Quantum interferometry using coherent beam stimulated parametric down-conversion

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Abstract: We show how stimulated parametric processes can be employed in experiments on beyond the diffraction limit to overcome the problem of low visibility obtained by using spontaneous down conversion operating in the high gain regime. We further show enhancement of the count rate by several orders when stimulated parametric processes are used. Both the two photon counts and the visibility can be controlled by the phase of the stimulating coherent beam.

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The question of beating the diffraction limit in optics has been the subject of extensive discussions recently [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. Dowling and coworkers proposed [1] a very new idea to improve the sensitivity of resolution by using detectors that work on two photon absorption and by using special class of entangled states called *NOON* states [2] . They showed that the diffraction limit can be beaten this way. The issue of the resolution in imaging continues to be addressed [17, 18, 19, 20, 21].

It is easy to produce *NOON* states experimentally with two photons by using a very low gain parametric down converter. In this case the resolution is improved by a factor of two. However the probability of two photon absorption is very low unless one could develop extremely efficient two photon absorbers. One alternative would be to work with down converters in the high gain limit [22] however then the visibility of two photon counts goes down asymptotically to 20% [4]. Clearly we need to find methods that can overcome the handicap of having to work with smaller visibility. Another difficulty is with the magnitude of two photon counts. One needs to improve the intensity of two photon counts considerably.

We propose a new idea using stimulated parametric processes along with spontaneous ones [23] to produce resolution improvement while at the same time maintaining high visibility at large gains of the parametric process. The stimulated processes enhance the count rate by several orders of magnitude. We use coherent beams at the signal and the idler frequencies. We further find that the phases of coherent fields can also be used as tuning knobs to control the visibility of the pattern. It may be borne in mind that the process of spontaneous parametric down conversion has been a work horse for the last two decades in understanding a variety of

issues in quantum physics and in applications in the field of imaging [24, 25, 26, 27, 28].

We expect that the use of stimulated processes along with spontaneous ones would change our landscape as far as fields of imaging and quantum sensors are concerned. We now describe the idea and the results of preliminary calculations that support the above assertion. Consider the scheme shown in Fig. 1. Here $\hat{a}_1$ and $\hat{b}_1$ are the signal and idler modes driven by the
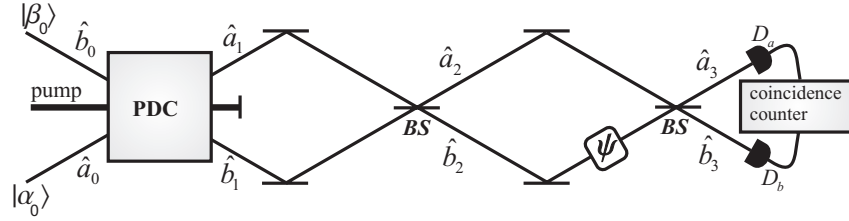

Fig. 1. Using an input from non-degenerate stimulated parametric down-conversion for determination of phase via photon-photon correlations.

coherent fields. The usual case of spontaneous parametric down conversion is recovered by setting $\alpha_0 = \beta_0 = 0$. The ψ is the phase introduced by the object or by an interferometer. For down conversion of type II the signal and idler would be two photons in two different states of polarization. In order to calculate the coincidence count it is good to work with Heisenberg operators. The fields reaching the detectors are related to the input vacuum modes $\hat{a}_0$ and $\hat{b}_0$ via

$$\begin{pmatrix} \hat{a}_3 \\ \hat{b}_3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\psi} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} \mu(\hat{a}_0 + \alpha_0) + v(\hat{b}_0^\dagger + \beta_0^*) \\ \mu(\hat{b}_0 + \beta_0) + v(\hat{a}_0^\dagger + \alpha_0^*) \end{pmatrix}, \quad (1)$$

where μ and v are given in terms of the gain parameter g,

$$\mu = \cosh(g), \quad (2)$$
$$v = e^{i\phi} \sinh(g). \quad (3)$$

and ϕ is the phase of the pump. We first note that in the absence of the object $\psi = 0$, the mean count say at the detector D_a is given by

$$I_{\hat{a}} \equiv \langle \hat{a}_3^\dagger \hat{a}_3 \rangle = \sinh^2(g) + |\alpha_0|^2 \left[1 + 2\sinh^2(g) + \sinh(2g)\cos(\phi - 2\theta) \right], \quad (4)$$

where for simplicity we assume that $\alpha_0 = \beta_0$. We denote θ as the phase of α_0. Note that the first term in Eq. (4) is the intensity of spontaneously produced photons. The g-independent term in the square bracket is just the intensity of the coherent beam and the rest of the terms result from stimulated parametric down conversion. Note further that the mean count depends on the phase of the coherent beams used to produce stimulated down conversion.

Now, using our basic equation (1) we calculate the two-photon coincidence counts as the following:

$$I_{\hat{a}\hat{b}} \equiv \langle \hat{a}_3^\dagger \hat{b}_3^\dagger \hat{b}_3 \hat{a}_3 \rangle = A \left\{ 1 + \frac{V}{1-V} \left(1 + \cos(2\psi) \right) \right\}. \quad (5)$$

Here V is the visibility of two-photon coincidence counts

$$V = \frac{B}{A+B}, \quad (6)$$

where

$$A = \sinh^4(g) + 2|\alpha_0|^2 \sinh^2(g) \left[1 + 2\sinh^2(g) + \sinh(2g)\cos(\phi - 2\theta) \right], \quad (7)$$

$$B = \frac{1}{2}\left\{ (1 + \sinh^2(g))\sinh^2(g) + |\alpha_0|^2 \sinh(2g)\left[\sinh(2g) + (1 + 2\sinh^2(g))\cos(\phi - 2\theta)\right] \right.$$
$$\left. + |\alpha_0|^4 \left[1 + 2\sinh^2(g) + \sinh(2g)\cos(\phi - 2\theta)\right]^2 \right\}. \tag{8}$$

Both A and B depend on the gain g, amplitude and phase of the stimulating beams. In Figs. 2(a) and 2(b) we display the fringes in two-photon counts under different conditions on the gain of the down-converter and the strength and phase of the stimulating beams. These figures clearly
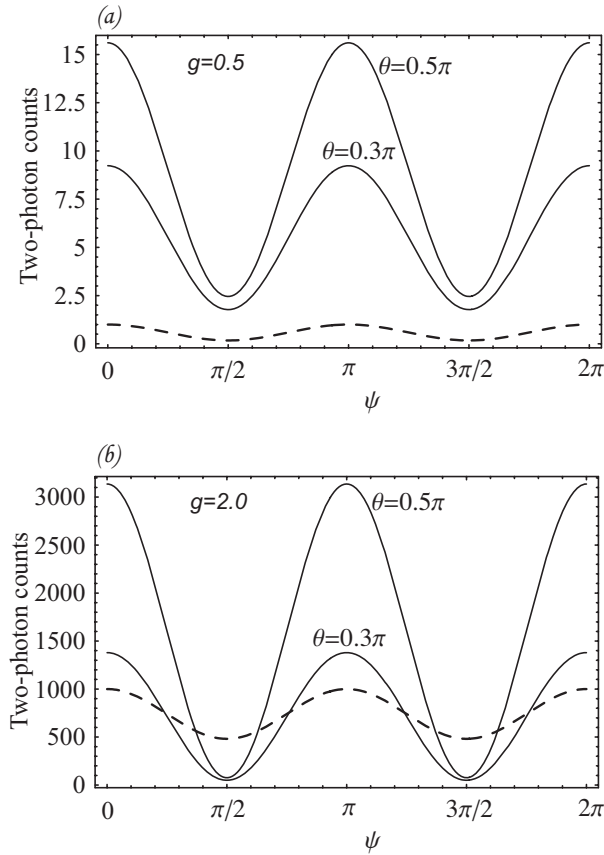

Fig. 2. (a) Stimulated emission enhanced two-photon counts for various phases of the coherent field at the gain $g = 0.5$. The horizontal line shows the interferometric phase. The pump phase ϕ is fixed at π. The counts are in units of two-photon coincidence rates coming from spontaneous down-conversion process. The modulus of the coherent field $|\alpha|$ is chosen such that the coincidences coming from SPDC and the coherent fields are equal to each other. The dashed line shows the two-photon counts for the case of spontaneous process. (b) The same with (a) at the gain $g = 2.0$. Here, the counts for the case of spontaneous process (dashed line) is multiplied by a factor of 10^3.

show the advantages of using stimulating parametric processes in quantum imaging. We next quantify these advantages.

We first note that in the absence of stimulating fields ($|\alpha_0| \to 0$)

$$V \longrightarrow \frac{1 + \sinh^2(g)}{1 + 3\sinh^2(g)}, \tag{9}$$

and the strength of the two-photon counts reduces to

$$I_{\hat{a}\hat{b}} \longrightarrow 2\sinh^4(g) + \sinh^2(g).$$ (10)

In the limit of large gain, the visibility drops to $1/3$ and the strength of two-photon counts goes as $\exp(4g)$. Next, we examine the effect of stimulated parametric processes on the visibility and the numerical strength of two-photon coincidence count. In the limit of large gain, the visibility of the stimulated process reads

$$V \longrightarrow \frac{\frac{1}{4} + |\alpha_0|^2 (1 + \cos(\Delta)) + |\alpha_0|^4 (1 + \cos(\Delta))^2}{\frac{3}{4} + 3|\alpha_0|^2 (1 + \cos(\Delta)) + |\alpha_0|^4 (1 + \cos(\Delta))^2},$$

(11)

where Δ is the phase difference, $\phi - 2\theta$, between the pump and stimulating (coherent) beams. Note that when $|\alpha_0| \to 0$ we recover the same result as Eq. (9). The visibility given in Eq. (11) has terms that arise from the interference between the spontaneous and the stimulated down-converted photons. Clearly we can control the value of the visibility by changing the amplitude of the stimulating beams. For example, we can obtain 60% visibility even for $|\alpha_0|^2 \sim 1$ if $\Delta = 0$, which should be compared with the 33% value in the absence of the stimulating beams. As we increase the stimulating beam intensity to ~ 10, we obtain 90% visibility. If we assume that the stimulating field's intensity of the order of the number of spontaneous photons produced by the down-converter, i.e. $|\alpha_0|^2 \sim \sinh^2(g)$, then the visibility of 100% can be reached at $g \simeq 2 - 2.5$ (For $\Delta = \pi$ we lose the advantage of stimulating beam to produce higher visibility.). In Fig. 3
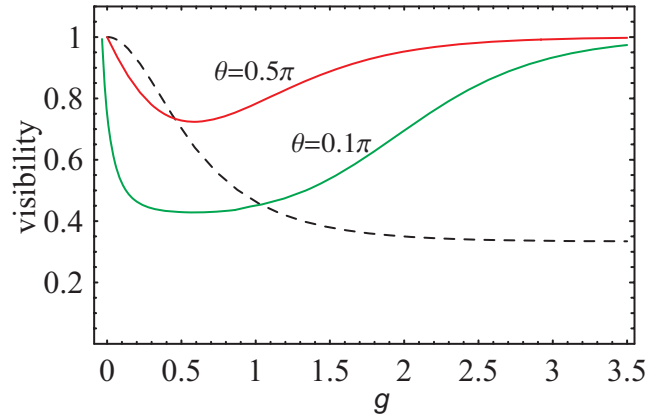

Fig. 3. (Color online) Stimulated emission enhanced visibility of two-photon counts for various phases (red and green lines) of the coherent field with respect to the gain g. The pump phase ϕ is fixed at π. The modulus of the coherent field $|\alpha_0|$ is chosen such that the coincidences coming from SPDC and the coherent fields are equal to each other. The dashed line shows the visibility of two-photon counts in the case of photons produced by spontaneous parametric down-conversion.

we show the visibility of two-photon coincidences with respect to the gain for different values of the stimulating beam phases. The results in the region of large gain follow the approximate results based on Eq. (11).

We next examine the strength of two-photon counts in the limit of high gain. This depends on the interferometric phase ψ. To get an estimate of the strength of two-photon counts let us
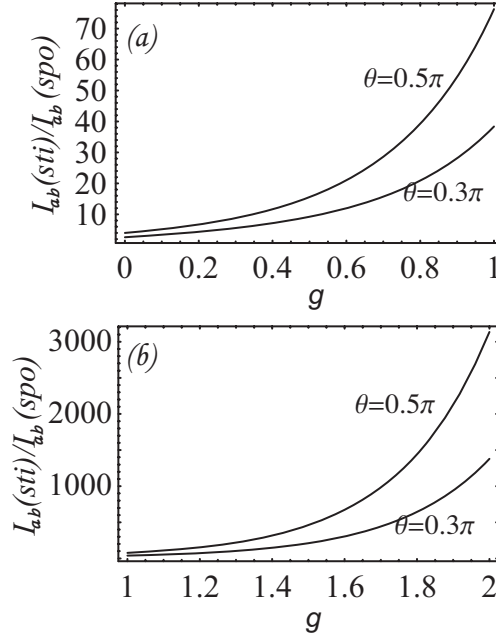
Fig. 4. The ratio of the two-photon coincidences coming from the stimulated process to the spontaneous process for various phases of the coherent beams at the (a) low and (b) high gain limits respectively. The pump phase is fixed at π and the modulus of the coherent field $|\alpha|$ is chosen such that the coincidences coming from SPDC and the coherent fields are equal to each other.

set $\psi = 0$:

$$I_{\hat{a}\hat{b}} \longrightarrow 2\sinh^4(g)\left\{1 + 4|\alpha_0|^2\left(1 + \cos(\Delta)\right) + 2|\alpha_0|^4\left(1 + \cos(\Delta)\right)^2\right\}. \qquad (12)$$

Note that when $\alpha_0 = 0$ we recover Eq. (10). For $\Delta = 0$, the highest order term in Eq. (12) goes as $\exp(4g)|\alpha_0|^4$, i.e. a factor of $|\alpha_0|^4$ appears here in compared to the spontaneous process. This then reduces to $I_{\hat{a}\hat{b}} \rightarrow \exp(8g)$ if we assume that the stimulating field's intensity of the order of the number of spontaneous photons produced by the down-converter, i.e. $|\alpha_0|^2 \sim \sinh^2(g)$. This leads to an enhancement by $\exp(4g)$ in the two-photon count rates compared to the case of spontaneous processes. In Figs. 4(a) and 4(b), we show the ratio of two-photon counts coming from the stimulated process to the spontaneous process both at the low and high gain limits respectively. It is shown that at $g \simeq 1.7$, three orders of magnitude rate enhancement is being reached. Therefore, in the determination of interferometric phase, we obtain a ground-breaking enhancement in both the visibility and the strength of the two-photon coincidence counts by controlling the phase and the amplitude of stimulating coherent beams. We show in Figs. 2(a) and 2(b), this cumulative enhancement in both the visibility and the strength in the low and high gain limits respectively.

A question that we have not investigated in the present paper concerns the minimum value of the phase $\Delta\psi$ that can be measured [13]. In the literature one has the well known shot noise limit ($\Delta\psi \sim 1/\sqrt{N}$; where N is the total number of photons) obtained with coherent sources. This is to be compared with the Heisenberg limit ($\Delta\psi \sim 1/N$) obtained with sources prepared in special states and with very special detection schemes [29, 30]. Thus to improve the sensitivity it would be especially interesting if one can do the latter with photon numbers of the same or-

der as in coherent sources. However so far one has achieved Heisenberg limit only with photon numbers of order few. Thus the real question is–what is the achievable phase uncertainty given the presently available sources and measurement techniques. This is something that needs to be studied at depth. We note that the original proposal of Dowling and collaborators employed the *NOON* states and measurements based on the observable $|N0\rangle\langle 0N| + |0N\rangle\langle N0|$. There have been other suggestions which enable one to achieve Heisenberg limit. Some of these are based on homodyne measurements [31] whereas others [32] make use of in principle measurements which would achieve Cramer-Rao lower bound on phase sensitivity. It would clearly be interesting to generalize the latter proposals when stimulating signal and idler fields are employed.

In conclusion, we have shown that using stimulated parametric processes along with spontaneous ones leads to resolution improvement and high signal values while at the same time maintaining high visibility at large gains of the parametric process. We use coherent beams at the signal and idler frequencies. We find that the phases of coherent fields can also be used as tuning knobs to control the visibility of the pattern. The use of stimulated parametric down-conversion also improves the rates of two-photon absorption in quantum lithography. The use of stimulated processes in multi-photon coincidence events is expected to produce even bigger advantages, for example in producing much higher count rates. We hope to examine these in future. Finally we believe that the use of stimulated processes along with spontaneous ones would change our landscape as far as fields of imaging and quantum sensors are concerned.