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Coherence matrices of light beams at dielectric interfaces and Goos–Hänchen effect

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The general relations among the coherence matrices of the incident, reflected, and transmitted beams at the surface of a dielectric are obtained for incident fields of arbitrary statistics and polarization. The Poynting vector is also expressed in terms of the coherence matrices and the general conditions for the nonvanishing of the lateral Goos–Hänchen shift are obtained.

Index Headings: Polarization; Reflection; Transmission; Coherence.

The concept of partial polarization is well known in coherence optics.[1] It has been formulated in terms of 2×2 matrices[2], or in the general case by 3×3 matrices.[3] Many properties of the coherence matrices,[4] such as transformation due to passage through an optical device, relation to Stokes parameters, and relation to the degree of optical coherence, have been studied in some detail.[5,6]

In this paper, we obtain the relations between the coherence matrices of the reflected and refracted beam at the surface of a dielectric when an electromagnetic beam of arbitrary statistics and polarization is incident on the dielectric. In the special case of nonfluctuating light beam, this provides an alternate method of discussing the well-known polarization effects at the dielectric interface. We also comment on the polarization effects in the present problem when electromagnetic correlations of order higher than two are considered.[7] We then work out several identities that relate the Poynting vector to coherence matrices. Such identities enable us to obtain both longitudinal and transverse Goos–Hänchen shifts, because as noted by Imbert[11] such shifts are closely related to the Poynting vector. We thus generalize the usual treatment of Goos–Hänchen shifts to light beams of arbitrary statistics and polarization. Our relations clearly exhibit the conditions under which lateral shifts can occur.

RELATIONS AMONG COHERENCE MATRICES OF INCIDENT, REFRACTED, AND REFLECTED BEAMS

Let us consider the case that when a plane wave of frequency ω and wave vector $\vec{K}_0$ is incident on the surface ($z=0$) of a dielectric, for the geometrical situation the region $-\infty \leq z \leq 0$ is occupied by a dielectric with dielectric function $\epsilon_0(\omega)$ and the region $0 \leq z \leq \infty$ is occupied by a dielectric with dielectric function $\epsilon(\omega)$. We will write the field in the analytic-signal form as

$$\vec{E}^{(i)}(\vec{r},t)=\vec{\mathcal{E}}^{(i)}e^{i\vec{K}_0\cdot\vec{r}-i\omega t}. \tag{1}$$

Our incident beam is in general a fluctuating beam (i.e., $\vec{\mathcal{E}}^{(i)}$ is a stochastic variable); its statistics will be given by a set of probability-distribution functions.[12] We first discuss the second-order correlation theory of such beams. Because the beam is propagating in the direction $\vec{K}_0$, we introduce a set of two orthogonal vectors defined by

$$\vec{S}_2^{(i)}=\frac{\hat{z}\times\vec{\kappa}}{\kappa},\quad \vec{S}_1^{(i)}=\frac{\vec{S}_2^{(i)}\times\vec{K}_0}{k_0\sqrt{\epsilon_0}},$$
$$\vec{K}_0=\vec{\kappa}+\hat{z}w_0,\quad w_0^2=k_0^2\epsilon_0-\kappa^2,\quad k_0=\omega/c, \tag{2}$$

where $\vec{\kappa}$ is the component of the wave vector parallel to the surface $z=0$. The vectors $\vec{S}_1^{(i)}$, $\vec{S}_2^{(i)}$, $\vec{K}_0/k_0\sqrt{\epsilon_0}$ form a right-hand coordinate system. These vectors are represented in Fig. 1. In this coordinate system, the coherence matrix of the incident beam will be a 2×2 matrix,[2,3]

$$J^{(i)}=\begin{pmatrix}\langle\mathcal{E}_1^{(i)*}\mathcal{E}_1^{(i)}\rangle & \langle\mathcal{E}_1^{(i)*}\mathcal{E}_2^{(i)}\rangle\\ \langle\mathcal{E}_2^{(i)*}\mathcal{E}_1^{(i)}\rangle & \langle\mathcal{E}_2^{(i)*}\mathcal{E}_2^{(i)}\rangle\end{pmatrix}, \tag{3}$$

where

$$\mathcal{E}_j^{(i)}=\vec{S}_j^{(i)}\cdot\vec{\mathcal{E}}^{(i)}. \tag{4}$$

In order to obtain the coherence matrices of reflected and refracted beams, we have to use the solution of the Maxwell equations for fluctuating incident beams with arbitrary polarization. The classical solution for nonfluctuating beams is well known.[1] So we take the classical solution and average it over the probability distribution of the incident field. Usually, the classical solution is obtained by resolving the beam into two polarization components. This is not necessary. Maxwell boundary conditions could be used directly, to obtain

$$\vec{E}^{(R)}(\vec{r},t)=\vec{\mathcal{E}}^{(R)}e^{i\vec{K}_0'\cdot\vec{r}-i\omega t},\quad \vec{E}^{(T)}(\vec{r},t)=\vec{\mathcal{E}}^{(T)}e^{i\vec{K}\cdot\vec{r}-i\omega t}, \tag{5a}$$

$$\vec{K}_0'=\vec{\kappa}-w_0\hat{z},\quad \vec{K}=\vec{\kappa}+w\hat{z},\quad w^2=k_0^2\epsilon-\kappa^2, \tag{5b}$$

$$\vec{\mathcal{E}}^{(T)}=\frac{2w_0}{(w+w_0)}\vec{\mathcal{E}}^{(i)}-\frac{2w_0}{k_0^2(w_0\epsilon+w\epsilon_0)}\vec{K}_0(\vec{K}\cdot\vec{\mathcal{E}}^{(i)}), \tag{6}$$

$$\vec{\mathcal{E}}^{(R)}=-\left[\left(\frac{w-w_0}{w+w_0}\right)\vec{\mathcal{E}}^{(i)}+\frac{2(w-w_0)}{k_0^2(w_0\epsilon+w\epsilon_0)}(w_0\vec{\kappa}-ww_0\hat{z})(\hat{z}\cdot\vec{\mathcal{E}}^{(i)})\right]. \tag{7}$$

In deriving Eqs. (5)–(7), we have also used the transverse nature of different fields. When $\epsilon_0=1$, such results could also be easily obtained from the Ewald–Oseen extinction theorem.[13] In analogy to Eq. (2), we introduce the unit vectors for reflected and transmitted beams

$$\vec{S}_2^{(T)}=\vec{S}_2^{(R)}=\vec{S}_2^{(i)},\quad \vec{S}_1^{(T)}=\frac{\vec{S}_2^{(T)}\times\vec{K}}{k_0\sqrt{\epsilon}},\quad \vec{S}_1^{(R)}=\frac{\vec{S}_2^{(R)}\times\vec{K}_0'}{k_0\sqrt{\epsilon_0}}. \tag{8}$$

Using Eqs. (2) and (6)–(8), we find that

$$(\vec{S}_2^{(T)}\cdot\vec{\mathcal{E}}^{(T)})=\frac{2w_0}{w+w_0}(\vec{S}_2^{(i)}\cdot\vec{\mathcal{E}}^{(i)}), \tag{9}$$

$$(\vec{S}_2^{(R)}\cdot\vec{\mathcal{E}}^{(R)}) = -\frac{w-w_0}{w+w_0}(\vec{S}_2^{(i)}\cdot\vec{\mathcal{E}}^{(i)}),$$

$$(\vec{S}_1^{(T)}\cdot\vec{\mathcal{E}}^{(T)}) = \frac{2w_0\sqrt{\epsilon\epsilon_0}}{(w_0\epsilon+w\epsilon_0)}(\vec{S}_1^{(i)}\cdot\vec{\mathcal{E}}^{(i)}), \tag{10}$$

$$(\vec{S}_1^{(R)}\cdot\vec{\mathcal{E}}^{(R)}) = \frac{w_0\epsilon-w\epsilon_0}{w_0\epsilon+w\epsilon_0}(\vec{S}_1^{(i)}\cdot\vec{\mathcal{E}}^{(i)}).$$

The coherence matrices of reflected and transmitted beams are defined by

$$J^{(R)} = \begin{pmatrix} \langle \mathcal{E}_1^{(R)*}\mathcal{E}_1^{(R)}\rangle & \langle \mathcal{E}_1^{(R)*}\mathcal{E}_2^{(R)}\rangle \\ \langle \mathcal{E}_2^{(R)*}\mathcal{E}_1^{(R)}\rangle & \langle \mathcal{E}_2^{(R)*}\mathcal{E}_2^{(R)}\rangle \end{pmatrix}, \quad J^{(T)} = \begin{pmatrix} \langle \mathcal{E}_1^{(T)*}\mathcal{E}_1^{(T)}\rangle & \langle \mathcal{E}_1^{(T)*}\mathcal{E}_2^{(T)}\rangle \\ \langle \mathcal{E}_2^{(T)*}\mathcal{E}_1^{(T)}\rangle & \langle \mathcal{E}_2^{(T)*}\mathcal{E}_2^{(T)}\rangle \end{pmatrix}, \tag{11}$$

where

$$\vec{\mathcal{E}}_j^{(R)} = \vec{S}_j^{(R)}\cdot\vec{\mathcal{E}}^{(R)}, \quad \vec{\mathcal{E}}_j^{(T)} = \vec{S}_j^{(T)}\cdot\vec{\mathcal{E}}^{(T)}. \tag{12}$$

Using Eqs. (9)–(11) and averaging over the distribution of $\vec{\mathcal{E}}^{(i)}$, we obtain the coherence matrices of the reflected and transmitted beams:

$$J^{(R)} = \begin{pmatrix} \left|\frac{w_0\epsilon-w\epsilon_0}{w_0\epsilon+w\epsilon_0}\right|^2 J_{11}^{(i)} & -\left(\frac{w_0\epsilon-w\epsilon_0}{w_0\epsilon+w\epsilon_0}\right)^* \frac{(w-w_0)}{(w+w_0)} J_{12}^{(i)} \\ -\left(\frac{w_0\epsilon-w\epsilon_0}{w_0\epsilon+w\epsilon_0}\right)\left(\frac{w-w_0}{w+w_0}\right)^* J_{21}^{(i)} & \left|\frac{w-w_0}{w+w_0}\right|^2 J_{22}^{(i)} \end{pmatrix}, \tag{13}$$

$$J^{(T)} = 4\begin{pmatrix} \left|\frac{w_0\sqrt{\epsilon\epsilon_0}}{w_0\epsilon+w\epsilon_0}\right|^2 J_{11}^{(i)} & \frac{|w_0|^2\sqrt{\epsilon^*\epsilon_0^*}}{(w_0\epsilon+w\epsilon_0)^*(w+w_0)} J_{12}^{(i)} \\ \frac{|w_0|^2\sqrt{\epsilon\epsilon_0}}{(w_0\epsilon+w\epsilon_0)(w+w_0)^*} J_{21}^{(i)} & \frac{|w_0|^2}{|w+w_0|^2} J_{22}^{(i)} \end{pmatrix}. \tag{14}$$

Equations (13) and (14) can also be written as

$$J^{(R)} = U^{(R)}J^{(i)}U^{(R)\dagger}, \quad J^{(T)} = U^{(T)}J^{(i)}U^{(T)\dagger}, \tag{15}$$

where $U^{(R)}$, $U^{(T)}$ represents the transformation matrices of the optical device (in the present case dielectric), given by

$$U^{(R)} = \begin{pmatrix} \frac{w_0\epsilon-w\epsilon_0}{w_0\epsilon+w\epsilon_0} & 0 \\ 0 & -\frac{w-w_0}{w+w_0} \end{pmatrix},$$
$$U^{(T)} = 2w_0\begin{pmatrix} \frac{\sqrt{\epsilon\epsilon_0}}{w_0\epsilon+w\epsilon_0} & 0 \\ 0 & \frac{1}{w+w_0} \end{pmatrix}. \tag{16}$$

The matrices $U^{(R)}$ and $U^{(T)}$ are nonunitary. If these matrices were unitary, then both the trace and determinant would have been conserved under the transformation and hence the polarization would not have changed. Note that the treatment given is valid for arbitrary wave vectors $\vec{K}_0$ (therefore, evanescent waves are included); arbitrary polarizations and also dampings of the dielectric functions are included throughout. We first discuss several special cases.

(1) Linearly polarized incident light with coherence matrix given by

$$J_{12}^{(i)} = J_{21}^{(i)} = J_{22}^{(i)} = 0, \quad J_{11}^{(i)} \neq 0. \tag{17a}$$

Using Eqs. (13), (14) and (17a), we obtain

$$J_{ij} = \delta_{i1}\delta_{j1}J_{11}, \quad J_{11}^{(R)} = \left|\frac{w_0\epsilon-w\epsilon_0}{w_0\epsilon+w\epsilon_0}\right|^2 J_{11}^{(i)},$$

$$J_{11}^{(T)} = 4\left|\frac{w_0\sqrt{\epsilon\epsilon_0}}{w_0\epsilon+w\epsilon_0}\right|^2 J_{11}^{(i)}, \tag{17b}$$

i.e., the reflected and transmitted beams are also linearly polarized. If $W_0\epsilon = W\epsilon_0$ (assuming ϵ, ϵ_0 = real), then there is no reflected field (Brewster effect), for angles of incidence that satisfy $w_0\epsilon = w\epsilon_0$.

(2) Unpolarized incident light

$$J_{12}^{(i)} = J_{21}^{(i)} = 0, \quad J_{11}^{(i)} = J_{22}^{(i)} = \tfrac{1}{2}\langle I_0\rangle \equiv \tfrac{1}{2}\langle \mathcal{E}^{(i)2}\rangle, \tag{18a}$$

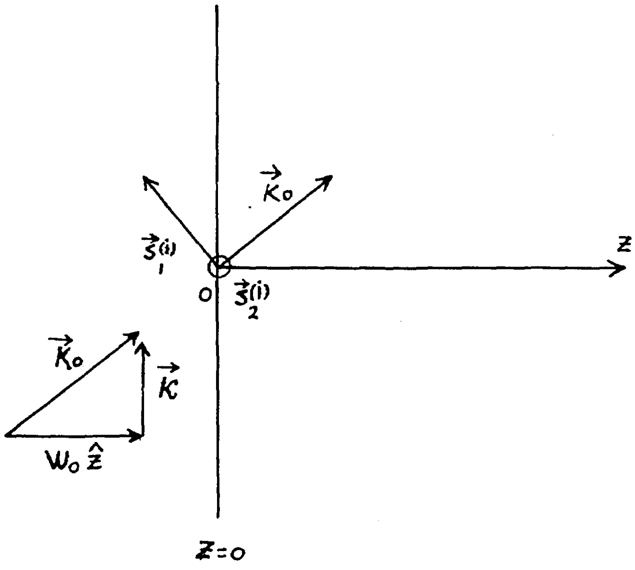

FIG. 1. Representation of various vectors introduced in Eq. (2).

$$J^{(R)} = \tfrac{1}{2}\langle I_0\rangle \begin{pmatrix} \left|\frac{w_0\epsilon - w\epsilon_0}{w_0\epsilon + w\epsilon_0}\right|^2 & 0 \\ 0 & \left|\frac{w-w_0}{w+w_0}\right|^2 \end{pmatrix}, \tag{18b}$$

$$J^{(T)} = \tfrac{1}{2}\langle I_0\rangle \begin{pmatrix} \frac{4|w_0|^2|\epsilon\epsilon_0|}{|w_0\epsilon + w\epsilon_0|^2} & 0 \\ 0 & \frac{4|w_0|^2}{|w+w_0|^2} \end{pmatrix}. \tag{18c}$$

The degree of polarization for the reflected beam is therefore

$$P^{(R)2} = 1 - \frac{4\det J^{(R)}}{(\operatorname{Tr} J^{(R)})^2} = \left(\frac{1 - \left|\frac{w-w_0}{w+w_0}\right|^2 \left|\frac{w_0\epsilon + w\epsilon_0}{w_0\epsilon - w\epsilon_0}\right|}{1 + \left|\frac{w-w_0}{w+w_0}\right|^2 \left|\frac{w_0\epsilon + w\epsilon_0}{w_0\epsilon - w\epsilon_0}\right|^2} \right)^2 . \tag{19}$$

Thus, the reflected light is in general partially polarized. For the Brewster-angle case $P^{(R)} = 1$.

(3) Normal incidence

$$J^{(R)} = \left|\frac{\sqrt{\epsilon}-1}{\sqrt{\epsilon}+1}\right|^2 \begin{pmatrix} J_{11}^{(i)} & -J_{12}^{(i)} \\ -J_{21}^{(i)} & J_{22}^{(i)} \end{pmatrix}, \quad \epsilon_0 = 1 . \tag{20}$$

Thus, the degree of polarization does not change; however, the ellipticity of polarization of the reflected beam changes.

(4) Total internal reflection ($\epsilon_0 = \text{real} > 1$, $\epsilon = 1$, $\kappa^2 > 1$),

$$\left|\frac{w_0\epsilon - w\epsilon_0}{w_0\epsilon + w\epsilon_0}\right|^2 = 1 ,$$

and hence the coherence matrix for the reflected beam is

$$J^{(R)} = \begin{pmatrix} J_{11}^{(i)} & -J_{12}^{(i)} e^{i\delta^{(R)}} \\ -J_{21}^{(i)} e^{-i\delta^{(R)}} & J_{22}^{(i)} \end{pmatrix},$$
$$e^{i\delta^{(R)}} = \left(\frac{w_0 - w\epsilon_0}{w_0 + w\epsilon_0}\right)^* \left(\frac{w-w_0}{w+w_0}\right). \tag{21}$$

Thus, the ellipticity of polarization of the reflected beam changes, whereas the degree of polarization does not change.

In the general case, we have, from Eq. (13),

$$\det J^{(R)} = \left|\frac{w_0\epsilon - w\epsilon_0}{w_0\epsilon + w\epsilon_0}\right|^2 \left|\frac{w-w_0}{w+w_0}\right|^2 \det J^{(i)} ; \tag{22}$$

therefore, in the case when damping ϵ is included, $\det J^{(R)}$ will be zero if $\det J^{(i)} = 0$. Thus, the fully polarized beam will remain fully polarized. However, its plane of polarization will change.

Because the relations between $\vec{E}^{(i)}$, $\vec{E}^{(R)}$, $\vec{E}^{(T)}$ are linear, the probability distributions of the different fields are simply related, e.g.,

$$P^{(R)}(\mathcal{E}_1^{(R)}, \mathcal{E}_2^{(R)}) = p^{(i)}(\mathcal{E}_1^{(i)}, \mathcal{E}_2^{(i)}) \left| \frac{d^2\mathcal{E}_1^{(i)} d^2\mathcal{E}_2^{(i)}}{d^2\mathcal{E}_1^{(R)} d^2\mathcal{E}_2^{(R)}} \right| . \tag{23}$$

If higher-order correlation properties of the field are taken into account, then unpolarized light is characterized by the probability distribution[9]

$$p^{(i)}(\mathcal{E}_1^{(i)}, \mathcal{E}_2^{(i)}) \equiv p^{(i)}(|\mathcal{E}_1^{(i)}|^2 + |\mathcal{E}_2^{(i)}|^2) ; \tag{24}$$

therefore, for unpolarized incident light, the distribution function for the reflected field is

$$P^{(R)}(\mathcal{E}_1^{(R)}, \mathcal{E}_2^{(R)}) = p^{(i)}\left[\left|\frac{w+w_0}{w-w_0}\right|^2 |\mathcal{E}_1^{(R)}|^2 + \left|\frac{w\epsilon_0 + w_0\epsilon}{w\epsilon_0 - w_0\epsilon}\right|^2 |\mathcal{E}_2^{(R)}|^2 \right] \times \left|\frac{w+w_0}{w-w_0}\right|^2 \left|\frac{w\epsilon_0 + w_0\epsilon}{w\epsilon_0 - w_0\epsilon}\right|^2 . \tag{25}$$

The distribution function for the case of fully polarized incident light can be obtained similarly if we use the fact[10] that for polarized light the probability distribution is given by

$$p^{(i)}(\mathcal{E}_1^{(i)}, \mathcal{E}_2^{(i)}) = p(|\mathcal{E}_1^{(i)}| \sec\alpha) \sec\alpha \times \delta^{(2)}[\mathcal{E}_2^{(i)} - \mathcal{E}_1^{(i)} \tan\alpha\, e^{i\eta}], \tag{26}$$

where α, η are related to the coherence matrix $J^{(i)}$ and p is an arbitrary distribution.

RELATION BETWEEN POYNTING VECTOR AND THE COHERENCE MATRIX

We now derive the relation between the Poynting vector and the coherence matrix of a fluctuating light beam. The Poynting vector, as usual, is defined by

$$\vec{S} = \frac{c}{8\pi} \operatorname{Re}\langle \vec{E} \times \vec{H}^* \rangle , \tag{27}$$

which, when the relation $\vec{H} = (\vec{K} \times \vec{E})/ik_0$ is used, becomes

$$\vec{S} = \frac{c}{8\pi k_0} \operatorname{Re}[\vec{K}^* \langle |\mathcal{E}|^2 \rangle + (w - w^*) \langle \mathcal{E}_z \vec{\mathcal{E}}^* \rangle] . \tag{28}$$

If we express $\vec{\mathcal{E}}$ as

$$\vec{\mathcal{E}} = \mathcal{E}_1 \vec{S}_1 + \mathcal{E}_2 \vec{S}_2 , \tag{29}$$

then

$$(\vec{\mathcal{E}} \cdot \vec{\mathcal{E}}^*) = |\mathcal{E}_1|^2 |\vec{S}_1|^2 + |\mathcal{E}_2|^2 ,$$
$$\mathcal{E}_z \vec{\mathcal{E}}^* = -\frac{\kappa}{k} \mathcal{E}_1 (\vec{S}_1^* \mathcal{E}_1^* + \vec{S}_2^* \mathcal{E}_2^*) , \tag{30}$$

and hence

$$\vec{S} = \frac{c}{8\pi k_0} \operatorname{Re}\left[\vec{K}^* (|\vec{S}_1|^2 J_{11} + J_{22}) - (w - w^*) \frac{\kappa}{k} (\vec{S}_1^* J_{11} + \vec{S}_2^* J_{21}) \right] . \tag{31}$$

It is clear from Eq. (31) that the component of the Poynting vector parallel to the surface is given by

$$\vec{S}_{\parallel} = \frac{c}{8\pi k_0} \left[\vec{\kappa} \left\{ |\vec{S}_1|^2 J_{11} + J_{22} + \frac{2J_{11}}{kk^*} (\operatorname{Im} w)^2 \right\} - i \frac{(\hat{z} \times \vec{\kappa})}{kk^*} (\operatorname{Im} w)(k^* J_{21} - k J_{12}) \right] . \tag{32}$$

Let us first examine the case of an evanescent wave in vacuum. We substitute in Eq. (32)

$k = k_0 = \text{real}, \quad w = i\sigma, \quad \sigma = \text{real}$

and obtain

$$\vec{S}_{\parallel} = \frac{c}{8\pi k_0}\left\{\vec{\kappa}\left[\frac{\kappa^2+3\sigma^2}{k_0^2} J_{11} + J_{22}\right] + \frac{i}{k_0}(\hat{z}\times\vec{\kappa})\sigma(J_{12} - J_{21})\right\} . \tag{33}$$

Moreover, if we consider the evanescent waves that propagate along the x axis and decay along the z axis ($\vec{\kappa} = \hat{x}\kappa$), Eq. (33) reduces to

$$S_x = \frac{c}{8\pi k_0}\kappa\left\{\frac{\kappa^2+3\sigma^2}{k_0^2} J_{11} + J_{22}\right\} , \tag{34}$$

$$S_y = \frac{c\kappa\sigma}{8\pi k_0^2} i(J_{12} - J_{21}) = -\frac{c\kappa\sigma}{4\pi k_0^2}\,\mathrm{Im}J_{12} . \tag{35}$$

From the work of Imbert,[11] it follows that the longitudinal and transverse Goos–Hänchen shifts will be given in terms of the integral of the Poynting vectors S_x and S_y, respectively. Note that Eqs. (34) and (35) give the values at $z = 0$. For an evanescent wave, the propagation of the Poynting vector can be expressed as

$$\vec{S}(z) = \vec{S}(0)\,e^{-2\sigma z} . \tag{36}$$

From Eq. (35), it can be seen that in order for the lateral shift to be nonzero,

$$\mathrm{Im}J_{12} \neq 0 . \tag{37}$$

The coherence matrices that appear in Eqs. (34) and (35) refer to the transmitted beam; therefore, from Eq. (14), we get

$$\mathrm{Im}J_{12} = 4|w_0|^2\,\mathrm{Im}\sqrt{\epsilon_0^*}\,J_{12}^{(i)}/(w+w_0)(w_0+w\epsilon_0)^* , \tag{38}$$

where we assume that the region $0 \leq z \leq \infty$ is in vacuum. Hence the condition for the nonvanishing of the lateral shift becomes

$$\mathrm{Im}(w_0 - i\sigma)(w_0 - i\sigma\epsilon_0)^*\sqrt{\epsilon_0^*}\,J_{12}^{(i)} \neq 0 . \tag{39}$$

Obviously, if the incident beam is linearly polarized (along $\vec{S}_1$ or along $\vec{S}_2$) or if unpolarized, then the lateral shift vanishes. Any other polarization will lead to lateral shifts that depend, of course, on the parameters ϵ_0, k_0, etc.

In deriving Eqs. (34) and (35), we have restricted our consideration to beams that propagate along the x axis. This we now relax. For lateral shifts, we want to show that $\vec{S}\cdot\vec{S}_2 \neq 0$. It is easily seen from Eq. (33) that

$$\vec{S}\cdot\vec{S}_2 = \frac{c}{8\pi k_0}\,\mathrm{Im}\left[\frac{2\kappa}{k} J_{21}(\mathrm{Im}w)\right] \neq 0 , \tag{40}$$

which is the most-general condition for the nonvanishing of the lateral shift. One obvious sufficient condition is $J_{12}^{(i)} \neq 0$.

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