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Helioseismology

H. M. Antia
Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400 005, India.
e-mail: antia@tifr.res.in

Abstract. The sun being the nearest star, seismic observations with high spatial resolution are possible, thus providing accurate measurement of frequencies of about half million modes of solar oscillations covering a wide range of degree. With these data helioseismology has enabled us to study the solar interior in sufficient detail to infer the large-scale structure and rotation of the solar interior. With the availability of high quality helioseismic data over a good fraction of a solar cycle it is also possible to study temporal variations in solar structure and dynamics. Some of these problems and recent results will be discussed.

Key words. Sun: oscillations—sun: interior—sun: rotation.

1. Introduction

The study of solar oscillations during the last three decades has provided a tool to study the solar interior in the same way as the study of seismic waves travelling through the earth have enabled the study of the earth's interior. The study of solar interior using oscillations has been referred to as helioseismology (Deubner & Gough 1984; Gough & Toomre 1991). Solar oscillations were discovered by Leighton *et al.* (1962) when they measured velocity at a point on the solar disk using the resulting Doppler shift. They found an oscillatory pattern with a period of around 5 minutes and hence these oscillations are also referred to as five-minute oscillations. The nature of these oscillations was not understood at that time and various theories were put forward to explain them. Ulrich (1970) and Leibacher & Stein (1971) suggested that these oscillations could be acoustic modes of oscillations of the entire sun which are trapped just below the photosphere. Subsequent observations with spatial resolution (Deubner 1975) confirmed this hypothesis as the power was found to be concentrated in a series of ridges in the k–ω diagram, where k is the spatial wavenumber and ω the temporal frequency of oscillations, in accordance with prediction from theoretical models. Once the nature of the modes was identified, it was realised that their properties are determined by the internal structure and dynamics of the sun. Hence, one can study the internal structure of the sun using these oscillations. It is now known that the observed oscillatory pattern on the solar surface arises from superposition of millions of independent modes of solar oscillations. Each of these modes provides an independent constraint on solar structure and dynamics.

Historically, stellar pulsation was first discovered on other stars where the amplitude of oscillations is much larger. Mira (*o* Ceti) is probably the first star which was established to be variable by Fabricius in 1596 and its oscillatory period was determined by

Holwarda in 1638, more than three hundred years before solar oscillations were discovered. While the luminosity in the visual band for this star varies by about an order of magnitude due to oscillations, for the sun the amplitude of oscillation of individual modes is of the order of the 10^{-6} of the solar luminosity. That is the main reason why solar oscillations were discovered only recently when appropriate technology to detect such low amplitude oscillations became available. Because of its proximity, the sun is the only star for which the disk can be easily resolved and hence it is possible to study a large number of modes, which may not be feasible for other stars.

The modes of solar oscillations can be characterized by 3 quantum numbers, the radial order n, the degree l and azimuthal order m. The radial order, n, is determined by the number of nodes along the radial direction, while l and m determine the horizontal structure of the eigenfunction which is defined by the spherical harmonic $Y_l^m(\theta, \phi)$ where θ is the colatitude and ϕ the longitude. Oscillations with $l \lesssim 3500$ have been detected on the sun. Early observations of solar oscillations had limited spatial and temporal resolution. It was soon recognised that in order to obtain high temporal resolution, one needs almost continuous observations covering long periods. This has been possible using a network of ground based observatories, e.g., the Global Oscillation Network Group (GONG) (Harvey *et al.* 1996) or from a suitably located satellite such as the Michelson Doppler Imager (MDI) instrument (Scherrer *et al.* 1995) on the Solar and Heliospheric Observatory (SOHO). The GONG network became operational in May 1995 and by now about 9 years of data have been analysed. The SOHO satellite was launched in December 1995 and the MDI instrument has been functioning since May 1996, except for two major breaks between July 1998 and February 1999, when the contact with the satellite was lost.

2. Seismic inferences of solar structure

If the sun were spherically symmetric, the frequencies $\nu_{n,l,m}$ of the modes of oscillations would be independent of m and these frequencies would be determined by the internal structure of the sun. The presence of rotation and magnetic field in the sun lifts the degeneracy of frequencies, but since the forces due to rotation and magnetic field are much smaller than those due to gravity and pressure gradient, we can treat rotation and magnetic field as small perturbations over the spherically symmetric structure. This gives rise to splitting of frequencies with the same n, l but different m. It is convenient to define the splitting coefficients by

$$\nu_{n,l,m} = \nu_{n,l} + \sum_j c_j^{n,l} P_j^{n,l}(m), \tag{1}$$

where, $\nu_{n,l}$ is the mean frequency of the multiplet, $c_j^{l,n}$ are the splitting coefficients and $P_j^{n,l}(m)$ are the set of orthogonal polynomials of degree j in m. The mean frequencies $\nu_{n,l}$ are determined by the horizontally averaged structure of the sun, while the splitting coefficients are determined by rotation, magnetic field and any other departure from spherical symmetry.

The mean frequencies $\nu_{n,l}$ of modes have been determined to very high accuracy. Figure 1 shows the l–ν diagram showing the frequencies as a function of l. The error bars in this figure show 1000σ errors in frequencies of each mode in the GONG data. For comparison, the frequencies of a standard solar model are also shown. It is clear that

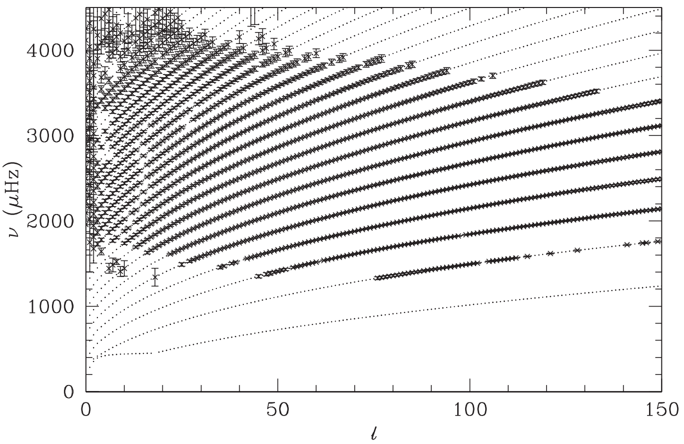

Figure 1. The l–ν diagram calculated using GONG data is compared with that for a standard solar model. The dots which have almost merged into a line represent frequencies of a solar model, while points with error bars are the observed frequencies. The error bars show 1000σ errors.

the model frequencies agree very well with the observed frequencies, thus giving us confidence in the solar model as well as the identification of observed modes with the acoustic modes of the sun. The various lines in this figure show the modes with different values of n starting with $n = 0$ as the lowest line. The $n = 0$ modes are referred to as the fundamental or f-mode and for large values of l these are essentially the surface gravity modes. The higher values of n correspond to the acoustic or p-modes where the pressure gradient provides the main restoring force. These are essentially sound waves propagating through the solar interior. Apart from these there is another series of modes known as gravity or g-modes with negative values of n. So far there is no reliable detection of these modes in the sun and hence these are not shown in Fig. 1.

Depending on the frequency and degree, the p-modes are trapped in different regions of the solar interior. As the sound waves travel inwards, the temperature and hence the sound speed increases and the waves are refracted away from the radial direction, until at some point they suffer a total internal reflection and are directed outwards. This defines the layer above which the mode is trapped. The critical layer depends on the angle of inclination to the radial direction at the surface, which is determined by the horizontal wavelength or the degree l. Thus modes with small values of l are closer to the vertical direction and hence penetrate to deeper layers before they get reflected outwards. While modes with large l are trapped in the layers immediately below the surface. The radial mode ($l = 0$) penetrates to the centre of the sun. This is an important property of acoustic modes which gives them the diagnostic power. Since different modes are trapped in different regions their frequencies are determined by the structure variation in the corresponding regions. Conversely, by studying the properties of a large number of acoustic modes we can infer the internal structure of the sun. A large number of inversion techniques have been developed for this purpose (Gough & Thompson 1991). Using these inversion techniques it is possible to determine the sound speed and density in the solar interior and the results are shown in Fig. 2. This figure shows the relative difference in sound speed and density between the sun and the standard solar model of Christensen-Dalsgaard *et al.* (1996).

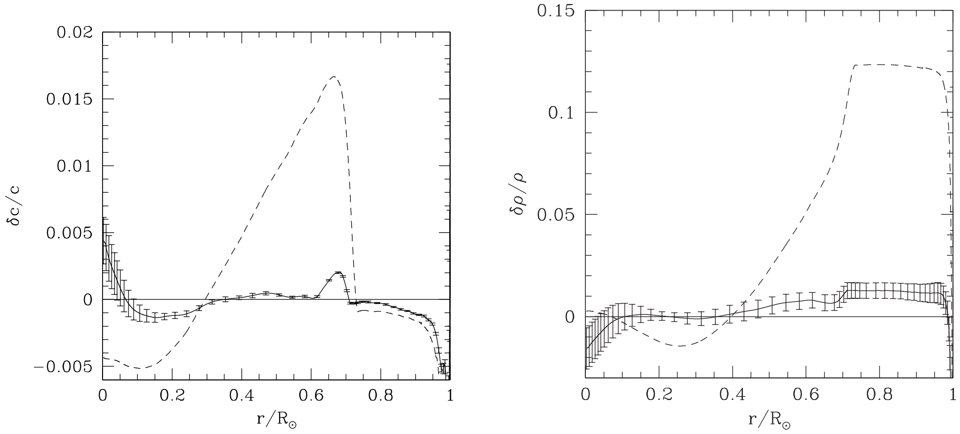

Figure 2. Relative difference in sound speed and density between the sun and the standard solar model of Christensen-Dalsgaard *et al.* (1996) is shown by *solid* lines with error bars. The *dashed* lines show the results obtained using a solar model with recent abundances.

From Fig. 2, it is clear that the sound speed and density in the standard solar model is close to that in the sun. This along with other considerations led to the conclusion that the low solar neutrino fluxes observed in the early experiments are due to neutrino oscillations (Bahcall *et al.* 2001 and references therein). Recently, these neutrino oscillations have been confirmed by neutrino detectors at the Sudbury Neutrino Observatory (Ahmad *et al.* 2002). This was regarded as a significant achievement of solar models and helioseismology. However, soon after this, the agreement between the solar model and the seismically inferred structure deteriorated when Asplund *et al.* (2004a, b) found that the oxygen abundance in the solar photosphere should be reduced by a factor of 1.5. Along with oxygen the abundances of other elements were also reduced. This reduces Z/X in the solar envelope from 0.023 (Grevesse & Sauval 1998) to 0.0165 (Asplund *et al.* 2004b), thus reducing the opacity of the solar material. This has introduced significant differences between the standard solar model and the seismically inverted structure profiles (Basu & Antia 2004; Bahcall *et al.* 2005) as well as in the depth of the convection zone (Bahcall & Pinsonneault 2004). Figure 2 also shows the difference in sound speed and density between a solar model constructed with revised abundances and the sun. It is clear that the differences are almost an order of magnitude larger than those with the earlier model using older abundances. The model with revised abundances also has a shallow convection zone. In order to get the correct depth of the convection zone, it is necessary to increase the opacity by about 20% as compared to the standard OPAL (Iglesias & Rogers 1996) values near the base of the convection zone (Basu & Antia 2004; Bahcall *et al.* 2004; Guzik & Watson 2005). It is unlikely that errors in theoretically computed opacities are so large. Recent independent computations by the OP project (Badnell *et al.* 2004) find a difference of only 2% between OP and OPAL opacities near the base of the convection zone. Increased diffusion of helium and heavy elements below the convection zone also does not solve the problem as in that case the resulting helium abundance in the solar envelope is much less than the seismically inferred value (Basu & Antia 2004).

The fact that a solar model using older abundances is remarkably close to the seismically inferred structure suggests that the revised abundances need to be checked independently. The revised abundances have been obtained using improved 3D hydrodynamic model atmosphere. Nevertheless, there could be some systematic errors due to inadequate treatment of turbulence or other effects. Antia & Basu (2005) have found that among all heavy elements the abundances of O, Ne and Fe are most important in determining the opacity near the base of the convection zone. Of these elements, the abundance of Ne has not been determined from photospheric lines and hence there could be larger uncertainties in its determination. If the abundance of Ne is increased by a factor of 2.5 as compared to the recently reduced value, it can compensate for much of the reduction in oxygen abundance (Antia & Basu 2005). It is not clear if this much increase in Ne abundance is permissible. It is clear that more work is required to verify the abundance determination as well as opacity calculations to resolve this discrepancy.

3. Rotation rate in the solar interior

From the observed rotational splitting of modes it is possible to determine the rotation rate in the solar interior (Thompson *et al.* 1996; Schou *et al.* 1998). Inversions of observed splitting coefficients have shown that the observed differential rotation at the solar surface continues throughout the convection zone, but near the base of the convection zone there is a transition to rotation rate that is independent of latitude. In the radiative interior the rotation rate is roughly constant. The transition region where the rotation rate changes from differential rotation to solid body like rotation is referred to as the tachocline (Spiegel & Zahn 1992). This is a region with strong shear and is generally believed to be the layer where solar dynamo is operating. This shear layer is also believed to cause some mixing in the tachocline region (Richard *et al.* 1996; Brun *et al.* 1999).

With the accumulation of seismic data over the last nine years it is possible to study the temporal variation in the solar structure and rotation rate. This may help us in understanding the operation of solar dynamo. The frequencies of solar oscillations are known to vary with solar activity and in fact, the variation is well correlated with solar activity (Libbrecht & Woodard 1990; Howe *et al.* 1999; Bhatnagar *et al.* 1999). These frequency differences can be inverted to study the variation in solar structure with solar cycle. These inversions show that almost all of the observed variation in solar oscillation frequencies is due to variation in surface layers and there is no significant variation in the solar interior. Basu and Mandel (2004) have found a marginally significant variation in the He II ionisation region close to the solar surface. However, inversion of rotational splittings show a significant temporal variation in the rotation rate in the solar interior (Howe *et al.* 2000; Antia & Basu 2000) which is similar to the observed torsional oscillations (Howard & LaBonte 1980) at the solar surface.

To study the temporal variations in the rotation rate we can define the residual rotation rate:

$$\delta\Omega(r,\theta,t) = \Omega(r,\theta,t) - \langle\Omega(r,\theta,t)\rangle, \tag{2}$$

where the angular brackets denote averaging over time. It is often more convenient to show the zonal flow velocity $\delta v_\phi = \delta\Omega r\cos\theta$ where θ is the latitude. Figure 3 shows the contours of constant zonal flow velocity at $r = 0.98R_\odot$ and at $\theta = 15^\circ$. These figures show a clear pattern with bands of faster (or slower) than average velocity

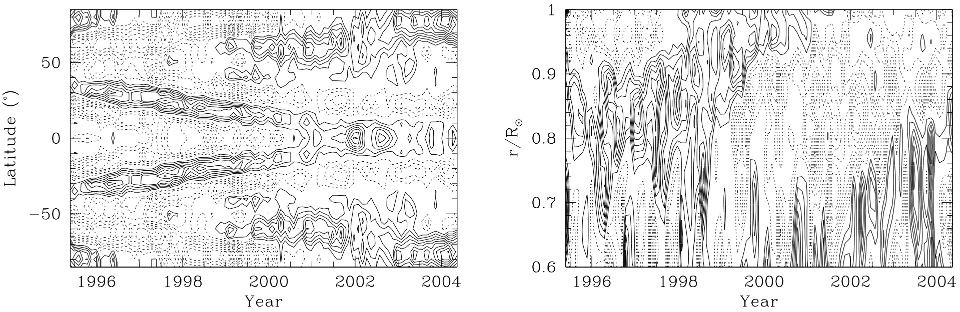

Figure 3. Contours of constant zonal flow velocity are shown as a function of time and latitude at $r = 0.98R_\odot$ (left panel) and as a function of time and radius at a latitude of 15° (right panel). The continuous lines show positive values and the dotted lines show negative values. The contour interval is 1 m s^{-1}.

moving with time. Further, these bands move towards the equator at low latitudes (Howe *et al.* 2000; Antia & Basu 2000) while at high latitudes these bands move towards the poles (Antia & Basu 2001). From the figure for $\theta = 15^\circ$ it can be seen that the bands are rising upwards with time and they continue almost to the base of the convection zone. Thus the zonal flow pattern continues throughout the convection zone.

4. The solar radius

The standard value of solar radius, $R_\odot = (695.99 \pm 0.07)$ Mm was first obtained by Auwers (1891). Since then a number of improved measurements have been made giving different results. Because the sun is not a solid object there is some ambiguity in the definition of solar surface and hence its radius. Conventional measurements use the point of inflexion in the intensity profile as the location of surface, while the solar models use a level where the optical depth (measured from outside) is of an order of unity. Using detailed atmospheric models Brown & Christensen-Dalsgaard (1998) have shown that there would be a difference of about 500 km between the two definitions of the solar radius, with the model definition being smaller. The solar oscillation frequencies are very sensitive to the solar radius and hence can be used to determine the solar radius. In particular, the f-modes, which are essentially surface gravity modes are well suited for this purpose. The frequencies of f-modes are essentially independent of stratification and are approximately given by the dispersion relation $\omega^2 \approx gk = GM\sqrt{l(l+1)}/r^3$ where g is the acceleration due to gravity at the surface, $k = \sqrt{l(l+1)}/r$ is the horizontal wave number, G is the gravitational constant. By comparing the observed frequencies with that in a solar model with given radius it is possible to estimate the radius (Schou *et al.* 1997; Antia 1998). Using observed frequencies from GONG and MDI data it is found that the solar model radius should be reduced by 200–300 km from the standard value. The exact value depends on the modelling of near surface layers in the solar model (Tripathy & Antia 1999). These systematic errors arising from the model are expected to be independent of time and hence the f-mode frequencies may be used to detect temporal variations in the solar radius at a level of 1 km.

There are conflicting reports about temporal variation in solar radius, with claims ranging from 0 to 700 km (Laclare *et al.* 1996 and references therein). Recent observations from MDI have put an upper limit of 5 km on possible temporal variation in solar radius during the current solar cycle (Kuhn *et al.* 2004). It can be easily seen that a variation in solar radius by δR will change the gravitational potential energy by $(GM_\odot^2/R_\odot^2)\delta R \approx 5 \times 10^{42}\delta R$ ergs, if δR is in km. If this variation is over a solar cycle of 11 years (3.5×10^8 sec) the energy is released or absorbed at the rate of $1.4 \times 10^{34}\delta R$ ergs per second. Thus even if $\delta R = 1$ km the rate of energy variation is more than the solar luminosity. Hence it is clear that if there is any significant variation in solar radius, it must be confined to outer layers which have little mass. Even if the radius variation extends till the base of the convection zone, the energy will be released or absorbed at a rate that is 2% of the above estimate, which is still quite large.

Thus it would be interesting to study these temporal variations using f-mode frequencies. Unfortunately, these studies have also given conflicting results with variations between 0 and 5 km (Dziembowski *et al.* 1998, 2000, 2001; Antia *et al.* 2000, 2001; Antia 2003). The dispersion in the results is because the actual variation in f-mode frequencies is more complicated with at least two independent components (Antia *et al.* 2001). One of the time varying components is found to have a period of almost exactly 1 year and is most likely to be an artifact in data due to the orbital motion of the earth. The second component appears to be correlated with solar activity but it has a steep dependence on degree l and hence is unlikely to be due to radius variation. Dziembowski *et al.* (2001, henceforth DGS) tried to decompose the f-mode frequency variation into two components, one arising from radius variation and another from some variation in the surface layer, which is expected to scale inversely with the mode inertia. Thus they assume

$$\Delta\nu_{l,0} = -\frac{3}{2}\frac{\Delta R}{R}\nu_{l,0} + \frac{\Delta\gamma}{I_{l,0}}, \tag{3}$$

where $\Delta\nu_{l,0}$ is the variation in f-mode ($n = 0$) frequency, ΔR is the variation in radius, $I_{l,0}$ is the mode inertia and $\Delta\gamma$ measures the variation in the surface contribution. This expression does not account for the oscillatory component with a period of one year. Nevertheless, DGS computed ΔR and $\Delta\gamma$ for each set of observed frequencies from MDI instrument and concluded that the radius is decreasing at the rate of (1.5 ± 0.3) km per year during the rising phase of solar activity. Figure 4 shows the result of the same exercise using an extended data set that is now available. It can be seen the χ^2 per degree of freedom in these fits are fairly large and it is clear that the data cannot be fitted by a simple expression given by equation (3). Further, the resulting variation in $\Delta\gamma$ is correlated to the solar activity, while radius does appear to decrease with time. Both these quantities show variation with a time period of 1 year, which has not been removed from the data. Apart from this oscillatory variation it appears that the radius has suddenly decreased by about 4 km around 1999, rather than a gradual decrease as claimed by DGS. If this variation is real then it would be difficult to explain. This time period coincides with the period when contact with SOHO satellite was lost and it is quite likely to be due to some systematic errors introduced during the recovery of the satellite.

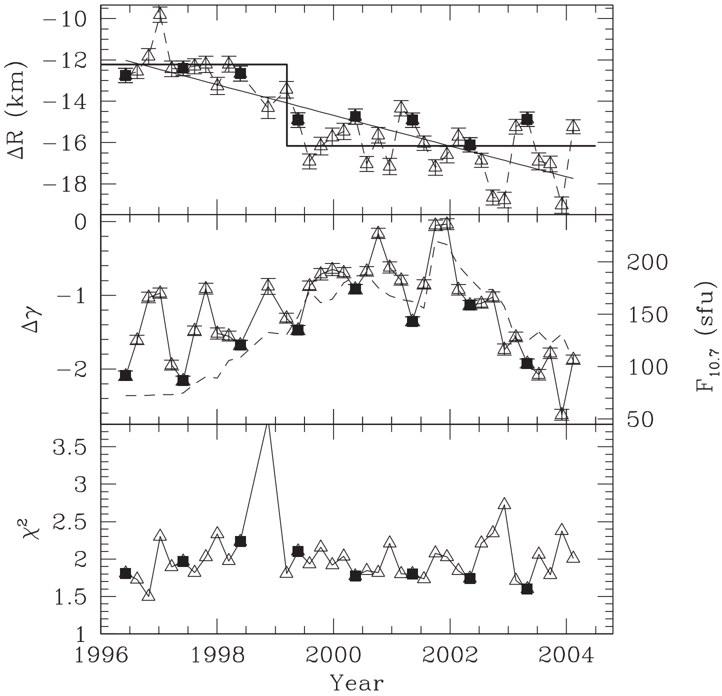

Figure 4. The estimated variation in the solar radius, ΔR, and the surface term, $\Delta\gamma$ from f-mode frequencies, obtained by fitting equation 3 to frequency difference between a given MDI set and a solar model. The χ^2 per degree of freedom for each set is shown in the lowest panel. In each panel the filled squares are the results for data sets at an interval of 360 days for which the fit is relatively good. The solid line in the top panel is a straight line fit to all points, similar to that obtained by DGS. The heavy line shows a step function fit to all points with discontinuity at 1999.2. The dashed line in the middle panel shows the 10.7 cm radio flux on a scale shown at the right.

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