Non-linear wave packet dynamics of coherent states of various symmetry groups

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Abstract: We present a comparative study of the non-linear wave packet dynamics of two-mode coherent states of the Heisenberg-Weyl group, the SU(1,1) group and the SU(2) group under the action of a model anharmonic Hamiltonian. In each case, we find certain generic signatures of non-linear evolution such as quick onset of decoherence followed by Schrödinger cat formation and revival. We also report important differences in the evolution of coherent states belonging to different symmetry groups.

OCIS codes: (190.0190) Nonlinear Optics, (270.0270) Quantum Optics, (270.1670) Coherent Optical Effects, (030.1640) Coherence

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Received September 20, 1999; Revised November 03, 1999

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1 Introduction

The quantum phenomena of revival and fractional revival (or the formation of Schrödinger cat and cat-like states) have been studied in many diverse systems and situations such as Rydberg atoms [1, 2, 3], the Jaynes-Cummings model [4], light propagation in Kerr media [5, 6] and even transient signals from multilevel quantum systems [7]. It is now apparent that these features are rather generic in that they are closely connected to the anharmonicity in the energy spectrum of the underlying Hamiltonian and to the periodicity of the time evolution operator.

The literature deals mostly with systems whose energy spectrum depends on a single quantum number with an associated revival time scale. However, there are many systems whose energy levels depend non-linearly on at least two quantum numbers [8, 9, 10]. In this paper, we focus on the non-linear wavepacket dynamics of coherent states under the action of a generic two-mode Hamiltonian.

Coherent states were originally defined for harmonic oscillators (H. O.) or the radiation field. From a group theoretic point of view, the H. O. coherent states arise in systems whose dynamical symmetry group is the Heisenberg-Weyl group. Coherent states of other symmetry groups also exist. Thus, for example, the much studied pair [11] and Perelomov [12, 13, 14] coherent states belong to the SU(1,1) group and are special cases of what may be called generalized SU(1,1) coherent states [10]. Coherent states of the SU(2) group have also been constructed [15, 16, 17, 18].

Our objective is to present a comparative study of how coherent states of various symmetry groups evolve under the action of the *same* two-mode generic Hamiltonian. Through a series of pictures and movies, we show how the initial coherent structure is lost due to quantum dephasing and then regained later on to form spectacular and varied quasi-coherent structures leading up to the formation of Schrödinger cats in some cases and to full revival in all cases.

2 Coherent states

H. O. coherent states $|\alpha,\beta\rangle$ are superpositions of number states $|m,n\rangle$ with m, $n = 0$, 1, 2, ... ∞. One can also construct 'even' (+) and 'odd' (-) coherent states

$$|\alpha,\beta\rangle_{\pm} = (|\alpha,\beta\rangle \pm |-\alpha,-\beta\rangle)/\sqrt{2P_{\pm}}, \quad P_{\pm} = 1 \pm \exp[-2(|\alpha|^2+|\beta|^2)]. \tag{1}$$

SU(1,1) coherent states $|\eta,\xi,q\rangle$ are formed by superposing number states of the form $|n+q,n\rangle$ where $q = a^\dagger a - b^\dagger b$ is an integer constant and $n = 0$, 1, 2, ... ∞:

$$\begin{aligned}|\eta,\xi,q\rangle &= N(\xi,q)\exp(-\xi\eta^*)\sum_{n'=0}^{\infty}\frac{\eta^{n'}}{\Gamma(n'+1)} \\ &\times\sum_{n=0}^{\infty}\frac{\xi^n(1-|\eta|^2)^{n+\frac{q+1}{2}}\sqrt{(n+n')!(n+n'+q)!}}{n!(n+q)!}|n+n'+q,n+n'\rangle,\end{aligned} \tag{2}$$

where

$$N(\xi,q) = \left[\sum_{n=0}^{\infty}\frac{|\xi|^{2n}}{n!(n+q)!}\right]^{-1/2}. \tag{3}$$

For pair coherent states $\eta \to 0$ whereas for Perelomov coherent states $\xi \to 0$. Since η and ξ are continuous and (in general) complex parameters, infinitely many other cases of $|\eta, \xi, q\rangle$ exist even for the same value of q.

In the Schwinger representation of the SU(2) algebra [17], SU(2) coherent states $|\theta, \phi, N\rangle$ can be formed by the superposition of number states of the form $|K, N-K\rangle$ where $N = a^\dagger a + b^\dagger b$ is an integer constant and $K = 0, 1, 2, \ldots N$:

$$\begin{aligned} |\theta, \phi, N\rangle &\equiv |\tau, N\rangle, \quad \tau = \tan\frac{\theta}{2} e^{-i\phi}; \\ &= \left(1 + |\tau|^2\right)^{-N/2} \sum_{K=0}^{N} \begin{pmatrix} N \\ K \end{pmatrix}^{1/2} \tau^K |K, N-K\rangle. \end{aligned} \tag{4}$$

The parameters θ and ϕ represent angles in the range $0 \leq \theta, \phi \leq 2\pi$.

3 Wave packet dynamics

To study the wave packet dynamics of these two-mode coherent states, we look at the evolution of their quadrature distributions under the action of a generic, phase insensitive, two-mode Hamiltonian (we use $\hbar = 1$):

$$H = c_1 \left[(a^\dagger a)^2 + (b^\dagger b)^2\right] - c_2 a^\dagger a b^\dagger b, \tag{5}$$

which can be readily diagonalised:

$$H = \frac{\pi}{4} \left[(a^\dagger a + b^\dagger b)^2/T_- + (a^\dagger a - b^\dagger b)^2/T_+\right], \quad \text{where,} \quad T_\pm = \pi/(2c_1 \pm c_2). \tag{6}$$

Recall that the quadrature distribution for a state vector $|\psi(t)\rangle$ is defined as $|\psi(x, y, t)|^2 = |\langle x, y|\psi(t)\rangle|^2$, where $|x, y\rangle$ is the eigenvector of $(a + a^\dagger)/\sqrt{2}$ and $(b + b^\dagger)/\sqrt{2}$ with eigenvalues x and y respectively. Details of analytical results on the evolution of H. O. and SU(1,1) coherent states for this Hamiltonian have been described elsewhere [9, 10]. In this paper we include the evolution of SU(2) coherent states as well. Our major results can be summarised as follows:

(a) The non-linearity of the Hamiltonian destroys the initial coherence of the wave packets. But the long-time evolution guarantees revival for all these states and at least one example each of cat formation by coherent states of all three dynamical groups considered.

(b) The details of their evolution are different. For H. O. coherent states, the evolution of the initial wave packet depends on its symmetry and on the ratio of the two times scales T_+ and T_-. For SU(1,1) coherent states, on the other hand, $q = a^\dagger a - b^\dagger b$ being constant, it is clear from Eq. (6) that T_+ and hence the ratio T_-/T_+ does not play an active role. However, the parity of q was found to be crucial in determining the revival features of SU(1,1) coherent states. Likewise, for SU(2) coherent states, $N = a^\dagger a + b^\dagger b$ is constant. In this case also only one of the time scales is significant, but the parity of N decides the revival time.

3.1 *H. O. Coherent States*

At $t = 0$, the H. O. coherent states $|\alpha, \beta\rangle$ and $|\alpha, \beta\rangle_\pm$ are represented by $\psi(x, y, 0) = \phi_-(x, y)$ and $\psi_\pm(x, y, 0) = [\phi_-(x, y) \pm \phi_+(x, y)]/\sqrt{2P_\pm}$ respectively, where $\phi_\pm(x, y)$ are Gaussians centered at $x = \pm\alpha\sqrt{2}$ and $y = \pm\beta\sqrt{2}$:

$$\phi_\pm(x, y) = \pi^{-1/2} \exp\left[-\frac{1}{2}\left\{|\alpha|^2 - \alpha^2 + |\beta|^2 - \beta^2 + \left(x \pm \alpha\sqrt{2}\right)^2 + \left(y \pm \beta\sqrt{2}\right)^2\right\}\right]. \tag{7}$$

The Gaussian pairs that represent the odd and even states at $t = 0$ are well separated for α, $\beta > 1$ since the interference between ϕ_- and ϕ_+ is negligible. However, as the wave packets spread, interference between the components of the wave packets soon destroys the initial coherent structure (**Movie 1**). But as shown in **Movie 2**, the same interference will bring back some coherence in the structure at some later instants of time for a given value of the ratio T_+/T_-. For example, when $T_+/T_- = 2/3$, a six-way fractional revival occurs for the odd state at $t = T_-/3$. Note that the odd state is *fully revived* at $t = 2T_-$ for $T_+/T_- = 2/3$ whereas the even state takes twice as long to revive. Although the evolution of the odd and even states may appear identical in the initial stages (**Movie 1**), it is clear that these states evolve quite differently at later times (**Movie 2**). Thus the symmetry of the initial state plays a crucial role in the long term dynamics.

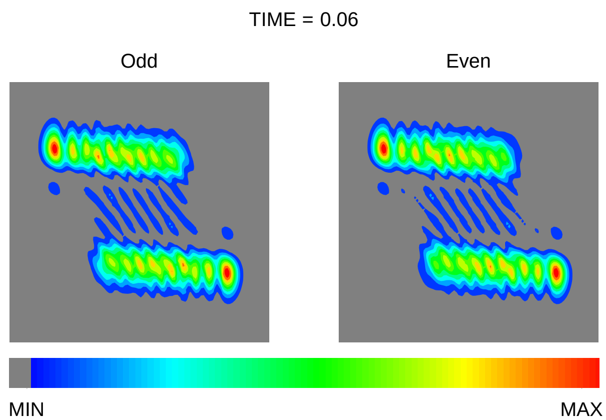

Fig. 1. [102 KB] A frame of Movie (1) showing the *erosion* of the initial coherent structures in the quadrature distributions of $|\alpha, \beta\rangle_-$ (Odd) and $|\alpha, \beta\rangle_+$ (Even). Shown here are the contour plots of these distributions as functions of x/α and y/β. The unit of time is T_-. We have set $\alpha = 2$, $\beta = 3$ and $T_+/T_- = 2/3$. All the frames in this paper have been drawn in *Mathematica* using a customised ColorFunction in which the base is set at RGBColor[0.5,0.5,0.5] for better contrast.

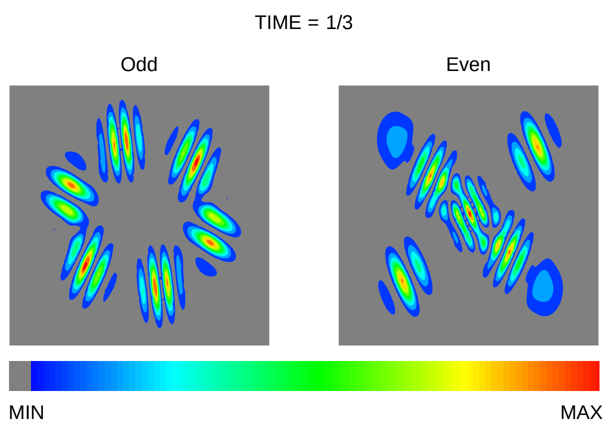

Fig. 2. [404 KB] A frame of Movie (2) showing the *restoration* of coherent structures in the quadrature distributions of $|\alpha, \beta\rangle_-$ (Odd) and $|\alpha, \beta\rangle_+$ (Even) at later instants of time. All other details are as in Fig. 1.

#13568 - \$15.00 US
Received September 20, 1999; Revised November 03, 1999

In **Movie 3**, we show that the ratio of T_+ and T_- is equally as important in determining the time evolution of the wave packets. Snapshots of the quadrature distribution for $|\alpha, \beta\rangle$ are shown for $T_+/T_- = 2/3$ (left picture) and $3/5$ (right picture). Note that the revival time is $16T_-$ in the former case whereas it is only $6T_-$ for the latter. Note also that for the same value of time, a change in the ratio T_+/T_- will drastically alter the characteristics of fractional revival. It can be shown that at $t = 4T_-$ and $T_+/T_- = 2/3$, $\psi(x, y, t) = [\exp(-i\pi/4)\phi_-(x, y) + \exp(i\pi/4)\phi_+(x, y)]/\sqrt{2}$. The components of the wave function do not interfere with each other as they are $\pi/2$ out of phase. The corresponding quadrature distribution represents a Schrödinger cat state, a two-way fractional revival.

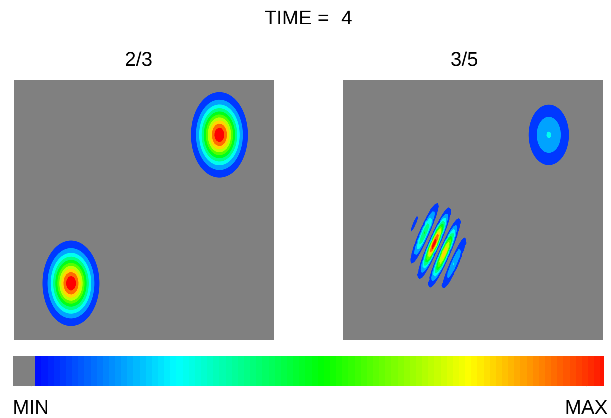

Fig. 3. [100 KB] A frame of Movie (3) showing how the ratio T_+/T_- affects the evolution of the quadrature distributions for $|\alpha, \beta\rangle$. Shown here are the contour plots of these distributions as functions of x/α and y/β for $T_+/T_- = 2/3$ (left) and $T_+/T_- = 3/5$ (right). The unit of time is T_-. We have set $\alpha = 2$ and $\beta = 3$.

3.2 *SU(1,1) Coherent States*

The SU(1,1) coherent states $|\eta, \xi, q\rangle$ will have different initial quadrature distributions for different values of η, ξ and q. The initial distributions are coherent structures. As the system evolves, this coherence is lost rather quickly but is restored partially (fully) at times of fractional (total) revival much as in the case of H. O. coherent states. The details are different. For odd values of q, $|\psi(x, y, T_-)|^2 = |\psi(x, y, 0)|^2$. This would be the case for even values of q also provided both η and ξ are pure imaginary, or one of them is zero while the other is pure imaginary. Otherwise, $|\psi(x, y, 2T_-)|^2 = |\psi(x, y, 0)|^2$ for even values of q. We illustrate these features in **Movies 4-5**.

In **Movie 4**, the system is a Perelomov coherent state ($\xi = 0$, $\eta = -i\tanh\pi/4$). In this case, the initial quadrature distribution is a Gaussian for $q = 0$ and a vortex [19, 20] for $q = 1$. But the revival time is T_- for *all* values of q. Note that the quadrature distribution for $q = 0$ develops hyperbolic dark fringes at $t = T_-/2$ whose origin can be traced analytically [10]. In **Movie 5**, the system is a pair coherent state ($\xi = 3$, $\eta = 0$). Since both ξ and η are real, the revival time is $2T_-$ when $q = 0$ and T_- when $q = 1$. Furthermore, this system allows the formation of Schrödinger cats at $t = T_-/2$ for $q = 0$. At $t = T_-$ the initial distribution rotates by $\pi/2$. At $t = T_-/2$, these two distributions add incoherently as the components of the wave function, being $\pi/2$ out of phase, do not interfere with each other.

#13568 - $15.00 US
Received September 20, 1999; Revised November 03, 1999

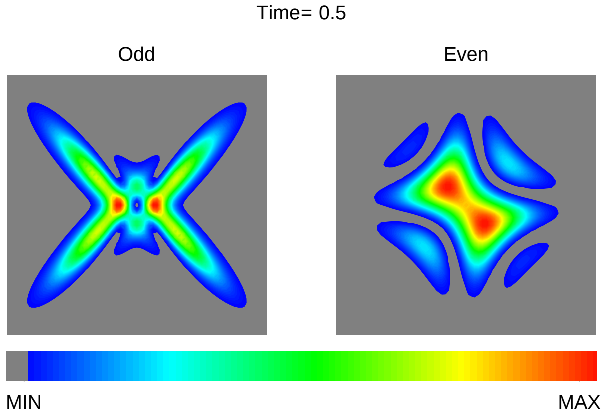

Fig. 4. [684 KB] A frame of Movie (4) showing the evolution of the quadrature distribution for a Perelomov coherent state ($\xi = 0$, $\eta = -i \tanh \pi/4$). for $q = 0$ (Even) and $q = 1$ (Odd). Shown here are the contourplots of these distributions. The unit of time is T_-. In order to erase any ambiguity from our nomenclature, we stress that the distribution for different values of q will be different even if the parity of q is the same.

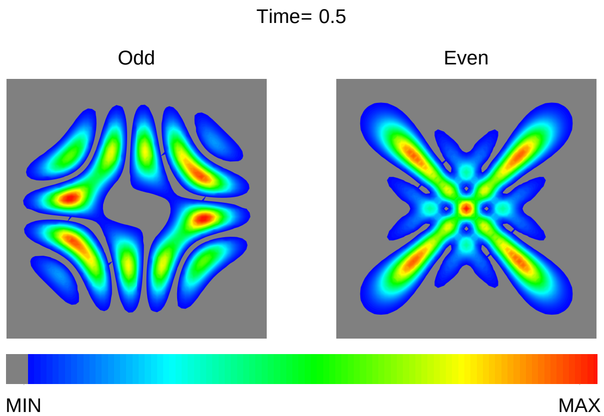

Fig. 5. [878 KB] A frame of Movie (5) showing the evolution of the quadrature distribution for a pair coherent state ($\xi = 3$, $\eta = 0$). All other details are as in Fig. 4.

3.3 *SU(2) Coherent States*

The SU(2) coherent states given by Eq. (4) are represented by the wave function

$$\psi(x, y, 0) = \frac{1}{\sqrt{\pi 2^N N!}} \left[\frac{1+\tau^2}{1+|\tau|^2}\right]^{N/2} e^{-(x^2+y^2)/2} H_N\left(\frac{\tau x + y}{\sqrt{1+\tau^2}}\right), \quad \tau = \tan\frac{\theta}{2} e^{-i\phi}. \tag{8}$$

Clearly the corresponding quadrature distribution will depend not only on N but also on the amplitude and phase of the parameter τ, that is , on the angles θ and ϕ. The distribution, which is a Gaussian modulated by the square of a Hermite polynomial, will have dark fringes at the nodes of the latter and will be lined up at an angle $\tan^{-1}(1/\tau)$ with respect to the positive x- axis. In the limit $\tau \to 0$ (or $\tau \to \infty$), all the N bosons are in one mode only (see also Eq. 4) and the quadrature distribution is aligned vertically (or horizontally). For $\tau = \pm 1$, the distribution is along the diagonals $x = \mp y$. An altogether different pattern arises in the limiting case $\tau \to \pm i$. One obtains a wave function with

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Received September 20, 1999; Revised November 03, 1999

vortex structure [19, 20]:

$$\psi(x,y,0)|_{\tau=\pm i} = \frac{(x^2+y^2)^{N/2}}{\sqrt{\pi N!}} \exp\left[-\frac{(x^2+y^2)}{2} \pm iN\eta\right], \quad \eta = \tan^{-1}\left(\frac{x}{y}\right). \tag{9}$$

The quadrature distributions corresponding to $\tau = 1$ and $\tau = i$ are shown in Fig. (6) for $N = 11$.

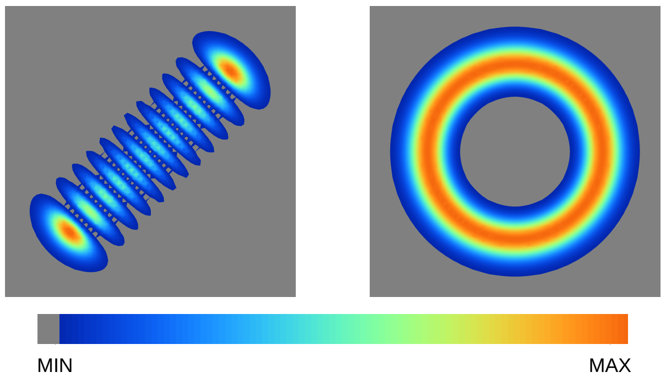

Fig. 6. Contour plots of the quadrature distributions for the SU(2) coherent state $|\tau, 11\rangle$ for $\tau = 1$ (left picture) and $\tau = i$ (right picture).

The remarkable transition from one pattern to another can be effected by changing the phase of τ while keeping $|\tau| = 1$. This is shown in **Movie 6**.

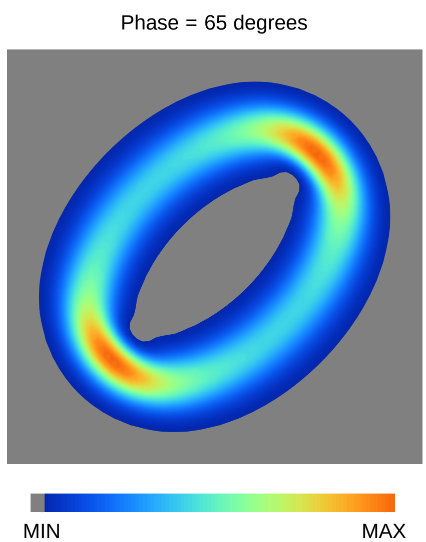

Fig. 7. [269 KB] A frame of Movie (6) showing how the pattern for the quadrature distribution of the SU(2) coherent state $|\tau, 11\rangle$ changes when one varies the phase of τ while keeping $|\tau| = 1$.

For $N \gg 1$, the quadrature distributions of $|\tau, N\rangle$ and $|\tau, N+1\rangle$ will have similar initial patterns and short-time evolutions. But their long-time evolutions and revival features will depend critically on the parity of N. In fact, one can show that under the action of the Hamiltonian H given by Eqs (5) and (6), $|\tau, N\rangle$ revives (but for an over-all phase factor) at *all* integer multiples of T_+ if N is odd, and at *even* multiples of T_+ if N is even. If τ is pure imaginary, then the revival time is the same for *all* values of N.

These findings are amply demonstrated in **Movies 7-8**. We consider two SU(2) coherent states with the same value of τ but with N values differing by unity. In movies (7) and (8) we compare the evolution of their quadrature distributions under the action of the Hamiltonian (5). In both movies, $N = 10$ and 11 whereas $\tau = 1$ in **Movie 7** and $\tau = i$ in **Movie 8**.

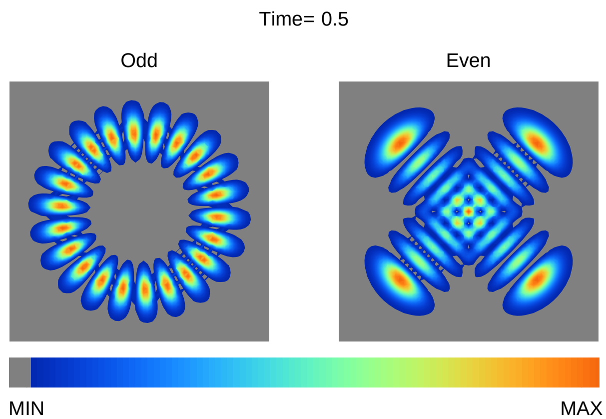

Fig. 8. [825 KB] A frame of movie (7) showing the evolution of the quadrature distributions of the SU(2) coherent state $|1, N\rangle$ for $N = 10$ (Even) and $N = 11$ (Odd). Shown here are the contourplots of these distributions. The unit of time is T_+. Lest there be any ambiguity in our nomenclature, we stress that the distribution for different values of N will be different even if the parity of N is the same.

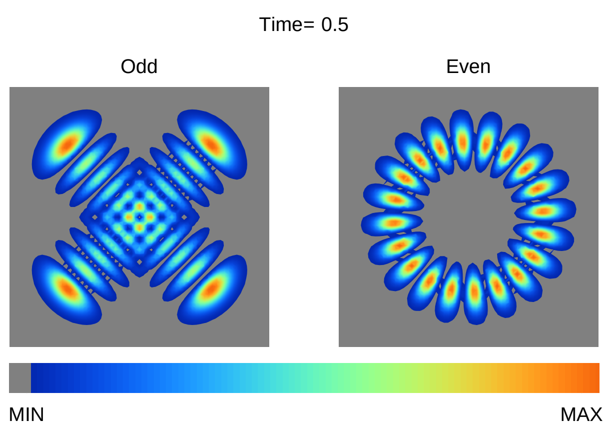

Fig. 9. [863 KB] A frame of movie (8) showing the evolution of the quadrature distributions of the SU(2) coherent state $|i, N\rangle$. All other details are as in Fig. 8.

Fractional revivals (if any) will occur at times $t = (r/s)T_+$, where r and s are mutually prime with $r < s$. Defining $\mathcal{U}(t) = \exp(-iHt)$ to be the time evolution operator, we can follow the method adopted earlier [10] to show that

$$\mathcal{U}(\frac{r}{s}T_+)|\tau, N\rangle = \exp\left[-\frac{i\pi}{4}\frac{r}{s}N^2\left(1+\frac{T_+}{T_-}\right)\right] \times \sum_{j=0}^{l-1} \alpha_j^{(r,s)} \left|\tau \exp\left(-i\pi\left[2j/l - rN/s\right]\right), N\right\rangle , \quad (10)$$

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Received September 20, 1999; Revised November 03, 1999

where

$$l = \begin{cases} s, & \text{if } r \neq s \pmod 2; \\ 2s, & \text{if } r = s = 1 \pmod 2). \end{cases} \tag{11}$$

The coefficients $\alpha_j^{(r,s)}$ are given by

$$\alpha_j^{(r,s)} = \frac{1}{l}\sum_{p=0}^{l-1} \exp(-i\pi r p^2/s + 2i\pi p j/l), \tag{12}$$

and can be evaluated analytically [21].

As an illustration of fractional revival, we will now explain the patterns obtained at $t = T_+/2$. From Eq. (10), one obtains

$$\mathcal{U}(T_+/2)|\tau, N\rangle \to |\tau e^{i\pi N/2}, N\rangle + i|-\tau e^{i\pi N/2}, N\rangle. \tag{13}$$

For **Movie 7**, we get

$$\mathcal{U}(T_+/2)|1, 10\rangle \to |-1, 10\rangle + i|1, 10\rangle, \tag{14}$$

and

$$\mathcal{U}(T_+/2)|1, 11\rangle \to |-i, 11\rangle + i|i, 11\rangle. \tag{15}$$

Note that the wave functions corresponding to $|\pm 1, N\rangle$ are real. In Eq. (14), these state vectors are added with a $\pi/2$ phase difference. The resulting state vector represents a cat state whose quadrature distribution is an incoherent sum of the distributions for $|1, 10\rangle$ and $|-1, 10\rangle$:

$$\frac{e^{-(x^2+y^2)}}{\pi 2^{N+1} N!}\left[H_N^2\left(\frac{y-x}{\sqrt{2}}\right) + H_N^2\left(\frac{y+x}{\sqrt{2}}\right)\right], \quad N = 10.$$

In Eq.(15), on the other hand, the state vectors $|\pm i, N\rangle$ are added with a $\pi/2$ phase difference. But the wave functions for $|\pm i, N\rangle$ are complex giving rise to strong interference. Using the wave functions for $|\pm i, N\rangle$, the quadrature distribution for $\mathcal{U}(T_+/2)|1, 11\rangle$ is found to be

$$(1 - \sin 2\eta N) \times \text{ the quadrature distribution for } |\pm i, N\rangle, \quad N = 11.$$

Dark fringes will appear whenever

$$\sin 2\eta N = 1, \quad \text{i.e.} \quad \eta = \frac{\pi}{4N} + \frac{\pi m}{N}; \tag{16}$$

where m is an integer. Since $0 \leq \eta \leq 2\pi$, the total number of such fringes will be $2N$. Although there will also be $2N$ bright patches, the phenomenon is not a $2N$-way fractional revival. Rather, it is more like 'slicing a doughnut' radially in $2N$ equal parts.

For **Movie 8**, we would get

$$\mathcal{U}(T_+/2)|i, N\rangle \to \begin{cases} |-i, N\rangle + i|i, N\rangle, & \text{if } N = 10; \\ |1, N\rangle + i|-1, N\rangle, & \text{if } N = 11. \end{cases}$$

The corresponding quadrature distributions can be explained in a similar fashion.

#13568 - $15.00 US
Received September 20, 1999; Revised November 03, 1999

4 Conclusion

In conclusion, we have studied the non-linear wave packet dynamics of two-mode (a) H. O. coherent states, (b) SU(1,1) coherent states and (c) SU(2) coherent states. The Hamiltonian used was generic, phase insensitive and quadratic in the photon number operators. Although the Hamiltonian was the same in each case, it was found that the states (a), (b) and (c) evolve quite differently which can be traced to their belonging to different symmetry groups. However, certain similarities in their evolution also emerged. Thus in *all* cases, the initial coherent structure is quickly lost but is regained later on to experience the quantum phenomenon of revival and the formation of Schrödinger cats.

#13568 - $15.00 US
Received September 20, 1999; Revised November 03, 1999