Ultraslow light in inhomogeneously broadened media

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We calculate the characteristics of ultraslow light in an inhomogeneously broadened medium. We present analytical and numerical results for the group delay as a function of power of the propagating pulse. We apply these results to explain the recently reported saturation behavior [Baldit *et al.*, Phys. Rev. Lett. **95**, 143601 (2005)] of ultraslow light in rare earth ion doped crystal.

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The usage of a coherent field to control the optical properties of a medium has led to many remarkable results such as enhanced nonlinear optical effects [1, 2], electromagnetically induced transparency (EIT) [3], lasing without inversion [4, 5, 6], ultraslow light [7, 8, 9, 10, 11], storage and retrieval of optical pulses [12] and many others [13, 14, 15, 16]. Most of these effects rely on quantum interferences which are created by the application of a coherent field. The coherent field opens up a new channel for the process under consideration. This interference effect produces the EIT dip or a hole in the absorption profile. The ultraslow light emerges as the EIT dip could be very narrow. It has been realized that in principle one could also use two level nonlinearities in presence of a strong pump. For a homogeneously broadened medium a hole can emerge if the transverse and longitudinal relaxation times are quite different. Under these conditions the hole has a width of the order of T_1 and this is being referred to as the effect of coherent population oscillation [17]. Bigelow *et al.* did experiments in this regime using ruby as the material medium which can be modelled as a homogeneously broadened system [18, 19]. Some studies on slowlight in inhomogeneous broadened medium exist [20, 21]. In an earlier paper the present authors had considered the case of inhomogeneously broadened gaseous medium where the Doppler effect is important [21]. We considered the case of saturation absorption spectroscopy. This leads to the well known hole in the Doppler profile. The width of this hole was of the order of $1/T_1$ which is about two times $1/T_2$. In the inhomogeneously broadened gaseous medium the group index of the order of 10^3 was obtained. The recent experiment of Baldit *et al.* reports group delays of the order of 1.1 s in rare earth ion doped crystal which has strong inhomogeneous broadening [22]. In this case all the relaxation times are quite different - $T_1 = 8$ ms; $T_2 = 3$ μs; inhomogeneous line width $\Gamma_{inh} = 1.3$ GHz. The width of the whole is essentially determined by T_1 and hence one gets very large delays. Baldit *et al.* did present a theoretical model based on homogeneous broadening of the medium where as to obtain agreement with

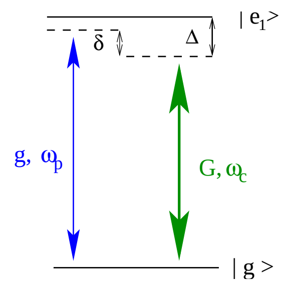

FIG. 1: A schematic diagram of two level atomic system with ground state $|g\rangle$ and excited state $|e_1\rangle$; The pump (ω_c) and probe (ω_p) fields are co-propagating.

experiments inhomogeneous broadening is to be included as alluded by them [23].

In this paper, we consider a system of inhomogeneously broadened two level atoms interacting with co-propagating pump and probe fields. We use the well known susceptibility [24] and average it over the inhomogeneous distribution to calculate the group index. We derive a number of analytical results and show how these can be used to understand the experimental results of Baldit *et al.* For example we show that in the limit of very small detuning of the probe from the pump the group delay goes as $\sqrt{S}$ for large S. The group delay also peaks at about S=0.9. The value of group delay increases as the detuning δ increases. We further present detailed numerical results.

In order to understand the experimental results of Baldit *et al*, we consider a two level system as shown in figure 1. Here we define all fields as

$$\vec{E}_i(z,t) = \vec{\mathcal{E}}_i(z,t)\ e^{-i(\omega_i t - kz)}\ +\ c.c., (i = p, c) \qquad (1)$$

where $\vec{\mathcal{E}}_i$ is the slowly varying envelope of the field. The pump field at frequency ω_c and the probe field at frequency ω_p are co-propagating through the medium. The linear susceptibility $\chi(\omega_p)$ is obtained by solving the density matrix equations for the two level system of figure 1, that is by calculating the density matrix element ρ_{eg} to the first order in the probe field but to all orders in the co-propagating pump field. The dynamics of population and polarization of the atoms in the two-level configura-

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tion are given by

$$
\begin{aligned}
\dot{\rho}_{ee} &= -\frac{1}{T_1}\rho_{ee} + i\left(G + ge^{-i\delta t}\right)\rho_{ge} - i\left(G^* + g^* e^{i\delta t}\right)\rho_{eg},\\
\dot{\rho}_{eg} &= -\left(i\Delta + \frac{1}{T_2}\right)\rho_{eg} + i\left(G + ge^{-i\delta t}\right)(\rho_{gg} - \rho_{ee}),\\
\rho_{ee} &+ \rho_{gg} = 1
\end{aligned}
\qquad (2)
$$

where T_1 and T_2 are the longitudinal and transverse relaxation times respectively. The density-matrix elements in the original frame are given by $\rho_{eg}e^{-i\omega_c t}, \rho_{gg}$, and ρ_{ee}. The detunings Δ, δ and the Rabi frequencies are defined by

$$
\Delta = \omega_{eg} - \omega_c; \delta = \omega_p - \omega_c; 2G = \frac{2\vec{d}_{eg}.\vec{\mathcal{E}}_c}{\hbar}; 2g = \frac{2\vec{d}_{eg}.\vec{\mathcal{E}}_p}{\hbar}, \qquad (3)
$$

where $\vec{d}_{eg}$ is the dipole matrix element. The susceptibility χ can be obtained by considering the steady state solution of Eq.(2) to the first order of g and write the solution as

$$
\rho = \rho^0 \; + \; g\; e^{-i\delta t}\; \rho^+ \; + \; g^*\; e^{i\delta t}\; \rho^- + ... \qquad (4)
$$

The eg element of ρ^+ will yield the linear susceptibility χ at the frequency ω_p as can be seen by combining Eqs.(2) and (4):

$$
\chi = -\frac{n|d|^2 T_2}{\hbar}\frac{1+\Delta^2 {T_2}^2}{(1+\Delta^2 {T_2}^2 + S)(\Delta T_2 + \delta T_2 + i)}\left[1 - \frac{S(\Delta T_2 - i)^{-1}(\delta T_2 + 2i)(\delta T_2 - \Delta T_2 + i)}{2(\delta T_1 + i)(\delta T_2 + \Delta T_2 + i)(\delta T_2 - \Delta T_2 + i) - S(\delta T_2 + i)}\right], \qquad (5)
$$

where n is the density of the atoms of the medium. The saturation parameter $S = 4|G|^2 T_1 T_2$ is defined as the ratio of the control field intensity and the saturation intensity. The average response of the susceptibility is given by

$$
\langle\chi\rangle = \frac{2\sqrt{ln2}}{\sqrt{\pi}\Gamma_{inh}}\int \chi(\Delta) e^{-\frac{4\ln 2\left[\Delta - (\overline{\omega}_{eg} - \omega_c)\right]^2}{\Gamma_{inh}^2}} d\Delta, \qquad (6)
$$

where $\overline{\omega}_{eg}$ is the central frequency of the atomic transition $|e\rangle \longleftrightarrow |g\rangle$. Here we consider the frequency of the control field ω_c is tuned to the line center $\overline{\omega}_{eg}$. We present the behavior of real and imaginary parts of the susceptibility as a function of the detuning of the probe field in Fig. (2). The real part of susceptibility gives normal dispersion. It is clear from Fig. (2a) that the slope of normal dispersion attains maximum when S $\sim$ 1 which leads to ultra slow light. The imaginary part of $\langle\chi\rangle$ exhibits the absorption dip which becomes deeper with the increase in the intensity of the control field as shown in Fig. (2b). The spectral width of absorption dip depends on the intensity of the control field. This dip is associated with coherent population oscillation[17].

In order to compare with experimental results of Baldit *et al.* we need to know the group index n_g which is defined by

$$
\begin{aligned}
n_g &= 1 + 2\pi\omega_p \frac{\partial}{\partial\omega_p}\mathrm{Re}\langle\chi\rangle\\
&= 1 - \frac{\alpha_{inh} c T_2}{2\pi}\langle D\rangle
\end{aligned}
\qquad (7)
$$

where

$$
D = \frac{i(\Delta + i)\left[S^2 + 2(\delta f + i)^2(i + \delta - \Delta)^2(1 + i\Delta) + S(i + \delta - \Delta)\left(-i + \delta + 2f(i + \delta - i\delta^2 - \Delta) + \Delta\right)\right]}{2\left(1 + S + \Delta^2\right)\left[S(\delta + i) - (i + \delta f)\left((i + \delta)^2 - \Delta^2\right)\right]^2}; \; f = \frac{T_1}{T_2}. \qquad (8)
$$

We denote the integration with respect to Δ has been denoted by $\langle\rangle$. The unsaturated inhomogeneous absorption coefficient of the two level atomic system is defined as

$$
\alpha_{inh} = \frac{4\pi\omega_p}{c}\langle Im\,[\chi]_{G=0}\rangle = \frac{8\pi^{\frac{3}{2}}\omega_p n|d_{eg}|^2\sqrt{\ln 2}}{c\hbar\Gamma_{inh}} \qquad (9)
$$

In the limit of very small detuning of the probe from the pump, the analytical expression for the group index for

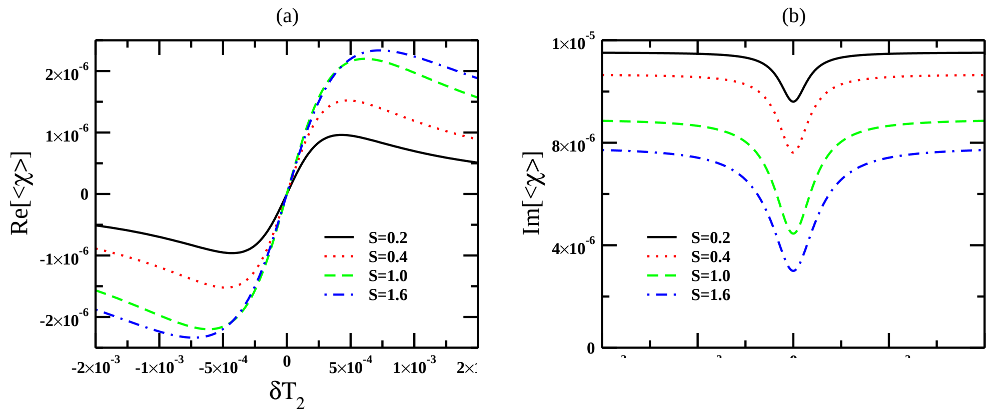

FIG. 2: (a) and (b) The real and imaginary parts respec presence of co-propagating control field G. The common pa inhomogeneous absorption coefficient $\alpha_{inh} = 6.5$ cm^{-1}; inhor T_1= 8 ms; transverse relaxation time T_2=3 μs.

inhomogeneous case can be expressed as

$$n_g \cong c\alpha_{inh}T_1 \left[\frac{S(4+S)}{16(1+S)^{5/2}}\right]; \quad \delta \rightarrow 0 \qquad (10)$$

It is clear from the above expression that the group index varies as $S^{-1/2}$ for large value of S. The group index attains the maximum value at $S = 0.9$. In case of homogeneously broadened two level system the group index is given by [22]

$$n_g \cong c\alpha_h T_1 \left[\frac{S}{2(1+S)^3}\right]; \quad \delta \rightarrow 0, \qquad (11)$$

where $\alpha_h = 4\pi\omega_p n|d_{eg}|^2T_2/c\hbar$ is the homogeneous absorption coefficient. For homogeneous two level system the group index varies as S^{-2} at large S and peaks at S=0.5. At large S, the group index for a two level system falls much slowly for an inhomogeneous medium as compared to the homogeneous case as shown in Fig. (3). We thus find an important difference between inhomogeneously and homogeneously broadened two level systems. Note that the ratio between inhomogeneous and homogeneous unsaturated absorption coefficient is $\alpha_{inh}/\alpha_h \approx \Gamma_{inh}T_2$. The behavior so obtained is consistent with the experimental observation. Figure (4) shows the variation of group index as function of the intensity of the control field at different probe detuning. As the detuning of the probe field is increased the peak of the group index shifts toward higher S. The maxima of the group index $n_g = .65 \times 10^8$ for $\delta = 10$ Hz occurs at $S = 1.16$ which corresponds to the group velocity $v_g = c/n_g = 4.61$ m/s which is higher than what is reported. Further the

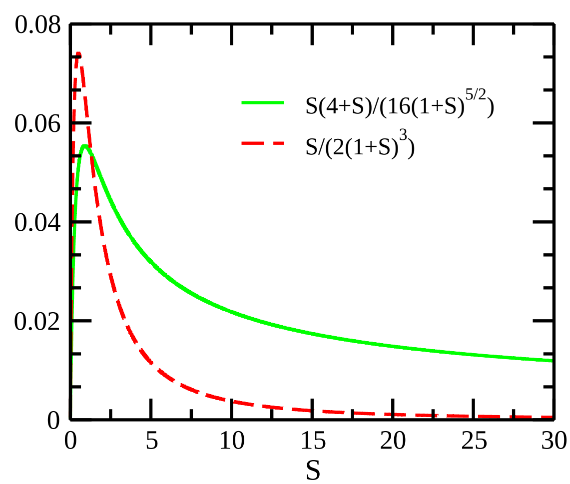

FIG. 3: Variation of the term in squared bracket of Eqs. (10) and (11) as a function of intensity of the control field for inhomogeneous and homogeneous cases of two level system.

to understand the nature of the experimental results of Baldit *et al.* However the results derived here are applicable to any system which can be modelled by inhomogeneously broadened two level system.

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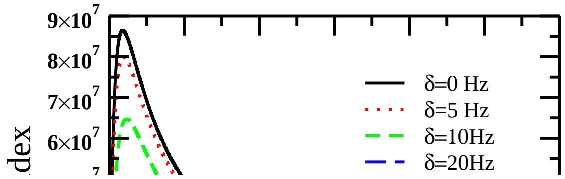

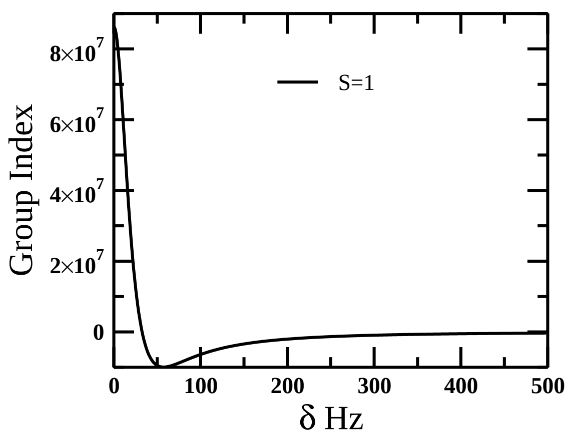

FIG. 5: Group index variation with the detuning of the probe field. The different parameters used in the numerical simulation of the Eq. (7) are as follows: α_{inh}=6.5 cm^{-1}, T_1=8 ms and T_2= 3 μs.

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