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Seismic Study of Magnetic Field in the Solar Interior

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Abstract. Magnetic field in the solar interior contributes to the even order splitting coefficients, but it is not possible to separate the effect of magnetic field from those due to other deviations from spherical symmetry. Results obtained using GONG and MDI data are discussed. Limits on possible magnetic field in the solar core and in the tachocline region are obtained. There is some signal from possible magnetic field in the convection zone, but evidence of possible temporal variation in the solar interior is only marginal.

Key words. Sun: oscillations, interior, magnetic field.

1. Introduction

During the last decade detailed observations of solar oscillations have provided a unique tool to study the structure and dynamics of the solar interior. The frequencies of solar oscillations are characterised by the three quantum numbers, the radial order n, the degree l and the azimuthal order m. If the Sun was spherically symmetric the frequencies would be independent of m. Departures from spherical symmetry due to rotation, magnetic field or other sources lift the degeneracy and give rise to frequency splittings in modes of same n, l. The mean frequency of the multiplet is determined by the spherically symmetric structure of the Sun while the frequency splittings are determined by rotation, magnetic field and any other aspherical perturbations to the solar structure. To the first order rotation affects only the odd splitting coefficients, i.e., odd function of m and hence its effect can be separated from those of magnetic field and other asphericities, which contribute only to even splitting coefficients. The mean frequencies have been successfully used to study the structure of the solar interior (Gough *et al.* 1996) while the odd splitting coefficients have been used to study the rotation rate as a function of radius and latitude in the solar interior (Thompson *et al.* 1996; Schou *et al.* 1998). The inferred rotation rate can be used to compute the second order contribution from rotation to the even order splitting coefficients. These can then be subtracted from the observed coefficients to get the effect of magnetic field and other contributions to asphericity. Unfortunately, it is not possible to distinguish between the contributions of a magnetic field and aspherical perturbation to the solar structure.

The even order splitting coefficients are fairly small, and it is difficult to detect any signature of possible magnetic field in the solar interior. Dziembowski & Goode (1989) claimed to find evidence for a mega Gauss field near the base of the convection zone, using data from the Big Bear Solar Observatory. Improved data from the Global Oscillation Network Group (GONG) project (Hill *et al.* 1996) and the Michelson Doppler Imager (MDI) instrument on board SOHO (Rhodes *et al.* 1997) have not confirmed these results (Antia *et al.* 2000). Instead of looking for signature of the magnetic field, we can assume that the even order splitting coefficients arise from asphericity in the solar structure. In this case, it is possible to apply an inversion technique to determine the latitudinal distribution of the sound speed and density (or other independent structure variables) in the solar interior (Antia *et al.* 2001). Of course, there is no way to distinguish whether the actual signal is due to magnetic field or asphericity. However, the advantage of this approach is that it is possible to identify the location of perturbation which is responsible for the even splitting coefficients.

The GONG and MDI instruments have been observing the Sun more or less continuously for the last 11 years and hence it is also possible to study possible temporal variation in the internal magnetic field or asphericity. It is well known that the frequencies of solar oscillations vary with time and this variation is correlated with the solar activity indices (Elsworth *et al.* 1990; Libbrecht & Woodard 1990; Bhatnagar *et al.* 1999). Similarly, the even splitting coefficients are also known to vary with time and their variation is correlated to the corresponding component of observed magnetic flux at the solar surface (Libbrecht & Woodard 1990; Woodard & Libbrecht 1993; Howe *et al.* 1999; Antia *et al.* 2001). However, most of these temporal variations are found to arise from perturbations near the solar surface (Basu & Antia 2000; Antia *et al.* 2001). There is only marginal evidence for any significant temporal variation in the solar interior. It is generally believed that the solar dynamo operates in the tachocline region and it would be interesting to study the possible presence of magnetic field and any associated temporal variations during the solar cycle.

2. Effect of magnetic field on solar oscillation frequencies

Since the forces due to rotation and magnetic field are much smaller than those due to gravity and pressure gradient, we can treat rotation and magnetic field as small perturbations over the spherically symmetric structure. This gives rise to splitting of frequencies with same n, l but different m. It is convenient to define the splitting coefficients by

$$\nu_{n,l,m} = \nu_{n,l} + \sum_j a_j^{n,l} P_j^{n,l}(m), \tag{1}$$

where, $\nu_{n,l}$ is the mean frequency of the multiplet, $a_j^{l,n}$ are the splitting coefficients and $P_j^{n,l}(m)$ are a set of orthogonal polynomials of degree j in m (Ritzwoller & Lavely 1991; Schou *et al.* 1994). The first order effect of rotation due to the Coriolis force affect only the odd order splitting coefficients, $a_1, a_3, a_5, \ldots$ and these have been used to infer the rotation rate in the solar interior. The even splitting coefficients arise from second order effects of rotation, through the centrifugal force and magnetic field or other departures from spherical symmetry. Since forces due to rotation or magnetic field in the Sun are smaller by about 5 orders of magnitude as compared to gravitational forces, it is possible to apply a perturbative treatment to calculate their contribution to frequency splittings (Gough & Thompson 1990). The rotation rate in the solar interior can be inferred from the odd splitting coefficients and this can be used to estimate its contribution to the even splitting coefficients. This estimated contribution can be subtracted from the observed coefficients to get the residuals which are due to magnetic field or other aspherical perturbations.

Following Gough & Thompson (1990) we consider a toroidal magnetic field of the form,

$$\mathbf{B} = \left[0, 0, a(r)\frac{dP_k(\cos\theta)}{d\theta}\right], \tag{2}$$

with the axis of symmetry coinciding with the rotation axis. Here, $P_j(x)$ is the Legendre polynomial of degree j and

$$a(r) = \begin{cases} \sqrt{8\pi p_0\beta_0}\left(1-\left(\frac{r-r_0}{d}\right)^2\right) & \text{if } |r-r_0| \le d \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where p_0 is the gas pressure, β_0 is the ratio of magnetic to gas pressure, which is assumed to be constant, r_0 and d are constants which determine the mean position and half-thickness of the layer where the field is concentrated. Using this magnetic field we calculate all second order contributions to the frequency splittings, which include the direct effect due to the Lorentz force and the contribution due to distortion from spherical symmetry introduced in the equilibrium state.

The splitting coefficients calculated from a given magnetic field configuration can be compared with observed splittings. We use the datasets from both GONG and MDI for this purpose. These datasets provide the mean frequency and the splitting coefficients for each n, l multiplet. We use 110 sets of GONG data each covering 108 days, starting from May 7, 1995 to April 14, 2006. The MDI data consist of 49 sets each covering 72 days, starting from May 1, 1996 to May 20, 2006. There was a break in MDI data between July 1998 and January 1999, when contact with SOHO was lost.

Since solar dynamo is believed to operate in the tachocline region we first consider magnetic field concentrated in the tachocline to calculate the splitting coefficients. These show a distinct signature in modes with lower turning point near the tachocline region. No such signature was found in observed splitting coefficients (Antia *et al.* 2000) and an upper limit of 300 kG in the tachocline region was estimated from these data. Even if the upper limit is reduced with accumulation of more data, it does not provide any significant constraint on theories of solar dynamo. The dynamo theories generally assume that the magnetic field is concentrated in flux tubes with a small filling factor, while in this work we have assumed an average field filling the whole tachocline region. If the dynamo field in flux tubes is averaged over the relevant region, its magnitude will decrease significantly, depending on the filling factor.

2.1 *Magnetic field in the solar core*

We can calculate the splitting coefficients due to a magnetic field in the solar core, but only a few modes penetrate to this region and the splitting coefficients of the low degree modes that penetrate to the core have large errors. Hence, it is difficult to make out any signal of possible magnetic field in the splitting coefficients. However, most of the solar mass is in the core and any magnetic field in the core will cause significant distortion which would be visible even at the solar surface, unless it is compensated by a suitably large distortion in the outer layers. Using the formalism of Gough & Thompson (1990) it is possible to calculate the distortion at the solar surface due to large scale magnetic field in the interior. If we assume that the magnetic field is given by equation (2) with $\beta_0 = 10^{-4}$, $k = 2$ and $d = 0.1\mathrm{R}_\odot$, we can calculate the resulting oblateness at the solar surface for different values of r_0. When the field is located in the core, the resulting distortion at the surface is comparable to β_0 and as the field is moved outwards the distortion decreases. The estimated distortion is 2×10^{-4}, 6×10^{-5}, 10^{-5}, 6×10^{-6} for $r_0 = 0.2\mathrm{R}_\odot, 0.4\mathrm{R}_\odot, 0.6\mathrm{R}_\odot, 0.8\mathrm{R}_\odot$, respectively (Antia 2002).

This distortion can be compared with observed distortion at the solar surface of $-(5.4 \pm 0.5) \times 10^{-6}$ (Kuhn *et al.* 1998). The seismically inferred rotation profile yields a distortion of -5.8×10^{-6} (Antia *et al.* 2000). Thus the unaccounted distortion at the surface is less than 10^{-6}. This can be compared with the expected distortion due to magnetic field to set an upper limit on the magnetic field in the core. The magnetic contribution will of course, depend on the form of the magnetic field and the region where it is located. Allowing for some uncertainties in that we can put a conservative upper limit of $\beta_0 = 10^{-5}$ in the solar core. This translates to a magnetic field of 7 MG at the centre, 3 MG at $r = 0.2\mathrm{R}_\odot$ and 0.8 MG at $r = 0.4\mathrm{R}_\odot$.

3. Asphericity in the solar structure

Apart from magnetic field and rotation, other departures from spherical symmetry in solar structure can also contribute to the even order splitting coefficients. Unfortunately, it is not possible to distinguish between these. Of course, the presence of magnetic field would yield some asphericity in solar structure, but that contribution is included in the magnetic field effects considered in the previous section. In this section we neglect the magnetic field and assume that even order splitting coefficients are due to asphericity in solar structure. Since the solar convection zone is not in hydrostatic equilibrium, such asphericities could arise due to anisotropy in turbulence. The advantage of this approach is that we can apply the inversion techniques to localise the perturbation (Antia *et al.* 2001). The splitting coefficients are sensitive only to the north-south symmetric component of asphericity and hence only this component can be determined. The differences in the sound speed, c, and the density, ρ, with respect to a spherically symmetric solar model can be expressed as (Antia *et al.* 2001)

$$\frac{l a_{2k}^{nl}}{\nu_{nl}} = \frac{Q_{lk}(4k+1)}{2} \int_0^R dr \int_0^\pi \sin\theta \, d\theta \left(\mathcal{K}_{c^2,\rho}^{nl} \frac{\delta c^2}{c^2} + \mathcal{K}_{\rho,c^2}^{nl} \frac{\delta\rho}{\rho} \right) P_{2k}(\cos\theta) + Q_{lk} \frac{F_k(\nu_{nl})}{E_{nl}}, \quad (4)$$

where E_{nl} is the mode inertia (Christensen-Dalsgaard 2002) and Q_{lk} is a geometric factor as defined by Antia *et al.* (2001). Here $F_k(\nu)$ are the surface terms which accounts for uncertainties in the treatment of surface layers. This would also include contributions from outer layers which cannot be resolved by the mode set that is used for inversion.

Equation (4) can be used for inversion to determine the sound speed and density as a function of radial distance and latitude. Further, using data collected at different times, it is also possible to study the temporal variations in these quantities. It is well-known that the mean frequencies as well as the even order splitting coefficients vary with time and it would be interesting to study if these imply any variation in solar structure with solar cycle. To bring out the temporal variations in the sound speed it is convenient to subtract the temporal average at each latitude and radius to obtain the residual which is the temporally varying component.

Figure 1 shows the results for aspherical component of $\delta c^2/c^2$ obtained from GONG data at $r = 0.96\mathrm{R}_\odot$ as a function of time and latitude. There is no clear pattern in the residuals, thus suggesting that there is no significant temporal variation in sound speed. Similar results are obtained at other depths and it appears that most of the temporal variations in the even splitting coefficients is due to the surface term. Figure 2 shows

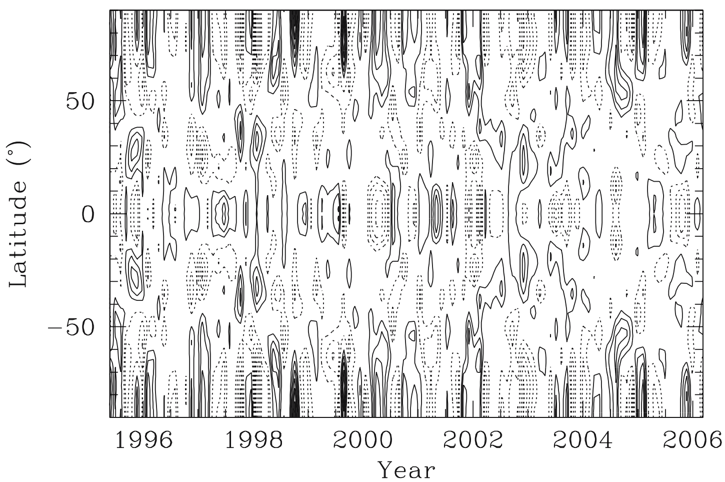

Figure 1. The contours of constant residual $\delta c^2/c^2$ at $r = 0.96\mathrm{R}_\odot$ as a function of time and latitude for GONG data. Solid contours represent positive (greater than average) values, while dotted ones show negative values. The contour spacing is 2×10^{-5}.

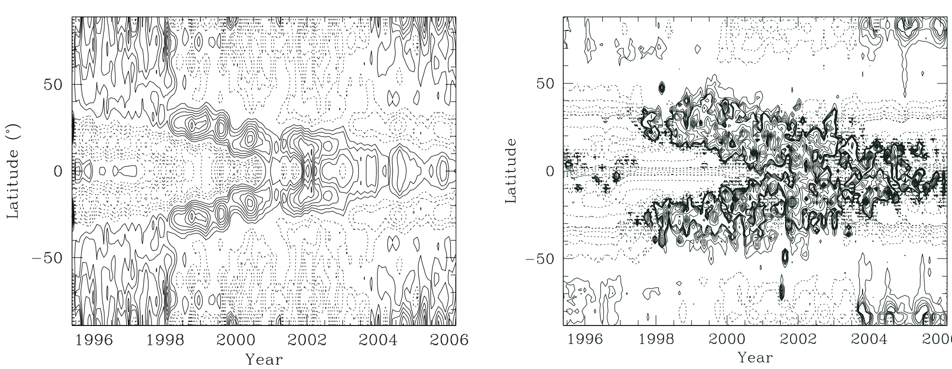

Figure 2. The surface term in asphericity inversion from GONG data (left panel) is compared with surface magnetic flux (right panel). Solid contours represent positive (greater than average) values, while dotted ones show negative (less than average) values.

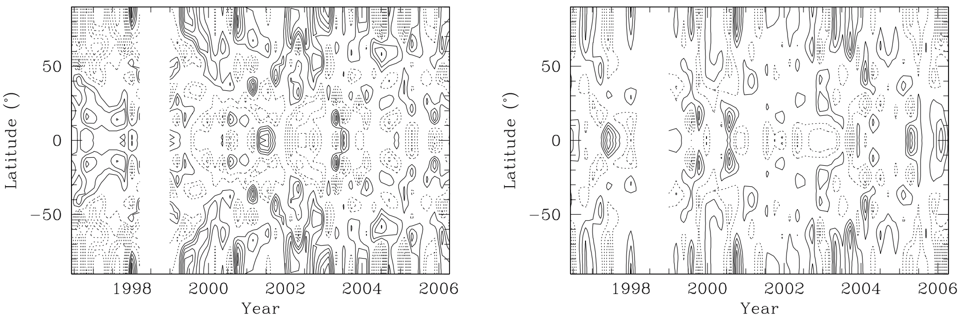

Figure 3. The contours of constant residual $\delta c^2/c^2$ at $r = 0.96\mathrm{R}_\odot$ as a function of time and latitude for MDI data. The left panel shows the results using all modes, while the right panel shows the results using only modes with $l < 110$. Format is the same as that in Fig. 1.

similar variation in the surface term compared with that in the observed magnetic flux at the solar surface. It is clear that the surface term is well correlated with the observed magnetic flux at the solar surface. Thus it is quite likely that observed asphericity near the solar surface is actually due to magnetic field.

The left panel in Fig. 3 shows $\delta c^2/c^2$ obtained using MDI data. These results clearly show some temporal variations, but a closer look suggests that most of the variation has occurred during the data gap. Before the gap $\delta c^2/c^2$ is positive at low latitude and negative at high latitudes, while after the gap the situation is reversed. There are two possibilities, either the Sun had some interesting transition exactly when MDI was not operational or the difference is due to some instrumental variation during the recovery of the satellite. Since the GONG data do not show this variation, it is likely to be due to instrumental effect. This is further confirmed by the right panel, which shows the same results obtained from MDI data using only the modes with $l < 110$, which doesn't show much variation. Only high l modes appear to be affected by this artifact. These systematic errors in MDI data are likely to affect other results too (e.g., Antia 2003).

Since there is no significant temporal variation in asphericity, we can take temporal average over all datasets to improve accuracy and the results using GONG data are shown in the left panel of Fig. 4. MDI data give similar results if we restrict to modes with $l < 110$. The typical error in these results is about 10^{-5}, which is the contour spacing. The errors increase with depth and latitude. The figure shows a broad peak around $r = 0.9\mathrm{R}_\odot$ and latitude of 60°. The maximum asphericity in c^2 is about 6×10^{-5}. This is well inside the convection zone and its origin is not clear. If this peak is due to magnetic field, then to an order of magnitude we can expect $\delta c^2/c^2 \approx v_A^2/c^2$, where v_A is the Alfvén speed. This would translate to a magnetic field of about 55 kG at $r = 0.9\mathrm{R}_\odot$. It is not clear how such a field would survive inside the convection zone. This asphericity or magnetic field doesn't appear to have significant variation with solar activity.

Although there is no clear temporal variation in asphericity, in order to improve the statistics we can take temporal averages during the high and low activity phases and look at the difference between these. The results are shown in the right panel of Fig. 4. Here the low activity period is taken to be when the 10.7 cm radio flux is less than 80 s.f.u. while the high activity period is considered to be when the flux is more

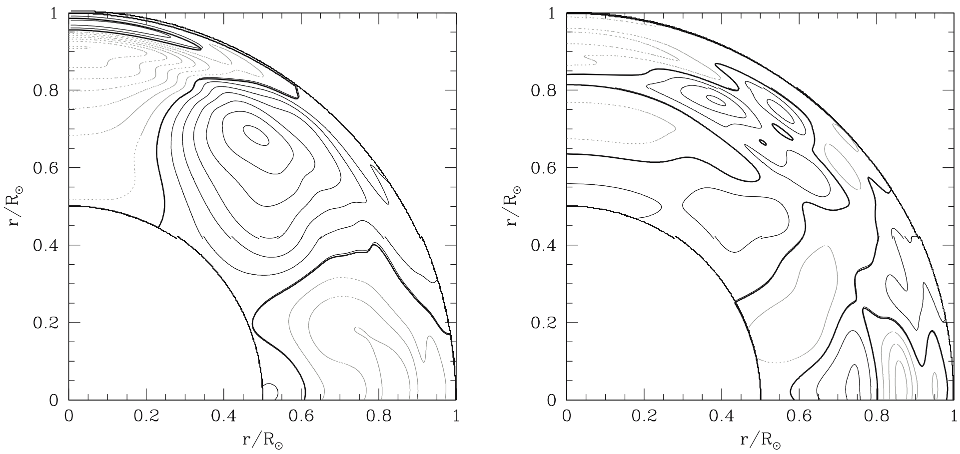

Figure 4. The contour diagram of the temporally averaged $\delta c^2/c^2$. Because of symmetry only one quadrant is shown. The horizontal axis represents the equator, while the vertical axis is the rotation axis. The dark contours represent positive values while light contours show negative value. The thick contour marks the zero level. The contour spacing is 10^{-5}. The left panel shows the results using temporal averaging over all GONG datasets. The right panel shows the difference between the high-activity (>140 s.f.u.) and low-activity (< 80 s.f.u.) periods.

than 140 s.f.u. There is a marginal variation in some parts which may be significant. Similar conclusions were obtained by Antia *et al.* (2003).

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