mean f-mode frequencies from the  $m$ -averaged spectra from GONG data covering 108 days. The use of  $m$ -averaged spectra enable us to determine the frequencies reliably. The relative frequency difference between observed frequencies and those calculated for a standard solar model are roughly independent of  $\ell$  in the range that was used in this study ( $100 < \ell < 200$ ). Hence it is likely that this difference is due to difference in solar radius, as other agents are likely to introduce a steep frequency dependence in relative frequency differences. The mean shift in f-mode frequencies appears to be correlated with solar activity indices like the mean daily sunspot number  $R_I$ .

If this frequency shift is due to change in solar radius then the radius decreases as activity increases. Further, the change in the solar radius during the solar cycle is approximately 5 km. This is one or two orders of magnitude smaller than the changes reported earlier (Delache, Laclare and Sadsaoud 1985; Wittmann, Alge and Bianda 1993; Fiala, Dunham and Sofia 1994; Laclare et al. 1996; Noeel 1997).

These results on the change in solar radius with solar cycle are much less than what has been deduced using MDI data (Dziembowski et al. 1998) and we believe the discrepancy is due to their using higher degree modes whose frequencies can be affected by a variety of other agents, since these modes are localized closer to the solar surface.

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