Normal Mode Splitting and Antibunching in Stokes and Anti-Stokes Processes in Cavity Optomechanics: Radiation Pressure Induced Four-Wave Mixing Cavity Optomechanics

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We study Stokes and anti-Stokes processes in cavity optomechanics in the regime of strong coupling. The Stokes and anti-Stokes signals exhibit prominently the normal-mode splitting. We report gain for the Stokes signal. We also report lifetime splitting when the pump power is less than the critical power for normal-mode splitting. The nonlinear Stokes processes provide a useful method for studying strong coupling regime of cavity optomechanics. We also investigate the correlations between the Stokes and anti-Stokes photons produced spontaneously by the optomechanical system. At zero temperature, our nanomechanical system leads to the correlations between the spontaneously generated photons exhibiting photon antibunching and those violating the Cauchy-Schwartz inequality.

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I. INTRODUCTION

The nonlinearities in a system can be studied using a number of optical methods. Among these, Stokes and anti-Stokes processes, and more generally four-wave-mixing processes are quite common tools used to understand the nonlinear nature of the system [1]. With this in view we study the stimulated Stokes and anti-Stokes processes in cavity optomechanics. As is well known, the nonlinearity in cavity optomechanics arises from the radiation pressure [2–7] on the moving mirror of the cavity. Thus, if the cavity is driven by a pump field of frequency ω_l and a Stokes field of frequency ω_s, then, due to radiation pressure, the output of the cavity would consist of fields at the applied frequencies ω_l and ω_s and a generated frequency $2\omega_l - \omega_s$. While some previous works [8–10] have explored the Stokes and anti-Stokes processes in the context of parametric oscillation instability, here we show how such processes can be conveniently used to study the phenomena of normal-mode splitting [11–19] arising from the strong coupling between the cavity and the mechanical mirror. Further, the system can act as an amplifier for the Stokes field. Needless to say, we work in a domain which is below the instability threshold.

Moreover, very interesting photon correlations between the Stokes and the anti-Stokes photons have been reported in atomic vapors under conditions of electromagnetically induced transparency [20]. Here we also discuss the correlations between the photons created spontaneously by the optomechanical system. The correlations are found to be nonclassical.

The article is organized as follows. In Sec. II, we introduce the model, obtain the equation of motion for the oscillator and the cavity field, and solve it. In Sec. III, we calculate the output fields and thus obtain nonlinear susceptibilities for Stokes and anti-Stokes processes. In Sec. IV, we show that the Stokes field is amplified, and find very prominent normal-mode splittings in the output fields. Thus, stimulated Stokes and anti-Stokes processes provide us with a new tool for studying the strong coupling regime of optomechanics. We find that normal-mode splittings are especially pronounced in the two quadratures of the output fields. In Sec. V, we analyze the correlations between the spontaneously generated photons in the four-wave-mixing processes in the optomechanical system. We show that such correlations are intrinsically quantum.

II. MODEL: STIMULATED GENERATION OF STOKES AND ANTI-STOKES FIELDS

We consider the system illustrated in Fig. 1, in which the cavity consists of two mirrors separated from each other by a distance L. The front mirror is fixed and partially transmitting; the end mirror is movable and perfectly reflecting. The cavity is driven by a pump field and a Stokes field obtained with lasers. Their frequencies are ω_l and ω_s, respectively. We would assume that the Stokes field is much weaker than the pump field. A radiation pressure produced by momentum transfer will act on the movable mirror, which is modeled as a harmonic oscillator with mass m, frequency ω_m, and momentum decay rate γ_m.

Considering a single-mode cavity ω_c, the Hamiltonian of the system in a frame rotating at the pump frequency ω_l is written as

$$H = \hbar(\omega_c - \omega_l)n_c - \hbar\omega_m\chi n_c Q + \frac{\hbar\omega_m}{4}(Q^2 + P^2) + i\hbar\varepsilon_l(c^\dagger - c) + i\hbar[\varepsilon_s e^{-i(\omega_s-\omega_l)t}c^\dagger - \varepsilon_s^* e^{i(\omega_s-\omega_l)t}c]. \tag{1}$$

Here Q and P are the dimensionless operators representing the oscillator's position and momentum, defined by $Q = q\sqrt{2m\omega_m/\hbar}$ and $P = p\sqrt{2/(m\hbar\omega_m)}$ with $[Q, P] = 2i$. In Eq. (1), the first term is the energy of the cavity field, $n_c = c^\dagger c$ is the number of the photons inside

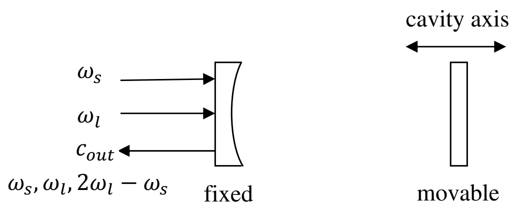

FIG. 1: Sketch of the studied system. A pump field with frequency ω_l and a Stokes field with frequency ω_s enter the cavity through the partially transmitting mirror. The output fields c_{out} have three components ($\omega_l, \omega_s, 2\omega_l - \omega_s$). No vacuum fields are shown here because we are examining only the mean response.

the cavity and c and $c^\dagger$ are the annihilation and creation operators, respectively, for the cavity field satisfying the commutation relation $[c, c^\dagger] = 1$. The second term describes the nonlinear coupling of the movable mirror to the cavity field via radiation pressure, where the dimensionless parameter $\chi = (1/\omega_m)(\omega_c/L)\sqrt{\hbar/(2m\omega_m)}$ is the optomechanical coupling constant between the cavity field and the movable mirror. The third term corresponds to the energy of the movable mirror. The last two terms give the interactions of the cavity field with the pump field and the Stokes field, ε_l and ε_s are, respectively, the amplitudes of the pump field and the Stokes field inside the cavity. They are defined by $\varepsilon_l = \sqrt{2\kappa\wp/(\hbar\omega_l)}$ and $|\varepsilon_s| = \sqrt{2\kappa\wp_s/(\hbar\omega_s)}$, respectively, where $\wp$ is the pump power, $\wp_s$ is the power of the Stokes field, and κ is the cavity decay rate due to the fixed mirror.

Let $\langle Q\rangle$, $\langle P\rangle$, $\langle c\rangle$, and $\langle c^\dagger\rangle$ be the expectation values of the operators Q, P, c, and $c^\dagger$, respectively. The time evolution of these expectation values can be derived by using the Heisenberg equations of motion and adding the damping terms:

$$\begin{aligned}
\langle\dot{Q}\rangle &= \omega_m\langle P\rangle, \\
\langle\dot{P}\rangle &= 2\omega_m\chi\langle n_c\rangle - \omega_m\langle Q\rangle - \gamma_m\langle P\rangle, \\
\langle\dot{c}\rangle &= -[\kappa + i(\omega_c - \omega_l - \omega_m\chi\langle Q\rangle)]\langle c\rangle + \varepsilon_l + \varepsilon_s e^{-i(\omega_s-\omega_l)t}, \\
\langle\dot{c}^\dagger\rangle &= -[\kappa - i(\omega_c - \omega_l - \omega_m\chi\langle Q\rangle)]\langle c^\dagger\rangle + \varepsilon_l + \varepsilon_s^* e^{i(\omega_s-\omega_l)t}.
\end{aligned} \tag{2}$$

The derivation of Eq. (2) uses the well-known mean-field assumption $\langle Qc\rangle = \langle Q\rangle\langle c\rangle$. As the field ε_s at the Stokes frequency ω_s is much weaker than the pump field ε_l, we derive the steady-state solution of Eq. (2) to first order in ε_s, that is, we find $t \to \infty$ limit of the solutions:

$$\begin{aligned}
\begin{pmatrix} \langle Q\rangle \\ \langle P\rangle \\ \langle c\rangle \\ \langle c^\dagger\rangle \end{pmatrix} &= \begin{pmatrix} Q_0 \\ P_0 \\ c_0 \\ c_0^* \end{pmatrix} + \varepsilon_s e^{-i(\omega_s-\omega_l)t} \begin{pmatrix} Q_+ \\ P_+ \\ c_+ \\ c_-^* \end{pmatrix} \\
&+ \varepsilon_s^* e^{i(\omega_s-\omega_l)t} \begin{pmatrix} Q_- \\ P_- \\ c_- \\ c_+^* \end{pmatrix}.
\end{aligned} \tag{3}$$

Thus Eq. (3) shows the cavity field $\langle c\rangle e^{-i\omega_l t}$ has three components, oscillating at the input frequencies ω_l and ω_s, and a new anti-Stokes frequency $2\omega_l - \omega_s$. By substituting Eq. (3) into Eq. (2), neglecting those terms containing ε_s^2, ε_s^{*2}, and $|\varepsilon_s|^2$ and equating coefficients of terms proportional to $e^{-i(\omega_s-\omega_l)t}$ and $e^{i(\omega_s-\omega_l)t}$, respectively, we find

$$\begin{aligned}
Q_0 &= 2\chi|c_0|^2, \\
P_0 &= 0, \\
c_0 &= \frac{\varepsilon_l}{\kappa + i\Delta}, \\
c_+ &= \frac{1}{d(\omega_s - \omega_l)}\{[\kappa - i(\Delta + \omega_s - \omega_l)] \\
&\quad \times[(\omega_s - \omega_l)^2 - \omega_m^2 + i\gamma_m(\omega_s - \omega_l)] \\
&\quad -2i\omega_m^3\chi^2|c_0|^2\}, \\
c_- &= -\frac{2i\omega_m^3\chi^2 c_0^2}{d^*(\omega_s - \omega_l)}.
\end{aligned} \tag{4}$$

where

$$\Delta = \omega_c - \omega_l - \omega_m\chi Q_0, \tag{5}$$

is the effective detuning, and where

$$\begin{aligned}
d(\omega_s - \omega_l) &= 4\omega_m^3\chi^2\Delta|c_0|^2 + [(\omega_s - \omega_l + \omega_m) \\
&\quad \times(\omega_s - \omega_l - \omega_m) + i\gamma_m(\omega_s - \omega_l)] \\
&\quad \times[\kappa + i(\Delta - \omega_s + \omega_l)][\kappa - i(\Delta + \omega_s - \omega_l)].
\end{aligned} \tag{6}$$

For brevity we do not write explicit expressions for $Q\pm$, $P\pm$, etc. because we do not need these in the discussion that follows.

III. THE OUTPUT FIELDS

To investigate normal-mode splitting of the output fields, we need to find the expectation value of the output fields. Using input-output relation [21] $\langle c_{out}\rangle + \varepsilon_l/\sqrt{2\kappa} +$

$\varepsilon_s e^{-i(\omega_s-\omega_l)t}/\sqrt{2\kappa}=\sqrt{2\kappa}\langle c\rangle$, we can obtain the expectation value of the output fields

$$\langle c_{out}\rangle=\sqrt{2\kappa}[c_0+\varepsilon_s e^{-i(\omega_s-\omega_l)t}c_+ +\varepsilon_s^* e^{i(\omega_s-\omega_l)t}c_-] \\ -\varepsilon_l/\sqrt{2\kappa}-\varepsilon_s e^{-i(\omega_s-\omega_l)t}/\sqrt{2\kappa}. \tag{7}$$

If we write $\langle c_{out}\rangle$ as

$$\langle c_{out}\rangle=c_l+\varepsilon_s e^{-i(\omega_s-\omega_l)t}c_s+\varepsilon_s^* e^{i(\omega_s-\omega_l)t}c_{as}, \tag{8}$$

where c_l is the response at the pump frequency ω_l, c_s is the response at the Stokes frequency ω_s, and c_{as} is the response at the four-wave-mixing frequency $2\omega_l-\omega_s$ (anti-Stokes frequency). Then we have

$$\begin{aligned} c_l&=\frac{\sqrt{2\kappa}\varepsilon_l}{\kappa+i\Delta}-\frac{\varepsilon_l}{\sqrt{2\kappa}},\\ c_s&=\frac{\sqrt{2\kappa}}{d(\omega_s-\omega_l)}\{[\kappa-i(\Delta+\omega_s-\omega_l)]\\ &\quad\times[(\omega_s-\omega_l)^2-\omega_m^2+i\gamma_m(\omega_s-\omega_l)]\\ &\quad-2i\omega_m^3\chi^2|c_0|^2\}-\frac{1}{\sqrt{2\kappa}},\\ c_{as}&=-\sqrt{2\kappa}\frac{2i\omega_m^3\chi^2c_0^2}{d^*(\omega_s-\omega_l)}. \end{aligned} \tag{9}$$

In the absence of the interaction between the cavity field and the movable mirror, one would expect the output fields to contain only two input components (ω_l and ω_s); no four-wave-mixing component appears. We can get this result from Eq. (9) by setting $\chi=0$, which gives

$$\begin{aligned} c_l&=\frac{\sqrt{2\kappa}\varepsilon_l}{\kappa+i\Delta}-\frac{\varepsilon_l}{\sqrt{2\kappa}},\\ c_s&=\frac{\sqrt{2\kappa}}{\kappa+i(\Delta-\omega_s+\omega_l)}-\frac{1}{\sqrt{2\kappa}},\\ c_{as}&=0, \end{aligned} \tag{10}$$

as expected. However, in the presence of the coupling with the oscillator ($\chi\neq 0$), [from Eq. (9), we have $c_l\neq 0, c_s\neq 0, c_{as}\neq 0$], the output fields contain three components. The generated signal would exhibit resonances whenever $\omega_s=\omega_l\pm\omega_m$. In addition, one would have the resonances produced by the cavity $\omega_s=\omega_l\pm\Delta$. These resonances are, of course, expected. The normal-mode splitting would arise as a result of strong coupling χ [17–19]. This is because the structure of the denominator in Eq. (9) depends on χ. We next present the roots of Eq. (6).

We use parameters which have been used in a recent experiment on the observation of the normal-mode splitting in the fluctuation spectra [17]: the wavelength of the laser $\lambda=2\pi c/\omega_l=1064$ nm, $L=25$ mm, $m=145$ ng, $\kappa=2\pi\times 215\times 10^3$ Hz, $\omega_m=2\pi\times 947\times 10^3$ Hz, the mechanical quality factor $Q'=\omega_m/\gamma_m=6700$, $\gamma_m=2\pi\times 141$ Hz, $\Delta=\omega_m$. In this range of parameters, no parametric instabilities occur.

Figure 2 shows the dependence of the real parts of the roots of $d(\omega_s-\omega_l)$ in the domain $\mathrm{Re}(\omega_s-\omega_l)>0$ on the pump power. Figure 3 shows the dependence of the imaginary parts of the roots of $d(\omega_s-\omega_l)$ on the pump power. For a small value of the pump power, the real parts of the roots of $d(\omega_s-\omega_l)$ have two equal values, so there is no splitting. However, there is lifetime splitting [22] as seen in the Fig. 3. If we increase the pump power to a certain value, the real parts of $d(\omega_s-\omega_l)$ in the domain $\mathrm{Re}(\omega_s-\omega_l)>0$ begin to have two different values, and the difference between two real parts of the roots of $d(\omega_s-\omega_l)$ in the domain $\mathrm{Re}(\omega_s-\omega_l)>0$ is increased with increasing pump power.

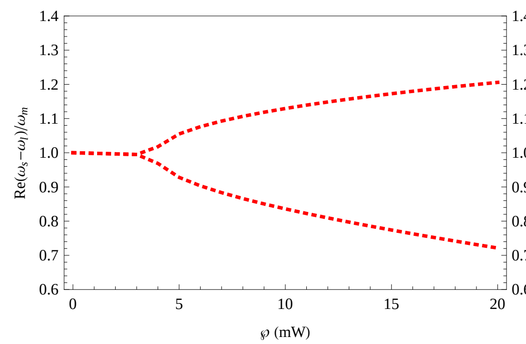

FIG. 2: (Color online) The roots of $d(\omega_s-\omega_l)$ in the domain $\mathrm{Re}(\omega_s-\omega_l)>0$ as a function of the pump power $\wp$.

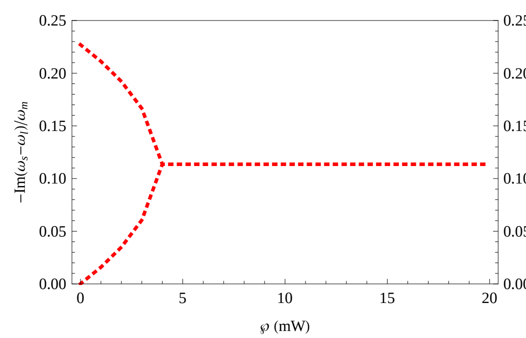

FIG. 3: (Color online) The imaginary parts of the roots of $d(\omega_s-\omega_l)$ as a function of the pump power $\wp$.

IV. NORMAL-MODE SPLITTINGS IN THE OUTPUT FIELDS

Before examining the normal-mode splitting in, say, the output anti-Stokes field, we examine Eq. (9) in the traditional limit of nonlinear optics; that is, we find the form of anti-Stokes field to lowest order in χ,

$$c_{as} = -\frac{2\sqrt{2\kappa}i\omega_m^3\chi^2\varepsilon_l^2}{(\kappa+i\Delta)^2[(\omega_s-\omega_l+\omega_m)(\omega_s-\omega_l-\omega_m)-i\gamma_m(\omega_s-\omega_l)][\kappa-i(\Delta-\omega_s+\omega_l)][\kappa+i(\Delta+\omega_s-\omega_l)]}, \quad (11)$$

which has resonances as discussed after Eq. (10) and which is proportional to the pump power.

We next discuss the normal-mode splitting in the generated Stokes and anti-Stokes fields. It is useful to normalize all quantities to the input Stokes power $\wp_s$. For simplicity, we assume ε_s to be real. For our plots we would give the output power at the Stokes frequency ω_s in terms of the input Stokes power

$$G_s = \frac{\hbar\omega_s|\varepsilon_s c_s|^2}{\wp_s} = |\sqrt{2\kappa}c_s|^2, \quad (12)$$

and the two quadratures of the output fields at the Stokes frequency ω_s in terms of the square root of the input Stokes power. Let us denote these normalized quadratures by v_s and $\tilde{v}_s$. These are defined as $v_s = \sqrt{2\kappa}\frac{c_s+c_s^*}{2}$ and $\tilde{v}_s = \sqrt{2\kappa}\frac{c_s-c_s^*}{2i}$. The quantity G_s is the gain of the cavity optomechanical four-wave mixer. In Figs. 4– 6, we have plotted v_s, $\tilde{v}_s$, and G_s, respectively, versus the normalized frequency $(\omega_s-\omega_l)/\omega_m$ for different pump powers. The quadrature v_s ($\tilde{v}_s$) exhibits absorptive (dispersive) behavior. As is known, there is a phase change on reflection and that is why the quadrature v_s shows absorptive behavior. The normal-mode splitting or the lifetime splittings are clearly seen depending on the input pump power in the quadratures v_s and $\tilde{v}_s$. The peak positions are in agreement with Fig. 2 for the case when the input pump power is such that normal-mode splitting occurs. The behavior of net gain as a function of ω_s is different due to the combination of absorptive and dispersive characteristics of the quadratures v_s and $\tilde{v}_s$. The gain shows normal-mode splitting for larger value of the pump power. Moreover, the maximum gain of the Stokes field is about 1.15. It should be borne in mind that the quadratures v_s and $\tilde{v}_s$ can be obtained by homodyne measurement.

Likewise, the output power at the anti-Stokes frequency $2\omega_l-\omega_s$ in terms of the input Stokes power is given by

$$G_{as} = \frac{\hbar(2\omega_l-\omega_s)|\varepsilon_s c_{as}|^2}{\wp_s} = |\sqrt{2\kappa}c_{as}|^2. \quad (13)$$

For brevity, we only show in Fig. 7 the function G_{as} against the normalized frequency $(\omega_s-\omega_l)/\omega_m$ for several values of the pump power. As can be seen in Fig. 7, increasing the pump power can make the signal of four-wave mixing evolve from one peak to double peaks. It is also seen that the maximum value of G_{as} is about 0.15 and the output power at the anti-Stokes frequency $(2\omega_l-\omega_s)$ is much less than the output power of the Stokes field. However, for larger pump powers, the maximum gain for Stokes and anti-Stokes fields are bigger.

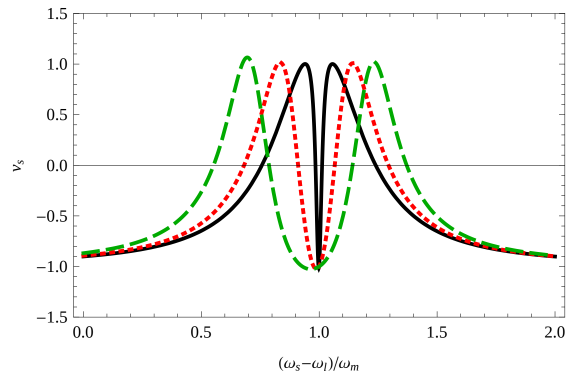

FIG. 4: (Color online) The normalized quadrature v_s plotted as a function of the normalized frequency $(\omega_s-\omega_l)/\omega_m$ for different pump power. $\wp$ = 1 mW (solid curve), 6.9 mW (dotted curve), and 20 mW (dashed curve).

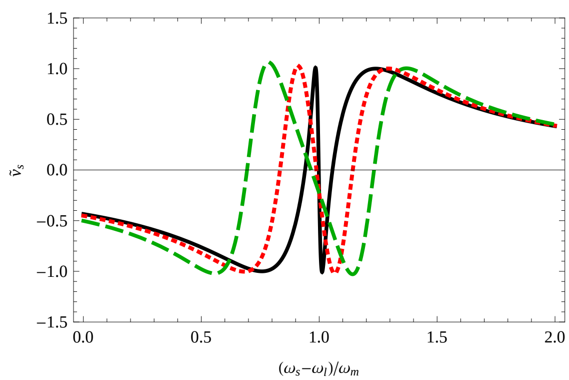

FIG. 5: (Color online) The normalized quadrature $\tilde{v}_s$ plotted as a function of the normalized frequency $(\omega_s-\omega_l)/\omega_m$ for different pump power. $\wp$ = 1 mW (solid curve), 6.9 mW (dotted curve), and 20 mW (dashed curve).

For example, for 40 mW pump power, the maximum of G_s and G_{as} are about 1.5 and 0.5, respectively.

V. SPONTANEOUS GENERATION OF STOKES AND ANTI-STOKES PHOTONS: QUANTUM CORRELATIONS

So far we have considered stimulated processes. The Stokes and anti-Stokes fields are also generated spontaneously. In this case we have to include input vacuum fields. These vacuum fields would be broad band. Thus the field at frequency ω_s in Fig. 1 is to be replaced by a

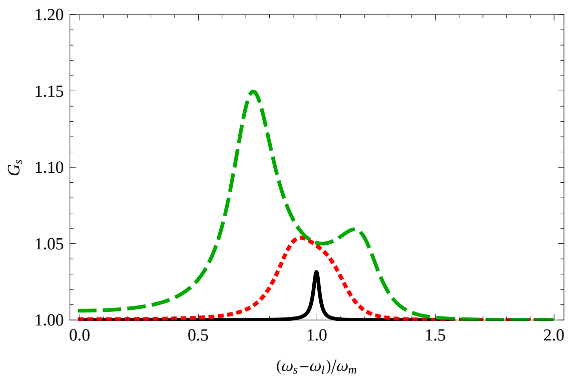

FIG. 6: (Color online) The normalized output power G_s plotted as a function of the normalized frequency $(\omega_s - \omega_l)/\omega_m$ for different pump power. $\wp = 1$ mW (solid curve), 6.9 mW (dotted curve), and 20 mW (dashed curve).

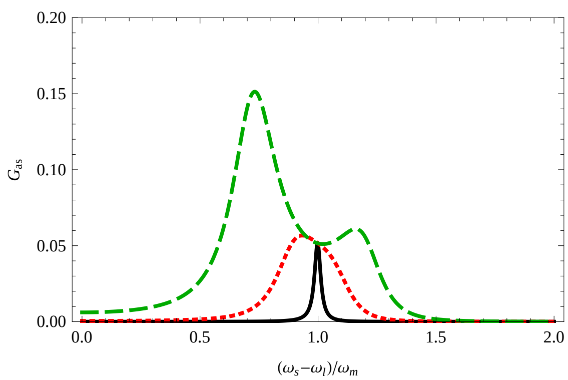

FIG. 7: (Color online) The normalized output power G_{as} plotted as a function of the normalized frequency $(\omega_s-\omega_l)/\omega_m$ for different pump power. $\wp = 1$ mW (solid curve), 6.9 mW (dotted curve), and 20 mW (dashed curve).

broad band quantum field c_{in} with zero mean value and with correlations $\langle \delta c_{in}(t)\delta c_{in}^{\dagger}(t')\rangle = \delta(t-t')$. The calculations of the output quantum fields are standard [23]. We have used these and introduced the Langevin force $\xi(t)$ which stems from the coupling of the movable mirror to the thermal environment having zero mean value with correlations [24]

$$\langle \xi(t)\xi(t^{'})\rangle = \frac{1}{\pi}\frac{\gamma_m}{\omega_m}\int \omega e^{-i\omega(t-t^{'})}\left[1+\coth\left(\frac{\hbar\omega}{2k_BT}\right)\right]d\omega, \tag{14}$$

where k_B is the Boltzmann constant and T is the temperature of the environment. The fluctuations of the output fields are obtained as

$$\delta c_{out}(\omega) = V(\omega)\xi(\omega) + E(\omega)\delta c_{in}(\omega) + F(\omega)\delta c_{in}^{\dagger}(-\omega), \tag{15}$$

where $\xi(\omega)$, $\delta c_{in}(\omega)$, and $\delta c_{in}^{\dagger}(-\omega)$ are the Fourier transform of the Langevin force $\xi(t)$ and the input vacuum fields $\delta c_{in}(t)$ and $\delta c_{in}^{\dagger}(t)$, respectively, and where

$$\begin{aligned}
V(\omega) &= -\frac{\sqrt{2\kappa}\omega_m^2\chi}{d(\omega)} i[\kappa - i(\omega+\Delta)]c_0,\\
E(\omega) &= \frac{2\kappa}{d(\omega)}\{-2\omega_m^3\chi^2 i|c_0|^2 + (\omega^2-\omega_m^2+i\gamma_m\omega)\\
&\quad \times[\kappa - i(\omega+\Delta)]\} - 1,\\
F(\omega) &= -\frac{4\kappa\omega_m^3\chi^2c_0^2}{d(\omega)} i.
\end{aligned} \tag{16}$$

in which

$$\begin{aligned}
d(\omega) &= 4\omega_m^3\chi^2\Delta|c_0|^2 + (\omega^2-\omega_m^2+i\gamma_m\omega)\\
&\quad \times[(\kappa-i\omega)^2+\Delta^2].
\end{aligned} \tag{17}$$

In Eq. (14), the first term containing $\xi(\omega)$ is the contribution of the Langevin force acting on the movable mirror, while the other two terms come from the input vacuum fields. So the fluctuations of the output fields depend on the Langevin force and the input vacuum fields. Further, we define time dependent $\delta c_{out}^{(s)}(t)$ and $\delta c_{out}^{(as)}(t)$, where $\delta c_{out}^{(s)}(t)$ represents the positive-frequency part of the fluctuations of the output fields, corresponding to Stokes component, and

$$\delta c_{out}^{(s)}(t) = \frac{1}{2\pi}\int_0^{\infty} \delta c_{out}(\omega)e^{-i\omega t}d\omega, \tag{18}$$

whereas $\delta c_{out}^{(as)}(t)$ represents the negative-frequency part of the fluctuations of the output fields, corresponding to anti-Stokes component, and

$$\delta c_{out}^{(as)}(t) = \frac{1}{2\pi}\int_{-\infty}^{0} \delta c_{out}(\omega)e^{-i\omega t}d\omega. \tag{19}$$

In the context of Stokes and anti-Stokes radiation generated by single atoms, several authors [20, 25–27] found important quantum correlations between the Stokes and anti-Stokes radiation. Such conclusions were drawn from the structure of photon-photon correlations. Motivated by these studies and the fact that we are dealing with a macroscopic system like a nanomechanical mirror; we examine photon-photon correlations in the generated radiation.

In the following, like in the work of Kolchin *et al.* [20], we do not differentiate between the Stokes and anti-Stokes photons. We calculate the coincidence probability defined by

$$g^{(2)}(\tau) = \frac{\langle 0|\delta c_{out}^{\dagger}(t)\delta c_{out}^{\dagger}(t+\tau)\delta c_{out}(t+\tau)\delta c_{out}(t)|0\rangle}{\langle 0|\delta c_{out}^{\dagger}(t)\delta c_{out}(t)|0\rangle\langle 0|\delta c_{out}^{\dagger}(t+\tau)\delta c_{out}(t+\tau)|0\rangle}, \tag{20}$$

in which τ is a time delay, and

$$\delta c_{out}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \delta c_{out}(\omega)e^{-i\omega t}d\omega. \tag{21}$$

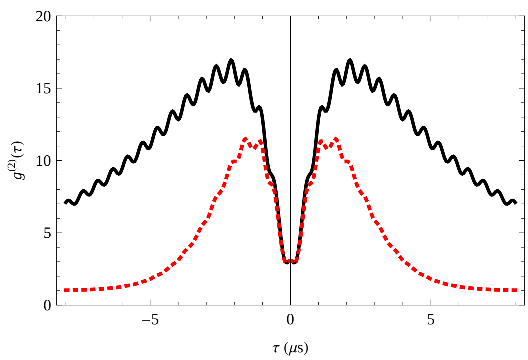

FIG. 8: (Color online) The normalized second-order correlation function $g^{(2)}(\tau)$ as a function of the time delay $\tau(\mu s)$ for different pump powers at $T = 0$K. $\wp$=1 mW (solid curve), and 4 mW(dotted curve).

Now we would evaluate the photon-photon correlations of the output fields numerically. We choose the pump power $\wp$=1 and 4 mW and the temperature of the environment $T = 0$ K; the other parameters are the same as those mentioned in Sec. III. The correlation function $g^{(2)}(\tau)$ between the spontaneously generated photons versus the time delay τ for different pump powers at a temperature of $T = 0$K is displayed in Fig. 8. We find that $g^{(2)}(\tau)$ is symmetric. It is also seen that $g^{(2)}(\tau) > g^{(2)}(0)$ as $\tau \neq 0$. This demonstrates the presence of photon antibunching, which is definitely of quantum origin. Further, we note the Cauchy-Schwartz inequality $g^{(2)}(\tau) \leq g^{(2)}(0)$ is violated, and the degree of the violation of the Cauchy-Schwartz inequality becomes smaller with increasing pump power. For pump power $\wp = 1$ mW, the peak value of $g^{(2)}(\tau)$ is about 17, and $g^{(2)}(0) \approx 3$; thus, $g^{(2)}(\tau)/g^{(2)}(0) \approx$5.6. However, for $\wp = 4$ mW, the peak value of $g^{(2)}(\tau)$ is about 11.5, and $g^{(2)}(0) \approx 3$, so $g^{(2)}(\tau)/g^{(2)}(0) \approx$3.8. Therefore, the spontaneously generated photons from the optomechanical system at $T = 0$ K are correlated nonclassically, and the nonclassical correlation becomes weaker with increasing pump power. This is reminiscent of the parametric downconversion process which at low pumping powers produces significant quantum correlations.

VI. CONCLUSIONS

We have shown that an optomechanical system driven by a pump field and a Stokes field can lead to generation of a four-wave-mixing signal. The Stokes field is amplified. We also find that normal-mode splitting occurs in both the generated fields, that is, in both Stokes and anti-Stokes fields. We also report lifetime splitting for pump power less than a critical power. Further, we have discussed the correlations of the photons generated from an optomechanical system by spontaneous processes. We find the correlations between these photons manifest the antibunching effect, and violate Cauchy-Schwartz inequality. Further, the violation of the Cauchy-Schwartz inequality becomes weaker with increasing pump power. Hence, the optomechanical system can be used to generate pairs of photons with quantum correlations. Thus the study of both stimulated and spontaneous Stokes and anti-Stokes signals provides us with a useful technique for studying the strong coupling regime of cavity optomechanics, as well as quantum fluctuations at macroscopic level.

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