Optical bistability with surface plasmons beyond plane waves in a nonlinear dielectric

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Optical bistability with surface plasmons is studied assuming nonlinear polaritons in a Kerr medium. Significant departures from the plane-wave results for the switching intensities and hysteresis curves are predicted.

Optical bistability continues to draw considerable attention,[1] and many different types of systems have been examined for its existence. Kaplan[2] showed that the light reflected at the boundary of a nonlinear medium can exhibit hysteresis. He examined the cases of both positive and negative nonlinearity of the medium. Wysin *et al.* suggested that the field enhancement can be useful if surface plasmons are excited in the metal film.[3] They made specific predictions of the possibility of observing the bistability with surface plasmons. Other geometrical arrangements[4,5] have also been suggested for observing bistability using surface plasmons.

Theoretical research in optical bistability with surface plasmons assumes that the waves in the nonlinear medium are plane waves with a propagation vector whose magnitude depends on the strength of the field in the nonlinear medium.[3,4] These calculations then use the Fresnel formulas for reflection and refraction from a layered medium[6] and replace the linear refractive index by the nonlinear refractive index, assuming that the field intensity in the nonlinear medium is homogeneous. If the field intensity is inhomogeneous, then the nonlinear refractive index becomes inhomogeneous. Thus the usual Fresnel formulas cannot be used, as these are obtained by assuming that all the media are homogeneous.[7] Note that in problems involving optical bistability with surface plasmons, the fields in the nonlinear medium are going to be inhomogeneous because of the surface character of the waves. Thus any spatial dependence of the field intensity E will make ϵ inhomogeneous, and therefore one should obtain the electric field in the nonlinear medium by solving the nonlinear wave equation. Such nonlinear solutions are now being studied in the context of nonlinear surface polaritons.[8–10] It is clear that one should investigate what happens to surface-plasmon bistability if the solutions in the nonlinear medium are taken to be one of the family of nonlinear polaritons. In this paper we present results for optical bistability assuming nonlinear polaritons in the dielectric material. Numerical solutions show that the hysteresis curves are quite sensitive to the plane-wave assumption. The bistability threshold is higher than what is predicted by plane-wave solutions in the nonlinear dielectric. The switching intensities are found to be about two times higher than those obtained by using plane-wave solutions in the nonlinear dielectric.

We assume Kretschmann configuration. A TM-polarized plane wave is incident from the prism of the dielectric constant ϵ_1 ($y > 0$) at an angle θ on the silver (metal) film with the dielectric function ϵ_2. The incident wavelength is such that surface plasmons are excited in the metal film. The nonlinear dielectric (Kerr medium) occupies the domain $y < -h$. The Kerr medium is characterized by a dielectric function,

$$\epsilon = \epsilon_3 + \alpha|E|^2. \quad (1)$$

Stegman and Seaton[8] suggested the use of the wave equation for the magnetic field $H\hat{z}$ in the Kerr medium, which they write as[11]

$$\nabla^2 H + k_0{}^2(\epsilon_3 + \alpha'|H|^2)H = 0, \qquad k_0 = \omega/c. \quad (2)$$

By writing

$$H = H_0(y)\exp[ixk_0(\epsilon_1)^{1/2}\sin\theta], \quad (3)$$

we get a simpler equation for H_0,

$$\mathrm{d}^2H_0/\mathrm{d}y^2 = k_0{}^2(g^2 - \alpha'|H_0|^2)H_0, \qquad g^2 = \epsilon_1\sin^2\theta - \epsilon_3. \quad (4)$$

If we write $H_0 = A\, e^{i\phi}$, then

$$\frac{\mathrm{d}^2A}{\mathrm{d}y^2} - A\left(\frac{\mathrm{d}\phi}{\mathrm{d}y}\right)^2 = k_0{}^2(g^2 - \alpha'A^2)A, \qquad \frac{\mathrm{d}}{\mathrm{d}y}\left(A^2\frac{\mathrm{d}\phi}{\mathrm{d}y}\right) = 0. \quad (5)$$

Assuming A to be constant and ϕ to be real, Eqs. (5) yield

$$\frac{\mathrm{d}\phi}{\mathrm{d}y} = k_0(\alpha'A^2 - g^2)^{1/2} \rightarrow \phi(y) = k_0 y\,(\alpha'A^2 - g^2)^{1/2}, \quad (6)$$

provided that

$$\alpha'A^2 - g^2 > 0. \quad (7)$$

Hence the magnetic field becomes

$$H_0 = A\exp[ik_0 y(\epsilon_3 + \alpha'A^2 - \epsilon_1\sin^2\theta)^{1/2}]. \quad (8)$$

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This gives propagating solutions in the nonlinear medium with a wave vector depending on the intensity of the field. To get solutions localized to the surface we put $d\phi/dy = 0$, and then we find the solution appropriate to surface waves,

$$H_0 = \left(\frac{2}{\alpha'}\right)^{1/2} \frac{g}{\cosh k_0 g(y_0 - y - h)} e^{iC}. \tag{9a}$$

Here C is a constant and y_0, which is real, is to be fixed by the boundary conditions. Solution (9a) is obtained under the condition

$$g^2 - \frac{\alpha'}{2}|H_0|^2 \geq 0. \tag{9b}$$

Note that in the limit of $y_0 \to \infty$, Eq. (9a) goes over to

$$H_0 = \left(\frac{2}{\alpha'}\right)^{1/2} 2g\, e^{iC} \exp[k_0 g(y + h - y_0)]$$

$$\equiv A_3 \exp[iC + k_0 g(y + h)], \tag{9c}$$

which is the correct surface waveform in the linear medium. It may thus be useful to introduce A_3 instead of y_0. The x component of the electric field in the nonlinear medium has the approximate form

$$E_x = i\left(\frac{2}{\alpha'}\right)^{1/2} \frac{g^2}{\epsilon_3} \frac{\sinh k_0 g[(y_0 - (h + y)]}{\cosh^2 k_0 g[(y_0 - (h + y)]} e^{iC}$$

$$\times \exp[ikx \sin\theta(\epsilon_1)^{1/2}]. \tag{10}$$

Since the other media are linear, we can take plane waves as solutions in the prism and the silver film. Using now the boundary conditions at the surfaces $y = 0$ and $y = -h$, we get equations giving the incident magnetic field A_1 and the reflected magnetic field B_1 in terms of the parameter y_0:

$$A_1 = \frac{1}{2}\left(\frac{2}{\alpha'}\right)^{1/2} g\, e^{iC} \left\{ \frac{\cos k_{2y}h}{\cosh k_0 g y_0} [(1 - i\beta_{12} \tan k_{2y}h) \right.$$

$$\left. + i\beta_{13} \tanh k_0 g y_0 (1 - i\beta_{21} \tan k_{2y}h)] \right\}, \tag{11}$$

$$B_1 = \frac{1}{2}\left(\frac{2}{\alpha'}\right)^{1/2} g\, e^{iC} \left\{ \frac{\cos k_{2y}h}{\cosh k_0 g y_0} [(1 + i\beta_{12} \tan k_{2y}h) \right.$$

$$\left. - i\beta_{13} \tanh k_0 g y_0 (1 + i\beta_{21} \tan k_{2y}h)] \right\}. \tag{12}$$

Here k_{iy} is the y component of the propagation vector in ith medium, assuming that all the media are linear,

$$k_{iy}^2 = \frac{\omega^2}{c^2}(\epsilon_i - \epsilon_1 \sin^2\theta), \qquad i = 1, 2. \tag{13}$$

The parameters β_{ij} are defined by

$$\beta_{ij} = \epsilon_i k_{jy}/\epsilon_j k_{iy}, \qquad k_{3y} = k_0 g. \tag{14}$$

In terms of the intensities defined by

$$U_i = \frac{\alpha'}{8g^2} A_1^2, \qquad U_r = \frac{\alpha'}{8g^2} B_1^2, \qquad U_t = \frac{\alpha'}{8g^2} A_3^2, \tag{15}$$

basic equations (11) and (12) can be written as

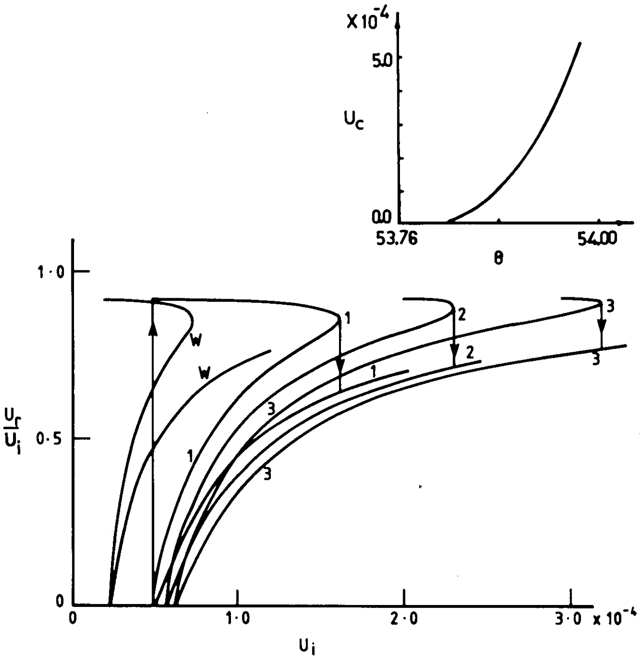

Fig. 1. The reflectivity U_R/U_i as a function of the incident field intensity U_i in the Kretschmann configuration for various values of the angles of incidence: θ = (1) 53.9°, (2) 53.92°, (3) 53.94°. The curve marked W is from the research of Wysin *et al.* for θ = 53.9°. The inset gives the switching intensity as a function of the angle of incidence.

$$U_i = \frac{U_t}{4} \left| \frac{\cos k_{2y}h}{(1 + U_t)} \left[(1 - i\beta_{12} \tan k_{2y}h) \right.\right.$$

$$\left.\left. + i\beta_{13} \frac{(1 - U_t)}{(1 + U_t)} (1 - i\beta_{21} \tan k_{2y}h) \right] \right|^2, \tag{16}$$

$$U_r = \frac{U_t}{4} \left| \frac{\cos k_{2y}h}{(1 + U_t)} \left[(1 + i\beta_{12} \tan k_{2y}h) \right.\right.$$

$$\left.\left. - i\beta_{13} \frac{(1 - U_t)}{(1 + U_t)} (1 + i\beta_{21} \tan k_{2y}h \right] \right|^2. \tag{17}$$

Equations (16) and (17) are numerically evaluated to eliminate the parametric dependence on U_t and to obtain U_r as a function of U_i. Numerical solutions are shown in Fig. 1 for the same parameters as those used by Wysin *et al.*: $\epsilon_1 = 3.6$, $\epsilon_2 = -57.8 + i0.6$ at $\lambda = 1.06$ μm, $\epsilon_3 = 2.25$. The nonlinear medium is taken to be CS_2. Figure 1 also shows the result of Wysin *et al.* obtained by using plane-wave assumption for fields in the nonlinear medium. The surface-plasmon angle θ_{sp} for $\lambda = 1.06$ μm, and, for the dielectric constants mentioned above, is found to be 53.76°. Figure 1 shows that departures from plane-wave results could be significant.[12] Using nonlinear solutions (9), switching intensities are almost twice those of the plane-wave case. The inset in Fig. 1 also gives the behavior of the switching intensity as a function of the angle of incidence.

Thus our analysis has shown the importance of the inhomogeneities in the field in the Kerr medium. These inhomogeneities result from the spatial dependence of E and the nonlinearity of the medium.

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Note added in proof: We recently investigated optical bistability with long-range surface plasmons. We found that, with a suitable choice of parameters, the switching intensities are much lower (approximately 2 orders of magnitude) than those required for optical bistability with short-range surface plasmons.

REFERENCES AND NOTES

1. See, for example, *Optical Bistability 2*, C. M. Bowden, H. M. Gibbs, and S. McCall, eds. (Plenum, New York, 1984).
2. A. E. Kaplan, IEEE J. Quantum. Electron. **QE-17**, 336–340 (1981); in *Optical Bistability*, C. M. Bowden, M. Ciftan, and H. Robl, eds. (Plenum, New York, 1981) p. 447–462.
3. G. M. Wysin, H. J. Simon, and R. T. Deck, Opt. Lett. **6**, 30–32 (1981).
4. P. Martinot, S. Laval, and A. Koster, J. Phys. (Paris) **45**, 597–600 (1984).
5. P. Martinot, A. Koster, S. Laval, and W. Carvalho, Appl. Phys. B **29**, 172–177 (1982).
6. M. Born and E. Wolf, *Principles of Optics* (MacMillan, New York, 1965), p. 62.
7. Generalizations of Fresnel formulas to inhomogeneous media are possible provided that such a media could be considered as being composed of a large number of homogeneous media.
8. G. I. Stegman and C. T. Seaton, Opt. Lett. **9**, 235–237 (1984).
9. N. N. Akmediev, Sov. Phys. JETP **56**, 299–303 (1982).
10. It may be added that there exists considerable literature on the question of the effect of Gaussian beams on various optical bistable systems [P. Drummond, IEEE J. Quantum. Electron. **QE-17**, 301–306 (1981); J. V. Moloney, M. R. Belic, and H. M. Gibbs, Opt. Commun. **41**, 379–382 (1982); **44**, 289 (1983); D. G. Hall, and T. G. Dziura, Opt. Commun. **49**, 146–150 1984]. Reflection of a two-dimensional Gaussian beam from a nonlinear interface has also been studied [D. Marcuse, Appl. Opt. **19**, 3130–3139 (1980); P. W. Smith and W. J. Tomlinson, IEEE J. Quantum. Electron. **QE-20**, 30–36 (1984)].
11. It should be borne in mind that Eq. (2) is an approximate wave equation, for $|E|^2$ has been replaced in Eq. (1) by $|H|^2/\epsilon_3$. It may be added that the use of nonlinear scalar-wave equations is in general not free from approximations.
12. A similar result has been obtained by V. J. Montemayor and R. T. Deck, J. Opt. Soc. Am. B **2**, 1010–1013 (1985)] for optical bistability with a waveguide mode.