

# Atomic Absorbers for Controlling Pulse Propagation in Resonators

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We consider pulse propagation through a Fabry-Perot cavity with silver mirrors containing a macroscopic sample of resonant absorbers. We show that the pulse velocity can be tuned from sub to superluminal in a strongly coupled atom-cavity system. We delineate the effects which arise from the interplay of cavity and absorbers. We demonstrate saturation of pulse advancement with increasing mirror thickness and atomic damping.

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## 1. Introduction

It is now well established that a clever manipulation of the dispersive properties of a medium can lead to both 'slow' (subluminal) and 'fast' (superluminal) light<sup>1,2</sup>. While there is nothing controversial about subluminal propagation (group velocity  $v_g$  less than the velocity of light in vacuum  $c$ ), there have been a flurry of discussions on the apparent 'superluminality' (transit time  $\tau_p$  through the system less than free space transit time  $\tau_f$ ). Superluminal transit has been reported in a variety of situations, including gain assisted systems<sup>3</sup>, in single and multi barrier tunnelling<sup>4-7</sup> in single cycle pulse transmission<sup>8</sup> and even in Fibonacci lattices<sup>9</sup>. In the context of superluminal tunnelling interesting saturation effects (independence of transit time on barrier width), known as Hartman effect<sup>10</sup> have been reported and studied in detail<sup>5,6,11</sup>. The effects can be even richer if the dispersive medium is kept inside a cavity. Typical examples of such systems can be Fabry-Perot (FP) cavity containing gaseous atoms<sup>12</sup>, semiconductor cavities with quantum wells<sup>13</sup>, microspheres doped with resonant atoms<sup>14</sup>, etc. In all the above cases the intracavity atoms or excitons can be modeled by resonant absorbers with familiar Lorentzian lineshape with a typical resonance frequency  $\omega_0$  and decay rate  $\gamma$ <sup>12</sup>. In the strong coupling regime, the atom-cavity (exciton-cavity) interaction leads to a splitting of the resonance into two resonances known as vacuum field Rabi splittings<sup>15,16</sup>. Recently such a strongly coupled system was studied for pulse control with a microscopic density of atoms in a high finesse ( $\sim 10^5$ ) cavity to show sub and superluminal propagation<sup>17</sup>. Superluminal transit was demonstrated at the outer edge of the split resonance. In this paper we probe the feasibility of pulse control in a less-sophisticated system, namely, in a low finesse FP cavity (with silver mirrors) which is easy to achieve. We consider macroscopic samples in the cavity. We specifically have solid state samples which have large relaxation times. We discuss in detail the cavity induced modifications of the transmission of a slab of

resonant absorbers. By means of a thorough analysis of the the phase time<sup>18</sup> (frequency derivative of the phases of the transmission and reflection coefficients) we demonstrate large tunability of the group velocity of narrow band pulses. We verify the predictions of the phase time calculations by explicit calculations of the transmitted and reflected pulse profiles. Finally, we show that the superluminal phase time exhibits saturation when the thickness of the metal mirrors (determining the cavity losses) and the atomic decay rates are varied.

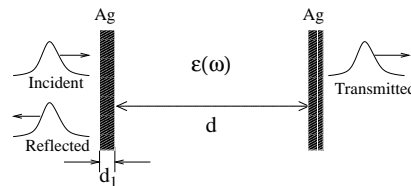


Fig. 1. Schematic view of the Fabry-Perot cavity with silver mirrors. The parameters are as follows:  $d_1 = 0.02\mu m$ ,  $d = 5.3\mu m$ ,  $\epsilon_{Ag} = -57.8 + 0.6i$ .

## 2. Numerical Results and Discussions

Consider the cavity shown in Fig.1, where a slab of resonant absorbers of length  $d$  is enclosed in between two identical silver mirrors of thickness  $d_1$ . All diffraction effects are neglected assuming an infinite extent in the transverse direction. We also assume the silver dielectric function to be independent of frequency, while that of the intracavity medium is given by

$$\epsilon(\omega) = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - 2i\gamma\omega}, \quad (1)$$

where  $\omega_0$ ,  $\omega_p$  are the atomic and plasma frequencies,  $\gamma$  is the atomic decay. The cavity length  $d$  is chosen such that the cavity resonance frequency  $\omega_{cav}$  coincides with the atomic frequency  $\omega_0$ . We apply the standard

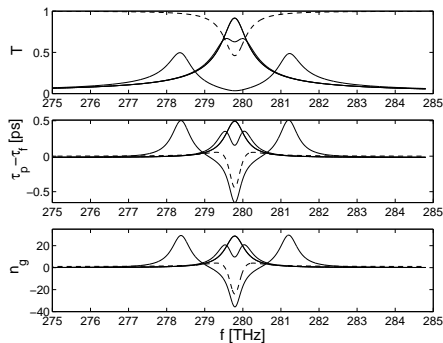


Fig. 2. (a) Absolute value of the amplitude transmission  $T$ , (b) delay  $\tau_p - \tau_f$  and (c) group index  $n_g$  as functions of frequency  $f$ . Curves from top to bottom (close to 280THz) are for  $(\omega_p/\omega_0)^2 = 0.0, 2.0 \times 10^{-6}$  and  $1.0 \times 10^{-4}$ , respectively, with  $\gamma/\omega_0 = 10^{-3}$ . The dashed line shows the results for the slab of resonant absorbers (no mirrors) for  $(\omega_p/\omega_0)^2 = 1.0 \times 10^{-4}$ . Other parameters are as in Fig.1

characteristic matrix approach<sup>19</sup> to evaluate the amplitude transmission  $T(\omega)$  and reflection  $R(\omega)$  through the structure at a given frequency  $\omega$ . The phase times for the reflected and transmitted pulse can be calculated as<sup>18</sup>

$$\tau_p^{T,R} = \left. \frac{\partial \phi_{T,R}}{\partial \omega} \right|_{\omega=\omega_c}, \quad (2)$$

where  $\phi_{T,R}$  is the phase of the transmission and reflection coefficients, respectively (i.e.,  $T, R = \text{Abs}(T, R) \exp(i\phi_{T,R})$ ). Both transmitted and reflected pulse profiles were calculated assuming a gaussian envelope of the input pulse with carrier frequency  $\omega_c$ .

The results for the amplitude transmission coefficient for three different values of  $(\omega_p/\omega_0)^2$  ( $\sim$ density of atoms), namely,  $0.0, 2.0 \times 10^{-6}$  and  $1.0 \times 10^{-4}$ , respectively, are shown in Fig.2a. The corresponding delay/advancement  $\tau_p - \tau_f$  and group index  $n_g$  are shown in Figs.2b and 2c, respectively. It is clear from Fig.2 that for an empty cavity ( $\omega_p/\omega_0 = 0.0$ ) the transmission of a pulse tuned at the cavity resonance can be highly subluminal with a group index of about 30. An increase in the density of atoms resulting in an increase in the atom-cavity coupling leads to the vacuum field Rabi splittings. The curve for  $(\omega_p/\omega_0)^2 = 2.0 \times 10^{-6}$  captures the onset of the splitting while for  $(\omega_p/\omega_0)^2 = 1.0 \times 10^{-4}$  one has well-split resonances. As a consequence of the strong coupling in the atom-cavity system, there is a significant change in the dispersive properties leading to the possibility of both sub and superluminal transmission. For example, for  $(\omega_p/\omega_0)^2 = 1.0 \times 10^{-4}$  and for a carrier tuned at  $\omega_0 = \omega_{cav}$ , the pulse transmission can be highly superluminal with an index  $n_g \sim -35$ . On the other hand for a pulse tuned at one of the side bands transmission is subluminal with group index of about +30. All these results were verified by explicit calculations of the transmitted pulse profiles (not shown). In order to appreciate the

role of the cavity in controlling the pulse delay we have shown the results for a dielectric slab (with no mirrors) of the same length  $d$  in Fig.2 by dashed lines. The value of  $(\omega_p/\omega_0)^2$  was taken as  $1.0 \times 10^{-4}$ . The transmission coefficient has a dip at resonance due to absorption in the slab. This is in contrast to the peak of transmission (close to unity) of an empty cavity or almost null transmission of the cavity filled with atoms with the same density. Note also that the slab can lead to superluminality (without much prospects for subluminal propagation) which can be easily surpassed if the same is enclosed in a cavity. Introduction of the cavity mirrors implies a decrease in the group index from -24 to -36 (see Fig.2c).

The inevitable question that arises is how to enhance the above effects. Since they are due to atom-cavity interaction, one of the ways is to increase the cavity finesse. The cavity of Fig.1 with no atoms has a finesse of about 46. One can increase the finesse, for example, by increasing the thickness of the silver mirrors. For mirror thickness  $d_1$ , say,  $0.03 \mu m$ , one has a finesse of about 125, leading to much sharper cavity resonances, albeit with slightly lower peak transmission. For an empty cavity, the increase in finesse from 46 to 125 implies an approximate increase in the group index from 30 to 80. The atom-cavity interaction in higher finesse cavities can lead to higher absolute values of group index leading to an enhancement of both sub- and super- luminality. For example, for the sideband of the split resonance for  $(\omega_p/\omega_0)^2 = 1.0 \times 10^{-4}$ , the group index can be enhanced from 30 to 46 corresponding to the same increase in finesse. In this context a better option would be to replace the silver mirrors by dielectric reflection coatings, which minimizes the mirror absorption. Next we discuss the be-

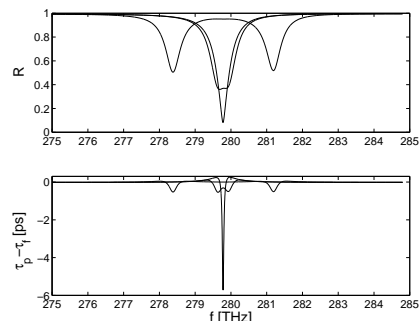


Fig. 3. (a) Absolute value of the amplitude reflection coefficient  $R$  and (b) delay  $\tau_p - \tau_f$  for reflected light as functions of frequency  $f$ . Curves from bottom to top are for  $(\omega_p/\omega_0)^2 = 0.0, 2.0 \times 10^{-6}$  and  $1.0 \times 10^{-4}$ , respectively. Other parameters are as in Fig.2

havior of the reflected pulse. In a one dimensional lossless system, the delay behavior (i.e., whether subluminal or superluminal) is expected to be the same in both transmitted and reflected pulses<sup>11</sup>. We now show that it could be very different in systems with losses. Note that our cavity is lossy on two counts, because of losses in silver and in absorbers. We plot the reflection coefficient  $R$

and pulse delay for reflected light in Fig.3, which also bears the signature of the vacuum Rabi splittings. It is clear from a comparison of Figs.2a, 2b and Fig.3 that the delay/advancement can be different in reflected and transmitted light. Even for an empty cavity a pulse resonant with the cavity gets delayed (advanced) in transmission (reflection). The same can be asserted for a pulse tuned to the side band of a cavity with atoms, for example, with  $(\omega_p/\omega_0)^2 = 1.0 \times 10^{-4}$ . For the same density a pulse tuned at unperturbed cavity/atomic frequency ( $\omega_c = \omega_0 = \omega_{cav}$ ), encounters a slight delay in reflection, while the transmitted pulse is highly superluminal with about 3.5% transmission.

We next discuss the sensitiveness of the phase times to the damping in the absorber as well as the losses in the mirror. Note that both the absorption and the reflectivity of the mirror increase and then saturate with an increase in the thickness  $d_1$ <sup>20</sup>. The results for a pulse tuned at the resonance of the empty cavity are shown in Fig.4. In all our calculations pulse width was taken as 20 ps for which transmitted pulse had nominal distortion. It can be easily seen from Fig.4a that for  $\gamma/\omega_0 = 10^{-4}$  the advancement is about 40 ps. This is quite a large effect for a cavity with a low finesse. Note also, for example, a change of phase time by two orders of magnitude as the damping of the absorbers goes down by a factor of ten (see Fig.4a). Another important feature is the saturation in both the curves of Fig.4. The saturation behavior is reminiscent of Hartman effect well known in tunnelling problems. The saturation in Fig.4b is a direct consequence of the saturation of mirror reflectivity and absorption with increasing  $d_1$ . The saturation in Fig.4a can be explained as follows. At the said frequency the transmission of the cavity is very low (due to exponentially decaying field envelope inside) and it acts like a barrier. An increase in the atomic damping leads to an increase in the effective barrier thickness resulting in the saturation of phase time like in Hartman effect.

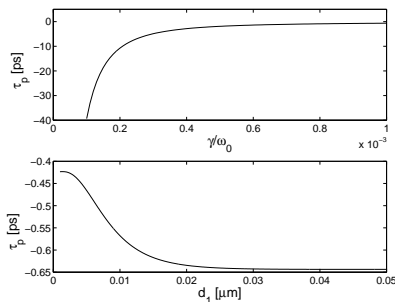


Fig. 4. Phase time  $\tau_p$  for the transmitted wave as functions of (a) normalized atomic decay  $\gamma/\omega_0$  and (b) mirror thickness  $d_1$  for  $(\omega_p/\omega_0)^2 = 1.0 \times 10^{-4}$ . Pulse carrier is tuned to the empty cavity resonance. Other parameters are as in Fig.1.

### 3. Conclusion

In conclusion, we studied a cavity with silver mirrors containing resonant atoms. We have shown that a proper knowledge of phase time can lead to a meaningful search for large sub and super luminal transit. We have demonstrated tunability of the group velocity with detectable levels of transmitted/reflected pulse for a strongly coupled atom-cavity system. All the phase time predictions have been verified by explicit calculations of the output pulse profiles. We have also shown that there are interesting saturation effects in the phase times as functions of the mirror thickness and atomic damping.

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