Dispersive bistability in coupled nonlinear Fabry–Perot resonators

S. Dutta Gupta and G. S. Agarwal

School of Physics, University of Hyderabad, Hyderabad-500 134, India

Received September 26, 1986; accepted December 10, 1986

The transmission properties of coupled nonlinear Fabry–Perot resonators with reflection coatings are studied. A general characteristic matrix approach for nonlinear layered media is developed for the case of normal incidence of linearly polarized light. Use of reflection coatings and coupled systems results in the lowering of the bistability threshold.

1. INTRODUCTION

The bistable behavior of a nonlinear Fabry–Perot resonator with cubic nonlinearity has been the subject of considerable theoretical and applied interest. Felber and Marburger[1,2] have given a thorough investigation of the bistable and multistable operation of a nonlinear Fabry–Perot resonator. Calculations in connection with ring cavities[3] have shown that the bistability threshold can be lowered by using a system of coupled ring cavities. In general, the threshold is determined by several parameters such as the reflectivity of the mirrors and the domain of the nonlinear material. It is important to investigate what happens in layered structures that are now extensively used in various switching devices. So far as a single nonlinear Fabry–Perot cavity is concerned, the theory is well developed.[1,2,4] However, a general theory for a sequence of nonlinear Fabry–Perot cavities is still missing. It is clear that for arbitrary polarization and angles of incidence it is rather difficult to have a general theory. If one restricts oneself to the case of linear polarization and normal incidence, it is possible to generalize the characteristic matrix approach[5] for a linear medium to the case when some or all of the layers in a layered structure are nonlinear. Once the characteristic matrix for the nonlinear layered structure is obtained, one can calculate the transmission coefficient.

It is known that the narrower the transmission resonances are, the easier it is to achieve bistable operation.[6] The Airy resonances in a Fabry–Perot resonator can be narrowed down by making use of reflection coatings. Keeping this in view, we investigate single and coupled nonlinear Fabry–Perot resonators with reflection coatings.

Thus, in Section 2, following Ref. 4, we discuss the reduced wave equation for a cubic nonlinear medium and its solutions. In Section 3 we develop the characteristic matrix for a nonlinear layered medium for TE polarization and normal incidence. Moreover, we present the results for the transmission coefficients in terms of the elements of the characteristic matrix. In Section 4 we present the numerical results for a single and two coupled nonlinear Fabry–Perot resonators with reflection coatings. Our results reveal the possibility of the realization of low-threshold bistable systems. The method developed here is also useful in the study of related problems concerning nonlinear mixing.[7,8]

2. DERIVATION OF THE WAVE EQUATION AND ITS SOLUTION

Let us consider a nonlinear medium with an arbitrary type of third-order nonlinearity. Assuming the time dependence to be of the type $e^{-i\omega t}$, the wave equation for such a medium can be written as

$$\Delta \mathbf{E} + k_0^2 \epsilon \mathbf{E} = -[k_0^2 \mathbf{D}^{NL} + \nabla \nabla \cdot (\mathbf{D}^{NL}/\epsilon)], \tag{2.1}$$

where $k_0 = \omega/c$ is the vacuum wave vector, E is the linear dielectric constant, and $\mathbf{D}^{NL}$ is the nonlinear component of electric displacement. For an arbitrary mechanism of nonlinearity $\mathbf{D}^{NL}$ can be expressed as[9]

$$\mathbf{D}^{NL} = \epsilon\chi [A\mathbf{E}(\mathbf{E} \cdot \mathbf{E}^*) + B\mathbf{E}^*(\mathbf{E} \cdot \mathbf{E})], \tag{2.2}$$

where A and B are nonlinearity constants and χ is a constant of nonlinear interaction. Equation (2.2) suggests a field-induced anistropy of χ^{NL} and can be written in a tensor form:

$$D_i^{NL} = \epsilon \chi_{ik}^{NL} E_k, \tag{2.3a}$$

with

$$\chi_{ik}^{NL} = \chi[B(E_i E_k^* + E_i^* E_k) + (A - B) I \delta_{ik}]. \tag{2.3b}$$

Here

$$I = |E_x|^2 + |E_y|^2 + |E_z|^2,$$

and δ_{ik} is the Kronecker delta symbol.

We now consider the one-dimensional case of two counterpropagating beams in the nonlinear medium and write the total field as

$$\mathbf{E}(x) = \mathbf{E}_1(x)\, e^{ikx} + \mathbf{E}_2(x) e^{-ikx}. \tag{2.4}$$

By using the slowly varying amplitude approximation (SVEA) with respect to $\mathbf{E}_1$ and $\mathbf{E}_2$ and dropping the $\nabla \cdot \mathbf{D}^{NL}$ term, Eq. (2.1) with Eq. (2.2) can be reduced to two coupled nonlinear equations:

$$\begin{aligned} 2ik_0\sqrt{\epsilon}\,\frac{d\mathbf{E}_1}{dx} &= -k_0^2\, \epsilon\chi\{A[\mathbf{E}_1(\mathbf{E}_1 \cdot \mathbf{E}_1^* + \mathbf{E}_2 \cdot \mathbf{E}_2^*) + \mathbf{E}_2\mathbf{E}_1 \cdot \mathbf{E}_2^*] \\ &\quad + B(\mathbf{E}_1^*\mathbf{E}_1 \cdot \mathbf{E}_1 + 2\mathbf{E}_2^*\mathbf{E}_1 \cdot \mathbf{E}_2)\}, \end{aligned} \tag{2.5a}$$

0740-3224/87/050691-05$02.00	

$$-2\,ik_0\sqrt{\epsilon}\frac{d\mathbf{E}_2}{dx} = -k_0^2\,\epsilon\chi\{A[\mathbf{E}_2(\mathbf{E}_2\cdot\mathbf{E}_2^* + \mathbf{E}_1\cdot\mathbf{E}_1) + \mathbf{E}_1\mathbf{E}_2\cdot\mathbf{E}_1^*)] + B(\mathbf{E}_2^*\mathbf{E}_2\cdot\mathbf{E}_2 + 2\mathbf{E}_1^*\mathbf{E}_2\cdot\mathbf{E}_1)\}. \quad (2.5b)$$

If both the waves are linearly polarized (say, TE polarized with $E_{1,2z} \neq 0$, $E_{1,2x} = 0$, $E_{1,2y} = 0$) the system of equations (2.5) can be simplified drastically and written as

$$2ik_0\sqrt{\epsilon}\frac{dE_{1z}}{dx} = -k_0^2\epsilon\chi(A + B)E_{1z}(|E_{1z}|^2 + 2|E_{2z}|^2), \quad (2.6a)$$

$$-2\,ik_0\sqrt{\epsilon}\frac{dE_{2z}}{dx} = -k_0^2\epsilon\chi\,(A + B)E_{2z}(|E_{2z}|^2 + 2\,|E_{1z}|^2). \quad (2.6b)$$

In the case of TE polarization χ_{ik}^{NL} also has a simpler form, and by using Eqs. (2.3) one can infer that there is no field-induced anisotropy or dispersion of axis. In this case $\nabla\cdot\mathbf{D}^{NL} = 0$.

$$\overline{M} = \frac{k_0}{k_+ + k_-}\begin{bmatrix} \frac{k_-}{k_0}\exp(-ik_+L) + \frac{k_+}{k_0}\exp(ik_-L) & \exp(-ik_+L) - \exp(ik_-L) \\ \frac{k_-k_+}{k_0^2}[\exp(-ik_+L) - \exp(ik_-L)] & \frac{k_+}{k_0}\exp(-ik_+L) + \frac{k_-}{k_0}\exp(ik_-L) \end{bmatrix}. \quad (3.3)$$

The solution of Eqs. (2.6) can be readily obtained, and it can be used to express the total field [Eq. (2.4)] as

$$E_z = A_+\exp(ik_+x) + A_-\exp(-ik_-x), \quad (2.7)$$

where A_+ and A_- are the forward- and backward-wave amplitudes, respectively; k_+ and k_- are the wave vectors of the forward and backward waves determined by the following expressions:

$$k_+ = k_0\sqrt{\epsilon}[1 + (\alpha/2)(|A_+|^2 + 2|A_-|^2)], \quad (2.8a)$$

$$k_- = k_0\sqrt{\epsilon}[1 + (\alpha/2)(|A_-|^2 + 2|A_+|^2)], \quad (2.8b)$$

where $\alpha = \chi(A + B)$.

It may be noted here that, for TE polarization, Eq. (2.1) can be solved without SVEA, yielding for k_+ and k_- the following relations[10]:

$$k_+ = k_0\sqrt{\epsilon}[1 + \alpha\,(|A_+|^2 + 2|A_-|^2)]^{1/2}, \quad (2.9a)$$

$$k_- = k_0\sqrt{\epsilon}[1 + \alpha\,(|A_-|^2 + 2|A_+|^2)]^{1/2}, \quad (2.9b)$$

3. TRANSMISSION THROUGH COUPLED NONLINEAR FABRY-PEROT RESONATORS

In this section we first obtain the characteristic matrix for a nonlinear slab and then use it to obtain the transmission coefficient for a combination of any number of linear and nonlinear layers.

Let us consider two planes $x = 0$ and $x = L$ perpendicular to the direction of propagation in the nonlinear medium. Making use of Eq. (2.7) and Maxwell's equations, one can express the tangential field components E_z and H_y at $x = 0$ and $x = L$ as

$$\begin{pmatrix} E_z \\ -H_y \end{pmatrix}_{x=0} = \begin{pmatrix} 1 & 1 \\ \frac{k_+}{k_0} & -\frac{k_-}{k_0} \end{pmatrix}\begin{pmatrix} A_+ \\ A_- \end{pmatrix}, \quad (3.1a)$$

$$\begin{pmatrix} E_z \\ -H_y \end{pmatrix}_{x=L} = \begin{bmatrix} \exp(ik_+L) & \exp(-ik_-L) \\ \frac{k_+}{k_0}\exp(ik_+L) & -\frac{k_-}{k_0}exp(-ik_-L) \end{bmatrix}\begin{pmatrix} A_+ \\ A_- \end{pmatrix}. \quad (3.1b)$$

Eliminating A_+ and A_- from Eqs. (3.1a) and (3.1b), we can relate the tangential field components at $x = 0$ and $x = L$:

$$\begin{pmatrix} E_z \\ -H_y \end{pmatrix}_{x=0} = \overline{M}\begin{pmatrix} E_z \\ -H_y \end{pmatrix}_{x=L}, \quad (3.2)$$

where $\overline{M}$ is the characteristic matrix for the nonlinear slab with width L:

It may be noted here that Eq. (3.3) in the linear case ($\chi = 0$, $k_+ = k_- = k_0\sqrt{\epsilon}$) reduces to the standard form of characteristic matrix[5] for TE waves. Henceforth the standard characteristic-matrix formalism can be applied to calculate the reflection and transmission coefficients for a medium consisting of N linear or nonlinear layers. If the jth layer has a width L_j and linear dielectric constant ϵ_j, the characteristic matrix M for the composite medium can be as follows:

$$M = \prod_{j=1}^{N}\overline{M}_j(L_j), \quad (3.4)$$

where $\overline{M}_j(L_j)$ is given by Eq. (3.3), with L replaced by L_j and ϵ by ϵ_j. The transmission and reflection coefficients are then given by

$$T = \left|\frac{2p_i}{(m_{11} + m_{12}p_f)p_i + (m_{21} + m_{22}p_f)}\right|^2, \quad (3.5)$$

$$R = \left|\frac{(m_{11} + m_{12}p_f)p_i - (m_{21} + m_{22}p_f)}{(m_{11} + m_{12}p_f)p_i + (m_{21} + m_{22}p_f)}\right|^2, \quad (3.6)$$

where $p_i = \sqrt{\epsilon_i}$, $p_f = \sqrt{\epsilon_f}$; ϵ_i is the dielectric constant of the medium from which the wave is incident and ϵ_f refers to the medium in which the wave is transmitted; m_{ij} are the elements of the characteristic matrix M. Though formulas (3.5) and (3.6) may appear straightforward, there are certain difficulties in the actual calculation of T and R. To be precise, let us assume that a particular jth medium is nonlinear. This implies that the wave vectors k_{j+} and k_{j-} will depend on the forward- and backward-wave amplitudes A_{j+} and A_{j-}. Therefore, to have the explicit form of $\overline{M}_j(L_j)$, one first has to solve two coupled nonlinear equations with respect to A_{j+} and A_{j-} for a given incident field amplitude A_{i+}.

In Section 4 we show how this can actually be done by treating the transmitted field amplitude as a parameter. We consider two specific cases: (1) one nonlinear slab with high-reflection coatings and (2) two nonlinear slabs with reflection coatings separated by a high- or a low-index $\lambda/4$ slab.

4. NUMERICAL RESULTS

A. Single Nonlinear Fabry-Perot Resonator with Reflection Coating

We consider a system (see Fig. 1) in which a nonlinear slab occupying $-L \leqslant x \leqslant 0$ is coated on both sides by alternating n low-index and $n+1$ high-index materials, each of optical thickness $\lambda/4$. Let the coating materials be linear and the total width of the coatings be L_n. Let the initial and final media be also linear and have dielectric constants ϵ_i and ϵ_f, respectively.

It is well established that, with an increase in the mirror reflectivities, the Airy resonances become sharper. Hence an increase in n will result in narrower resonances. As was noted in our earlier work,[6] the sharper the resonance, the easier it is to have bistable operation. Keeping this in mind, we first examine the linear characteristics of the systems for different n, namely, $n = 1, 2, 3$.

The characteristic matrix for the composite medium in this case can be written as a product of three matrices:

$$M = M_n \times \overline{M} \times M_n, \tag{4.1}$$

where $\overline{M}$ is the characteristic matrix for the medium occupying $-L \leqslant x \leqslant 0$, with $\chi = 0$, and

$$M_n = \begin{bmatrix} 0 & -(i/\sqrt{\epsilon_a})(-\sqrt{\epsilon_b/\epsilon_a})^n \\ -i\sqrt{\epsilon_a}(-\sqrt{\epsilon_a/\epsilon_b})^n & 0 \end{bmatrix} \tag{4.2}$$

is the characteristic matrix of the coatings occupying $0 < x \leqslant L_n$ or $-(L_n + L) \leqslant x < L$. Here ϵ_a is the dielectric constant of the high-index material and ϵ_b that for the low-index coating material. A straightforward calculation using Eqs. (3.5), (4.1), and (4.2) yields the transmission coefficient. The T versus $k_0\sqrt{\epsilon}\,L$ curves for $n = 1, 2, 3$ are shown in Fig. 2(a). The resonance half-widths are found to be $\Delta = 0.113$ for $n = 1$, $\Delta = 0.04$ for $n = 2$, and $\Delta = 0.018$ for $n = 3$. The resonances are indeed sharper for larger n.

If the middle slab is nonlinear ($\chi \neq 0$), then we express all the field amplitudes as parametrically given functions of the transmitted field amplitude. Let the transmitted electric-field amplitude at $x = L_n^+$ be equal to A_f. The tangential field components at $x = L_n^+$ can be written as

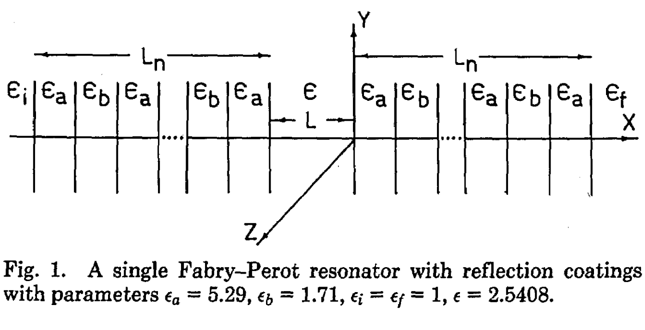

Fig. 1. A single Fabry–Perot resonator with reflection coatings with parameters $\epsilon_a = 5.29$, $\epsilon_b = 1.71$, $\epsilon_i = \epsilon_f = 1$, $\epsilon = 2.5408$.

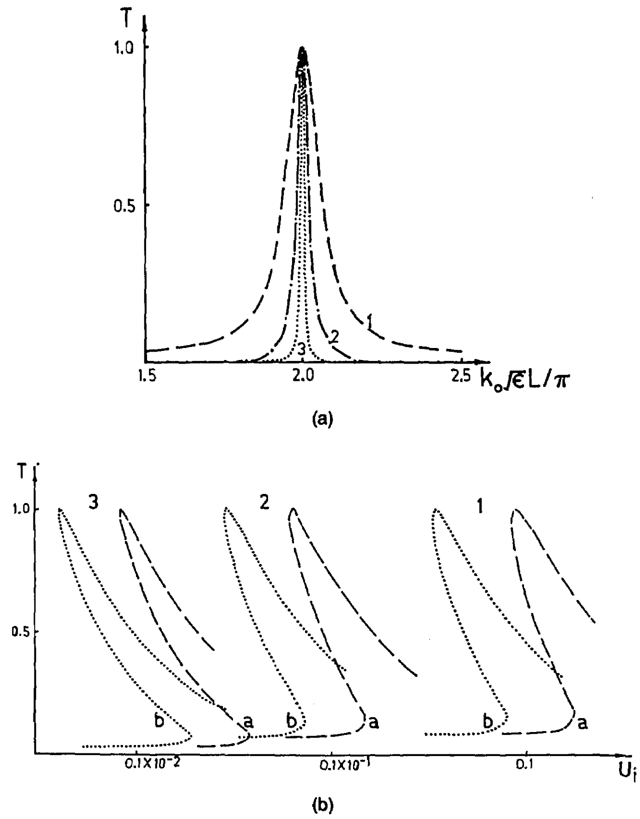

Fig. 2. (a) Linear transmission coefficient T as a function of $k_0\sqrt{\epsilon}\,L/\pi$ for the system in Fig. 1. Different curves are marked by different values of n. (b) Transmission coefficient T as a function of the incident intensity U_i: a, $k_0\sqrt{\epsilon}\,L = (2 - 2\Delta)\pi$; b, $k_0\sqrt{\epsilon}\,L = (4 - 2\,\Delta)\pi$. Different curves are labeled by different values of n, and Δ values are given in the text following Eq. (4.2).

$$\begin{pmatrix} E_z \\ -H_y \end{pmatrix}_{x=L_n^+} = \begin{pmatrix} A_f \\ \sqrt{\epsilon_f}A_f \end{pmatrix}. \tag{4.3}$$

Let the forward and backward electric-field amplitudes in the nonlinear medium be A_+ and A_-, respectively. For the tangential field components at $x = 0^-$ we can write

$$\begin{pmatrix} E_z \\ -H_y \end{pmatrix}_{x=0^-} = \begin{bmatrix} A_+ + A_- \\ (k_+/k_0)A_+ - (k_-/k_0)A_- \end{bmatrix}. \tag{4.4}$$

Making use of the characteristic matrix (4.2), one can connect the tangential components at $x = 0^-$ and $x = L_n^+$. This when combined with Eqs. (4.3) and (4.4) leads to

$$\begin{bmatrix} A_+ + A_- \\ (k_+/k_0)A_+ - (k_-/k_0)A_- \end{bmatrix} = M_n \begin{pmatrix} A_f \\ \sqrt{\epsilon_f}A_f \end{pmatrix}. \tag{4.5}$$

Equation (4.5) can be rewritten in terms of the intensities U_+ ($=\alpha|A_+|^2$) and U_f ($=\alpha|A_f|^2$) in the form

$$U_\pm = |[1/(p_+ + p_-)][p_-(m_{11} + m_{12}p_f) \pm (m_{21} + m_{22}p_f)]|^2\, U_f, \tag{4.6}$$

with

$$p_+ = \sqrt{\epsilon}(1 + U_+ + 2U_-)^{1/2}, \tag{4.7a}$$

$$p_- = \sqrt{\epsilon}(1 + U_- + 2U_+)^{1/2}. \tag{4.7b}$$

Here m_{ij} are the elements of the characteristic matrix M_n.

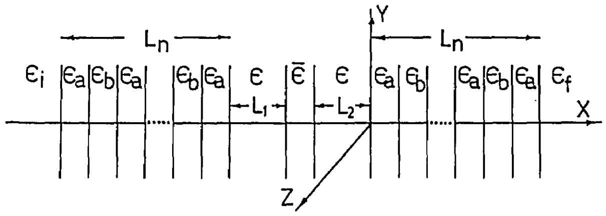

Fig. 3. Two coupled Fabry–Perot resonators with reflection coatings separated by a $\lambda/4$ layer of dielectric constant $\bar{\epsilon}$ with $\epsilon_a = 5.29$, $\epsilon_b = 1.71$, and $\epsilon = 2.5408$.

Equation (4.6) with Eqs. (4.7) defines the intensities $U_\pm$ in the nonlinear slab in terms of the transmitted intensity U_f. We solve the nonlinear coupled Eqs. (4.6) by using fixed-point iteration for a given value of U_f. We take $U_\pm = 0$ as the initial value for finding the fixed points. Once the $U_\pm$ are determined, $k_\pm$ can be calculated by using Eqs. (2.9). The next step is a straightforward calculation of the total characteristic matrix, using Eqs. (3.3) and (4.2). Once the total characteristic matrix is evaluated, the incident field can be expressed as a function of U_f as follows:

$$U_i = |(m_{11} + m_{12}p_f)p_i + (m_{21} + m_{22}p_f)|^2 U_f, \tag{4.8}$$

and the transmission coefficient T can be directly evaluated by making use of Eq. (3.5). Next we plot T as a function of U_i, treating U_f as the parameter.

With a view to obtaining bistable behavior of T as a function of the input intensity U_i, we detune the system slightly by an amount -2Δ from the peak position of the linear transmission curve of Fig. 2(a). The nonlinear transmission is shown in Fig. 2(b). As is evident from Fig. 2(b), with an increase in the number of coatings the bistability threshold decreases. We have also presented in the same figure T versus U_i curves when the width of the nonlinear slab is increased twofold. An increase in the region of nonlinear interaction of forward and backward waves results in lower values of the bistability threshold. It may be noted here that a nonlinear slab with width $2L$ with the coatings is equivalent to two periods of the system of Fig. 1. This is so because, for the chosen parameters of the coating materials, the characteristic matrix between the nonlinear slabs is a unit matrix ($M_n \times M_n = -I$). Therefore the results for the system with width $2L$ are identical to the case when we have two coupled nonlinear Fabry–Perot resonators (each with n low-index and $n + 1$ high-index alternating coatings on both sides) placed side by side.

B. Two Coupled Nonlinear Fabry-Perot Resonators with Reflection Coatings

Next we consider the system shown in Fig. 3. The system consists of two nonlinear slabs of widths L_1 and L_2, respectively, separated by a $\lambda/4$ plate of width L with dielectric constant $\bar{\epsilon}$. The left end of the first and the right end of the second nonlinear slab are coated with alternating n low-index and $(n + 1)$ high-index $\lambda/4$ layers with total width L_n.

We adopt a similar approach, as outlined in Subsection 4.A. First we investigate the linear characteristics of the system. Detuning the system slightly, we investigate the dependence of T on U_i. We examine two specific cases when the middle slab has (1) high refractive index and (2) low refractive index.

The linear curves for cases (1) and (2) are shown in Figs. 4(a) and 4(b), respectively. In calculating the linear transmission curves, we have fixed the width of the second resonator L_2 ($k_0 \sqrt{\epsilon} L_2 = \pi/2$) and varied L_1. Here, as in the case of one Fabry–Perot resonator, the resonances are sharper with larger n. Figures 4(a) and 4(b) differ; for example, in their peak values showing a lower (higher) peak for a low- (high-) index middle layer.

In the present context the calculation of the transmission

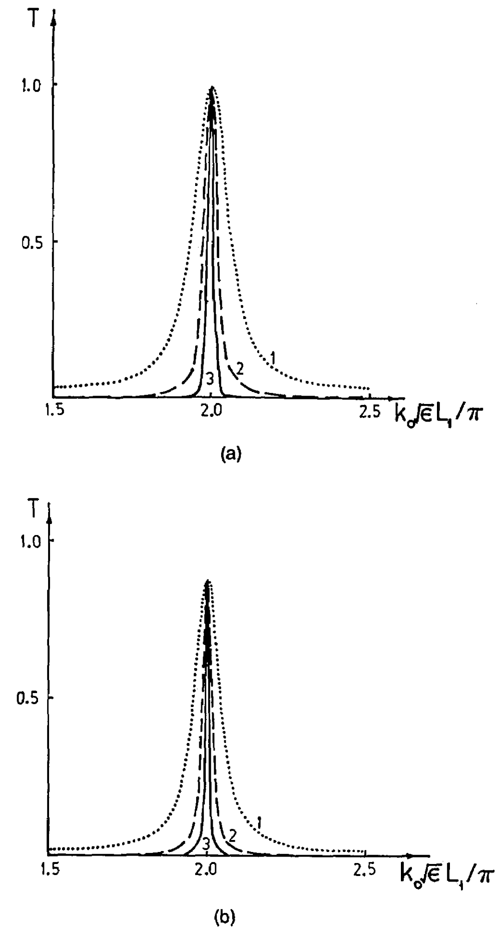

Fig. 4. (a) Linear transmission coefficient T as a function of $k_0 \sqrt{\epsilon}\, L_1/\pi$, with fixed $k_0 \sqrt{\epsilon}\, L_2 = 2.5\pi$ for $\bar{\epsilon} = 2.58$ and different n values. (b) Same as in (a) but for $\bar{\epsilon} = 5.29$.

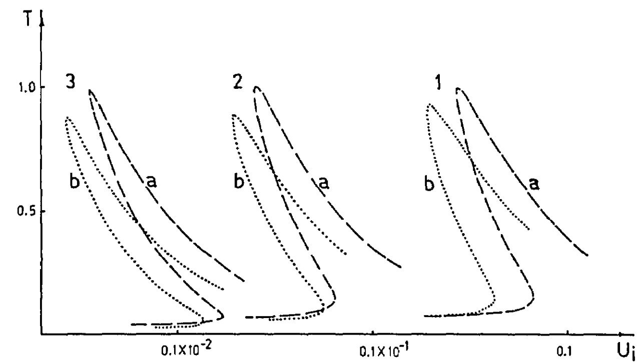

Fig. 5. Transmission coefficient as a function of U_i for two different values of $\bar{\epsilon}$: a, $\bar{\epsilon} = 2.58$; b, $\bar{\epsilon} = 5.29$. The values of $k_0 \sqrt{\epsilon}\, L_{1,2}$ are taken to be $k_0 \sqrt{\epsilon}\, L_1 = (2 - 2\Delta)\pi$ and $k_0 \sqrt{\epsilon}\, L_2 = 2.5\,\pi$. Detunings Δ are chosen from the curves in Figs. 4(a) and 4(b): a, $\Delta = 0.13, 0.04, 0.018$ for $n = 1, 2, 3$, respectively; b, $\Delta = 0.082, 0.032, 0.012$ for $n = 1, 2, 3$, respectively.

coefficient and the incident intensity for given transmitted intensities is more complicated because of the presence of two nonlinear slabs. One has to carry out fixed-point iteration twice. As a first step one must evaluate $U_{2\pm}$, the forward- and backward-wave intensities in the second nonlinear slab for a given U_f. A knowledge of $U_{2\pm}$ defines the forward- and backward-wave vectors $k_{2\pm}$ in the second slab and hence the characteristic matrix of the second slab. Knowing the total characteristic matrix for layers occupying $-(L_1 + L) \leqslant x \leqslant L_n$, one can obtain the coupled nonlinear equations with respect to the forward- and backward-wave intensities $U_{1\pm}$ in the first nonlinear slab. This defines $k_{1\pm}$ as well as the characteristic matrix of the first nonlinear slab. The rest of the procedure is identical to the preceding case.

We present the results of our study for two coupled nonlinear Fabry–Perot resonators in Fig. 5. Initial detuning is chosen to be -2Δ. As in the previous case, the bistability threshold is lowered for higher reflectivities of the end mirrors. It may be noted that, for high value of the refractive index of the middle layer, the thresholds are even lower than in the case of one Fabry–Perot resonator with width $2L$.

In conclusion, we have presented a study of single and coupled nonlinear Fabry–Perot resonators and shown the feasibility of low-threshold bistable systems by making use of reflection coatings. We have used the complete set of boundary conditions at the nonlinear interface. Our approach differs from most of the existing treatments, which use only the continuity of the electric field at the boundary. We have developed a general characteristic matrix approach for nonlinear layered media for normal incidence and linear polarization. Our study reveals the possibility of further lowering the threshold by using various combinations for the layered structure. In the future we hope to develop the general characteristic-matrix approach and present an analysis of the nonlinear layered media for oblique incidence and arbitrary polarization.

ACKNOWLEDGMENT

G. S. Agarwal would like to thank the University Grants Commission, Government of India, for partially supporting this research.

REFERENCES

1. F. S. Felber and J. H. Marburger, Appl. Phys. Lett. **28**, 731 (1976).
2. J. H. Marburger and F. S. Felber, Phys. Rev. A **17**, 335 (1978).
3. For coupled ring-cavity systems see G. S. Agarwal and S. Dutta Gupta, Phys. Rev. A **30**, 2764 (1984); J. S. Satchell, C. Parigger, and W. J. Sandle, Opt. Commun. **47**, 230 (1983); M. Orenstein, S. Speiser, and J. Katriel, IEEE J. Quantum Electron. **QE-21**, 1513 (1985).
4. A. E. Kaplan, Opt. Lett. **8**, 560 (1983).
5. M. Born and E. Wolf, *Principles of Optics* (Pergamon, New York, 1970), Sec. 1.6.2.
6. G. S. Agarwal and S. Dutta Gupta, Phys. Rev. B **34**, 5239 (1986).
7. A. E. Kaplan and C. T. Law, IEEE J. Quantum Electron. **QE-21**, 1529 (1985).
8. H. G. Winful and J. H. Marburger, Appl. Phys. Lett. **36**, 613 (1980).
9. P. D. Maker, R. W. Terhune, and C. M. Savage, Phys. Rev. Lett. **12**, 507 (1964).
10. J. H. Marburger, in *Optical Phase Conjugation*, R. A. Fisher, ed. (Academic, New York, 1983), p. 102.