DC field induced enhancement and inhibition of spontaneous emission in a cavity

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We demonstrate how spontaneous emission in a cavity can be controlled by the application of a dc field. The method is specially suitable for Rydberg atoms. We present a simple argument based on Stark shifts for the control of emission.

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The manipulation of spontaneous emission has been extensively studied. Methods involving either external fields [1, 2, 3, 4, 5, 6, 7, 8] or appropriate cavities [9, 10] have been suggested. Use of external fields enables one to control spontaneous emission via quantum interference effects [5, 6]. Purcell [11] recognized how the emission rate in a single mode cavity can be much higher than in free space. Several demonstrations of the cavity enhanced spontaneous rates exists in the literature [9, 10]. Kleppner [12] discovered that the radiation rate in a cavity can be inhibited by choosing the transition frequency such that the density of states at this frequency is insignificant. Quantum interference between various channels [3, 4, 6, 7] could also result in the inhibition of emission. Further very interesting experiments [1, 2, 6, 7, 8] on the field induced inhibition of emission in a cavity were reported. In these experiments the applied fields were resonant with atomic transitions. In this paper we show how a possible control of spontaneous emission can be obtained by using dc fields. We treat the case of atoms in a cavity and explain in rather simple terms the origin of the control produced by the dc fields. To be precise we are considering only the effect of dc field on the part of decay which is due to the emission in the cavity mode. The decay of the atom depends on the detuning between the atomic frequency and the cavity frequency. The application of the dc field makes the detuning dependent on the field (Stark effect) and thus the dc field provides a control of the spontaneous emission. The dc field induced modification of spontaneous emission in free space is treated in reference [3].

We next describe how to calculate the dc field induced modification of the decay characteristics in a cavity. For our purpose we consider a two level atom placed in a cavity and a dc field (or low frequency field) is injected inside the cavity . The Hamiltonian of the system can be written as,

$$\begin{aligned} H &= \hbar\omega_0 S^z + \hbar\omega_c a^\dagger a + \hbar g\left(aS^+ + S^- a^\dagger\right) \\ &\quad + \hbar\mathcal{E}\cos\Omega t\left(S^+ + S^-\right), \end{aligned} \tag{1}$$

where ω_0 is atomic transition frequency, ω_c is cavity mode frequency and g is the atom cavity coupling constant. The term $\mathcal{E}\cos\Omega t$ corresponds to a low frequency field if Ω is chosen to be very small. Note that $\mathcal{E}$ has dimensions of frequency. The cavity field has been expressed in terms of annihilation and creation operators a, $a^\dagger$ and S^+, S^-, S^z are usual atomic spin operators. We perform master equation calculation for atom-cavity system. The density matrix of the system ρ will evolve as,

$$\dot\rho = -\frac{i}{\hbar}[H,\rho] - \kappa\left(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a\right), \tag{2}$$

where 2κ gives the leakage of photons. It is related to the cavity Q via $\kappa = \omega_c/2Q$. We will work in a frame rotating with atomic frequency ω_0 The density matrix in this frame is given by

$$\tilde\rho = e^{i\omega_0(S^z+a^\dagger a)t/\hbar}\rho e^{-i\omega_0(S^z+a^\dagger a)t/\hbar}. \tag{3}$$

Using Eq(2) and (3) we obtain the equation for $\tilde\rho$

$$\begin{aligned} \dot{\tilde\rho} &= -\frac{i}{\hbar}[H_a,\tilde\rho] - \kappa\left(a^\dagger a\tilde\rho - 2a\tilde\rho a^\dagger + \tilde\rho a^\dagger a\right) \\ &\quad -\frac{i}{\hbar}[H_d,\tilde\rho], \end{aligned} \tag{4}$$

where

$$\begin{aligned} H_a &= -\hbar\Delta a^\dagger a + \hbar g\left(aS^+ + S^- a^\dagger\right), \\ H_d &= \hbar\frac{\mathcal{E}}{2}\left\{S^+\left(e^{i(\omega_0+\Omega)t} + e^{i(\omega_0-\Omega)t}\right)\right. \\ &\quad \left. + S^-\left(e^{-i(\omega_0+\Omega)t} + e^{-i(\omega_0-\Omega)t}\right)\right\}, \end{aligned} \tag{5}$$

and $\Delta = \omega_0 - \omega_c$ is the detuning. We first note that the experiments of Lange and Walther correspond to using a microwave field and thus $\Omega \sim \omega_0$. The results of Purcell and Kleppner also follow from the master equation (4). For $\mathcal{E} = 0$ and $g << \kappa$, we can derive an equation for the atomic density matrix $\tilde\rho_a$

$$\tilde\rho_a = Tr_c\tilde\rho, \tag{6}$$

where Tr_c is trace over cavity field, by adiabatically eliminating cavity variables. This leads to

$$\dot{\tilde\rho}_a = -i[\delta_0 S^z,\tilde\rho_a] - \Gamma_0\left(S^+S^-\tilde\rho_a - 2S^-\tilde\rho_a S^+ + \tilde\rho_a S^+S^-\right), \tag{7}$$

where

$$\Gamma_0 = \frac{g^2\kappa}{\kappa^2+\Delta^2}, \quad \delta_0 = \frac{g^2\Delta}{\kappa^2+\Delta^2}. \tag{8}$$

For resonant cavity $\omega_c = \omega_0$, $\delta_0 = 0$ and the decay rate $\Gamma_0 = g^2/\kappa$. There is cavity induced enhancement if g^2/κ

is greater than the free space decay rate. Note that as the cavity is detuned ($\Delta \neq 0$) Γ_0 decreases which is Kleppner's result for a single mode cavity. The first experimental observation of the Purcell effect was made by Goy *et al* [9]. Next we investigate the effect of the applied dc or low frequency field. Note that the last term in the master equation (4) is highly oscillating. We do time averaging for this as such terms oscillating at the cavity frequency would not be normally observed. The time averaging is well justified here as all other relevant time scales g^{-1}, κ^{-1}, Δ^{-1} are much larger than $(\omega_0 \pm \Omega)^{-1}$. The inequality $\omega_0 >> g,\ \kappa,\ \Delta$ enables us to do the time averaging in a much simpler fashion *i.e.* we can essentially ignore the terms having H_a and κ in (4). We relegate the details of time averaging to the appendix. The calculation leads to the following time averaged master equation

$$\begin{aligned}\dot{\tilde{\rho}} \;&=\; i\left[\Delta_e a^\dagger a, \tilde{\rho}\right] - ig\left[\left(aS^+ + S^- a^\dagger\right), \tilde{\rho}\right] \\ &\quad -\kappa\left(a^\dagger a\tilde{\rho} - 2a\tilde{\rho}a^\dagger + \tilde{\rho}a^\dagger a\right), \end{aligned} \tag{9}$$

where

$$\Delta_e = \Delta + 2\omega_0\mathcal{E}^2/(\omega_0^2 - \Omega^2). \tag{10}$$

We note that the dc field contributes to the Stark shift of the two levels in question. We further note that these two atomic levels can also be shifted because of the interaction of the dc field with other levels. These can be accounted for by introducing the polarizabilities α_e and α_g of the levels $|e\rangle$ and $|g\rangle$ [13, 14], we can rewrite Eq(10) as

$$\Delta_e = \Delta + \alpha_0\mathcal{E}_d^2\ ;\quad \alpha_0 = \alpha_e - \alpha_g\ ; \tag{11}$$

where $\mathcal{E}_d$ is now the dc field in esu. The formulation of the appendix can also be used to produce the well known expressions for the $\alpha^{,s}$. The value of α_0 is known for many low lying as well as Rydberg transitions. The values of α_0 have been calculated in the literature by converting infinite sums into the solution of differential equations.

The Eq(9) can be solved assuming that the atom is initially excited and the cavity field is in vacuum state. The Eq(9) can be converted into a set of coupled equations in terms of the states $|e,0\rangle$, $|g,1\rangle$ and $|g,0\rangle$. The results of the numerical integration are shown in the Fig1 for different values of the parameter Δ_e. Clearly there is inhibition as Δ_e increases. The effective detuning Δ_e changes due to the applied dc field . For a fixed cavity detuning Δ the dc field can make Δ_e larger or smaller depending on the sign of Δ. The results can be understood by deriving analytical results in the bad cavity limit [15] $g << \kappa$ (and more precisely $g^2 << \kappa^2 + \Delta_e^2$). In this limit we can obtain a simpler equation for the atomic density matrix $\tilde{\rho}_a$ defined by Eq(6). The final result for the atomic system is

$$\dot{\tilde{\rho}}_a = -i[\delta_e S^z, \tilde{\rho}_a] - \Gamma_e\left(S^+S^-\tilde{\rho}_a - 2S^-\tilde{\rho}_a S^+ + \tilde{\rho}_a S^+S^-\right) \tag{12}$$

where

$$\Gamma_e = \frac{g^2\kappa}{\kappa^2 + \Delta_e^2},\quad \delta_e = \frac{g^2\Delta_e}{\kappa^2 + \Delta_e^2}. \tag{13}$$

Here Γ_e is the dc field modified decay parameter and δ_e is the net frequency shift. The ratio η of the decays in the presence and absence of dc field is given by

$$\eta = \frac{\Gamma_e}{\Gamma_0} = \frac{\kappa^2 + \Delta^2}{\kappa^2 + \Delta_e^2}. \tag{14}$$

Clearly the dc field modifies the decay rate which depends on the detuning. For the cavity resonant to the atomic transition ($\Delta = 0$), using Eq(10), η reduces to

$$\eta = \frac{\kappa^2}{\kappa^2 + \alpha_0^2\mathcal{E}_d^4} \;\approx \left(1 + \frac{4\mathcal{E}^4}{\kappa^2\omega_0^2}\right)^{-1},\ \text{for } \Omega = 0. \tag{15}$$

It is clear from the Eq(15) that dc field inhibits the decay rate. Note that the inhibition starts becoming significant for

$$\alpha_0\mathcal{E}_d^2 \sim \kappa. \tag{16}$$

Let us estimate the condition (16) for Na Rydberg transition $23S_{1/2} \rightarrow 22P_{3/2}$ whose frequency is $340GHz$. For the sake of argument we also assume $\alpha_0\mathcal{E}_d^2 \sim 2\mathcal{E}^2/\omega_0$. This transition has a dipole moment $d \sim 10^{-15}esu$. The atom is placed in the cavity having one mode resonant to the atomic transition. Let us choose the cavity decay rate $\kappa = 1MHz$. The condition (16) then leads to a Rabi frequency $\mathcal{E}$ of the order $400MHz$ which in turn requires a dc field of the order of $10^{-2}esu$. We note that the required dc field is small enough so that the perturbative results for the Stark shift hold. We further note that the scalar and tensor polarizabilities are available for some S and P levels of Na [13, 14] though the absolute values for both $23S_{1/2}$ and $22P_{3/2}$ level are not available in Fabre *et al* [13]. However the reported polarizabilities for say $23P$ level are of the order of few $MHz/(Volt/cm)^2$.

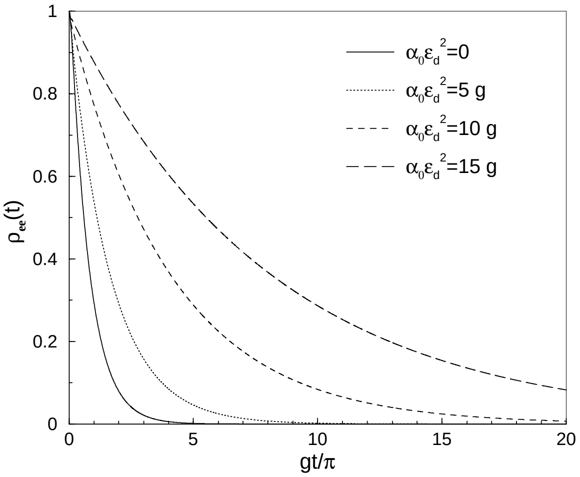

FIG. 1: The probability of the atom remaining in its excited state, $\rho_{ee} \equiv \langle e,0|\rho|e,0\rangle$ vs time, for $\kappa = 5g$, $\Delta = 0$, $\Omega = 0$, and for the different values of the dc field $\mathcal{E}_d$.

Thus the condition (16) is realistic and our finding that the dc field can be used to control spontaneous emission, can be implemented by the appropriate choice of the Rydberg transitions, [*cf* the condition (16)]. We emphasize that we are discussing the inhibition or enhancement of spontaneous emission on a given transition which is resonant with the cavity. This, for example, is the transition $23S \rightarrow 22P$ in the experiments of Goy *et. al* [9]. The authors of ref. [9] emphasize this as well and it is in the spirit of the original suggestion of Purcell [11]. It must be noted that the field ionization techniques enable one to study transitions selectively [16].

In the case of cavities detuned from the atomic transition, spontaneous decay is smaller and the decay rate is given by $\Gamma = g^2\kappa/\left(\kappa^2+\Delta^2\right)$. Further inhibition of decay rate is possible by applying dc field. When cavity is tuned below the atomic transition frequency (Δ is positive) then there is significant inhibition of spontaneous decay, which increases further as the applied dc field is increased. On the other hand when cavity is tuned above the atomic frequency (Δ is negative) there is enhancement in the atomic decay *i.e.* on increasing the value of applied dc field the atom decays faster. In Fig2 we show the behavior of the factor η as a function of Δ for different values of the dc field. The enhancement as well as inhibition of spontaneous decay occurs depending on whether the cavity is tuned above or below the atomic frequency. The results shown in the Fig2 are consistent with the results obtained by direct solution of the Eq(9).

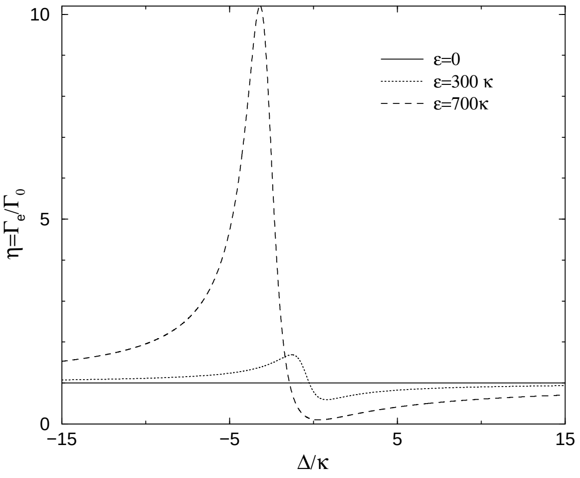

FIG. 2: The ratio (η) of the decays in the presence and the absence of dc field vs Δ/κ . The parameters are $\omega_0 = 3.4 \times 10^5\kappa$ and $\Omega = 0$.

In conclusion we find that in presence of dc field spontaneous emission can be inhibited significantly in the case of cavities resonant to atomic transition. In the case of cavities having negligible mode density around atomic frequency spontaneous emission itself is smaller and the presence of dc field shows significant inhibition or enhancement depending on cavity is tuned below the atomic transition frequency or above the transition frequency.

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APPENDIX A

We outline how the time averaging is to be done. Let us consider schrodinger equation

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -\frac{i}{\hbar}V(t)|\psi(t)\rangle, \tag{A1}$$

where $V(t)$ consists of rapidly oscillating terms only so that the time average of $V(t)$ is zero. Let $|\psi\rangle$ be written as

$$|\psi\rangle = |\bar{\psi}\rangle + |\phi\rangle, \tag{A2}$$

where $|\bar{\psi}\rangle$ is time averaged part and $|\phi\rangle$ is the rapidly oscillating part. On substituting (A2) in Eq (A1) we find that to the lowest order in $V(t)$

$$|\phi\rangle = -\frac{i}{\hbar}\int_0^t V(\tau)d\tau|\bar{\psi}\rangle, \tag{A3}$$

and

$$\frac{\partial}{\partial t}|\bar{\psi}(t)\rangle = -\frac{i}{\hbar}\bar{V}(t)|\bar{\psi}\rangle, \tag{A4}$$

where

$$\bar{V}(t) = -\frac{i}{\hbar}\overline{V(t)\int_0^t V(\tau)d\tau}. \tag{A5}$$

The field induced shift term in (9) is obtained by using Eq(A5).

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