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Opacities of high temperature high Z plasmas*

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Abstract. A procedure is described for computation of opacities for high Z plasmas. Bound-bound, bound-free, free-free and scattering processes are considered. The inputs for these have been obtained by solving IEEOS form of Saha's equation. Detailed calculations of opacities have been done for tungsten and uranium up to 10 keV of temperature.

Keywords. High Z plasma; opacity; bound-bound; bound-free, free-free and scattering.

1. Introduction

The calculation of radiation transport through a heated high Z plasma (by the equation of radiation transport) requires the knowledge of opacities or absorption (emission) coefficients. In laser, electron and ion beam-driven fusion schemes high Z elements have been shown to offer many advantages† (Mason and Morse 1975; Winterberg 1968; Chu 1972; Moses 1977), when used as pusher-tamper shells surrounding the D-T fuel. Apart from increasing the containment time after the thermonuclear burn starts, they aid in compression efficiency. For example, in laser fusion, they enhance the implosion by shielding the fuel from fast suprathermal (energies in excess of 100 keV) electron and high frequency x-ray preheat. Also it has been pointed out that the requirements on pulse shaping may be relaxed if heavy high Z shells are used. Again, enhanced energy transport observed in recent experimental studies of Mizui *et al* (1977) through high Z metal foils, may lead to improved coupling efficiency of laser energy to the D-T fuel by passing the laser beam through a high Z plasma. In this paper, we present a method for calculation of the opacities for high Z plasmas. The results are presented for two typical elements** tungsten ($Z=74$) and uranium ($Z=92$).

2. Calculational procedure

As is well known (Pomaraning 1973), the absorption (emission) of radiation in a plasma depends upon the photon energy and the various photon-electron processes—

*Preliminary results have been presented at the Nucl. Phys. and Solid State Phys. Symp., Dept. of Atom. Energy, held at Ahmedabad, India (December 1976).

†A minor advantage of the use of high Z shells is that because of their high density, they provide structural rigidity and containment of initially gaseous fuel at room temperature.

**The procedure can however be used for any high Z plasma at any density.

the important ones being bound-bound, bound-free and free-free transitions and scattering. The relative importance of each is decided by the temperature of the plasma which determines its composition (different ion concentrations and free electron density) and populations of the various energy levels for a given ion. Mainly for applications in astrophysics (see Carson 1971), many procedures have been evolved for obtaining this information in calculation of opacities for low Z elements, say up to iron (Z=26). One of these is the 'mean ion' model in which the free electron density of the plasma at a given temperature is obtained by the Thomas-Fermi-Dirac equation and the mean ion of charge Z_M postulated. The energy levels of such a mean ion can then be described by a method such as given by Huebner (1970). The other method makes use of the solutions of Saha's ionization equation (or its modified form e.g. CSCP-IEEOS form of Rouse 1971). The latter method is obviously superior to the average ion model as it gives quantitatively the various ion densities present in the plasma. We adopt this method here because as already shown by us (Godwal and Sikka 1977), IEEOS Saha's equation can be reliably solved for high Z elements, with free ion ionization potentials derived from Bohr's formula and charges for various ions obtained from modified Slater type screening constants. The resultant equation of state for W and U was in very good agreement with Thomas-Fermi-Dirac (TFD) equation of state for temperatures from 0·2 to 10 keV (see figure 3 and 4 of Godwal and Sikka 1977).

3. Formulae for different processes

First we give the opacity formula for bound-free (b–f), free-free (f–f) and scattering (TS) processes. In the hydrogenic approximation, b–f absorption coefficient is (Pomaraning 1973)

$$\chi_{\mathrm{b-f}} = \sum_i \sum_n n_n^i \, \sigma_{\mathrm{b-f}}^i \tag{1}$$

$$\sigma_{\mathrm{b-f}}^i = \begin{cases} \dfrac{64\pi^4 e^{10} m_e Z_i^4}{3\sqrt{3} c h^6 \nu^3 n^5} \, g_{\mathrm{b-f}} & h\nu \geqslant I_n \\ 0 & h\nu < I_n \end{cases} \tag{2}$$

Here n_n^i is the number of bound electrons/cc for a given ion i in quantum state n and Z_i is the effective charge of the ion which is taken as

$$Z_i = Z - S \tag{3}$$

where S is the Slater type screening constant due to the inner electrons. The values of S are given by Burns (1964). For hydrogen-like atoms with charge Z_i (R=Rydberg constant)*

$$I_n = R Z_i^2 / n^2. \tag{4}$$

The bound-free Gaunt factor $g_{\mathrm{b-f}}$ has been taken as unity.

*Actually one should not use the isolated ion energy levels, but at normal density, for which we have done calculations, the change is estimated to be negligible (following Rouse 1971, eq. 5A, p. 173).

The free-free absorption coefficient is

$$\chi_{\mathrm{f-f}} = N\sigma_{\mathrm{f-f}} \tag{5}$$

$$\sigma_{\mathrm{f-f}} = 3{\cdot}69 \times 10^{8} \frac{Z_{\mathrm{av}}^{2} N_{e}}{T^{1/2}\nu^{3}} g_{\mathrm{f-f}} \tag{6}$$

Here N_e=free electron density in the plasma and

$$Z_{\mathrm{av}} = \frac{\sum_{i=1}^{Z} C_i Z_i}{\sum_{i=1}^{Z} C_i} \tag{7}$$

Where C_i is the concentration** of the ith ion in the plasma at a given temperature T. The free-free Gaunt factor $g_{\mathrm{f-f}}$ is again set equal to one.

For scattering, for the range of temperatures considered here, the absorption coefficient is taken to be that for Thampson scattering as

$$\chi_{\mathrm{sc}} = 0{\cdot}665 \times 10^{-24} N_e \tag{8}$$

For bound-bound transitions the absorption coefficient is (Carson 1971)

$$\chi_{\mathrm{b-b}} = \sum_i \sum_l \frac{\pi e^2}{m_e C} f_{il} F(\nu) N_i \tag{9}$$

Where f_{il} is the oscillator strength of the ion i between quantum states i and l and $F(\nu)$ is the line profile. The various mechanisms which contribute to the line shape of a spectral line in hot dense matter have been described by Rozsnyai (1977). The estimation of this broadening is complicated but the total absorption of an isolated spectral line is independent of the line profile $(\int F(\nu)d\nu = 1)$. Hence it is easier to obtain mean b–b absorption coefficient than the spectral absorption coefficient. Some jutification for this will be given in the next section.

The summation over i in eq. (9) was limited to the few ionic species which are present in the plasma at the temperature T (see table 1, taken from our Saha's

Table 1. Concentration of various ions in uranium plasma (ρ/ρ_0=1.0) for different temperatures*

T	Ion number 85*	86*	87*	88*	89*	90*
5 keV	0·239	0·589	1·39	2·93	5·91	—
6	0·0034	0·0278	0·205	1·42	9·48	—
8	—	—	0·010	0·73	10·76	0·007
10	—	—	—	0·112	10·8	0·171

*Concentration of the given ion is defined in foot-note on page 5 and each value is multiplied by the factor 10^{-3}—means values less than 10^{-6}.

**$C_i = N_i/N$ where N_i is the number/cc of ion i and N the total number of free particles/cc $= N_e + \sum_{i=1}^{Z} N_i$

equation solution outputs). This simplified the calculations as the oscillator strengths were required for these ion types only. These are readily available in literature for various isoelectronic series both for non-relativistic and relativistic cases (Johnson and Lin 1976; Armstrong *et al* 1976; Lin *et al* 1977; Cheng *et al* 1976). The length form of f_{il}'s was used in preference to the velocity form for reasons as given by Cheng and Johnson (1976) and Starace (1971, 1973). However the differences between the two forms did not produce any significant change in the results. For a given ion, the contributions for all important transitions were considered.

4. Results and discussion

The bound-free, free-free and scattering absorption coefficients (cm^{-1}) were evaluated for normal density tungsten and uranium plasmas for temperatures between 0·5 and 10 keV. The n_n^i's, N_e, C_i's and N_i's were taken from Saha's solutions. Figures 1 and 2 display the variations of χ_{b-f} and χ_{f-f} with photon energies at T=1 keV. As expected, the bound-free opacity shows jumps corresponding to various absorption edges in the ions and the free-free absorption coefficient varies smoothly from high values at low photon energies to low values at high photon energies. The scattering coefficient is of course constant with photon energy.

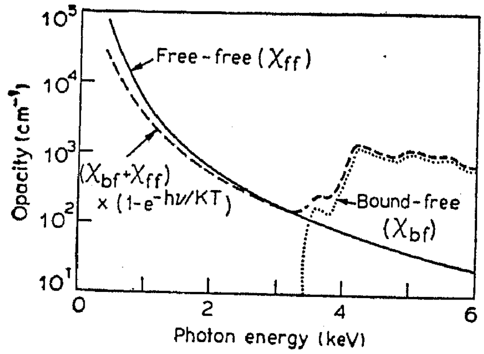

Figure 1. Variation of various contributions to opacities with photon energy for uranium plasma (ρ/ρ_0=1·0) at T=1 keV.

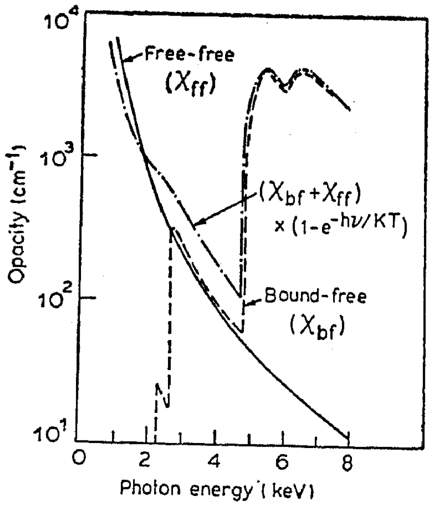

Figure 2. Variation of various contributions to opacities with photon energy for tungsten plasma (ρ/ρ_0=1·0) at T=1 keV.

The mean bound-bound contributions were computed only for temperatures higher than 5 keV. This was because of two reasons. One is that for lower temperatures, the values of b–f, f–f and TS opacities were such that radiation transport would become of secondary importance compared to hydrodynamic transport (e.g. for uranium case [$\rho/\rho_0=1$], the estimated radiation front velocity from these opacities become less compared to sound velocity near about 3 keV) and hence the computation of χ_{b-b} would be of little value. Secondly, the ionic composition of the plasma at these temperatures is such that photons of very high energies would be required to cause b–b transitions and since their number density, assuming LTE in the plasma, would become significant only beyond $T=5$ keV, little error will result in opacities by not including χ_{b-b} below this temperature.

The Rosseland and Planck mean opacities were calculated from b–f, f–f and TS separately (b–f, f–f corrected for stimulated emission) and then these corrected for the b–b process by the methods of Armstrong and Nicholls (1972) and Sibulkin (1968) respectively. Figures 3 and 4 illustrate the variations of Rosseland and Planck means (without b–b contribution) with temperature and tables 2 and 3 show the comparison of χ_{b-b} with total means. The rise in opacities for the uranium case in figure 3 beyond 5 keV is due to increasing b–f contribution due to the *K*-shell and the constant value of Rosseland mean beyond 8 keV for W case (figure 4) is due to the fact that the only important contribution here is due to scattering.

In order to check the method presented in section 2 for opacity calculation, we did calculations for N_i (Z=28) plasma at normal density, which was experimentally

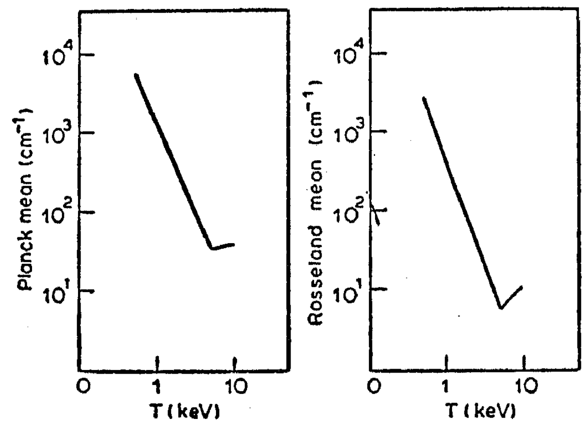

Figure 3. Variation of Planck and Rosseland mean opacities (without b–b contribution) with temperature for uranium.

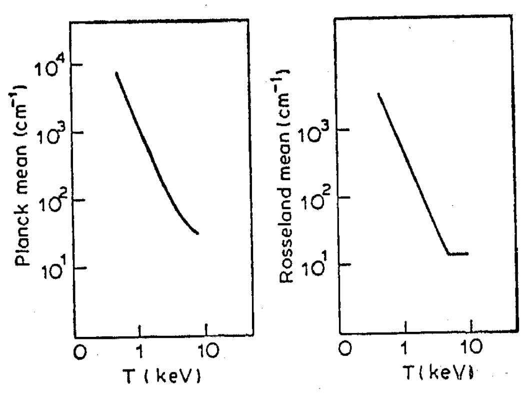

Figure 4. Variation of Planck and Rosseland mean opacities (without b–b contribution) with temperature for tungsten.

Table 2. Mean opacities (cm^{-1}) with bound-bound contribution for uranium ($Z=92$) and $\rho/\rho_0 = 1{\cdot}0$

Temperature keV	Rosseland mean cm^{-1}	Bound-bound contribution to Rosseland mean cm^{-1}	Planck mean cm^{-1}	Bound-bound contribution to Planck mean cm^{-1}
5·0	6·50	9·7	33·83	20·80
8·0	9·70	4·67	39·43	19·00
10·0	11·60	6·02	39·60	18·40

Table 3. Mean opacities (cm^{-1}) with bound-bound contribution for tungsten ($Z = 74$) and $\rho/\rho_0 = 1{\cdot}0$

Temperature keV	Rosseland mean cm^{-1}	Bound-bound contribution to Rosseland mean cm^{-1}	Planck mean cm^{-1}	Bound-bound contribution to Planck mean cm^{-1}
5·0	13·8	7·90	49·30	17·41
8·0	13·10	6·81	32·00	15·87
10·0	7·38	7·60	13·64	8·74

produced by Mizui *et al* (1977) using high power Nd-glass lasers. The temperature of the plasma was estimated to be 200 eV. It was argued by the authors that x-ray radiations, emitted from the laser irradiated region of the foil (between critical density to solid density), were responsible for the enhanced transmission of the laser light and qualitatively showed that the size of the x-ray source satisfies the black body condition. The length of the x-ray source [$X_l = Ll_n(Z_{av}n_s/n_c)$, $L = C\triangle t$, Z_{av}=average charge, n_s=solid density, n_c=critical density c=isothermal sound velocity, $\triangle t$= half duration of the incident laser pulse] is found by us to be $67{\cdot}1\times10^{-4}$ cm from $Z_{av}=19{\cdot}3$ at 200 eV (defined by eq. (7)). The free-free Planck and Rosseland mean free paths are $7{\cdot}0\times10^{-4}$ cm and $2{\cdot}2\times10^{-2}$ cm respectively. Thus, the condition for the source to behave as a black body is satisfied very well (e.g. $l_{PL} < X_l < l_{RO}$).

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