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# Energy dependence of multiplicity in proton-nucleus collisions and models of multiparticle production

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**Abstract.** This is a continuation of our earlier investigation (Gurtu *et al* 1974 *Phys. Lett.* **50 B** 391) on multiparticle production in proton-nucleus collisions based on an exposure of emulsion stack to 200 GeV/*c* beam at the NAL. It is found that the ratio $R_{em} = \langle n_s \rangle / \langle n_{ch} \rangle$, where $\langle n_{ch} \rangle$ is the charged particle multiplicity in pp-collisions, increases slowly from about 1 at 10 GeV/*c* to 1·6 at 68 GeV/*c* and attains a constant value of $1 \cdot 71 \pm 0 \cdot 04$ in the region 200 to 8000 GeV/*c*. Furthermore, $R_{em} = 1 \cdot 71$ implies an effective *A*-dependence of $R_A = A^{0 \cdot 13}$, *i.e.*, a very weak dependence. Predictions of $R_{em}$ on various models are discussed and compared with the emulsion data. Data seem to favour models of hadron-nucleon collisions in which production of particles takes place through a *double step* mechanism, *e.g.*, diffractive excitation, hydrodynamical and energy flux cascade as opposed to models which envisage instantaneous production.

**Keywords.** Proton-nucleus collisions; charged particle multiplicity; nuclear emulsion; hadron-nucleus models.

## 1. Introduction

Not very long ago multiparticle production in hadron-nucleus collisions used to be ignored as being complex and rather a messy affair. This situation has now been reversed and recent years have seen an ever increasing interest in the study of hadron-nucleus collisions. There are a variety of reasons for this spurt of interest and we shall list a few of them. First and foremost is the phenomenal success of Glauber theory of multiple scattering (Glauber 1967) which has made it possible to incorporate correctly the nuclear effects. The second reason is the possibility of measuring hadron-nucleon cross sections for hadrons which decay *via* the electromagnetic and strong interactions (Kolbig and Margolis 1968, Trefil 1969). The third reason is the realisation that it may be possible to test the different models of multiparticle production in hadron-nucleon collisions by confronting their predictions for multiparticle production in hadron-nucleus collisions with the experimental data (Fishbane and Trefil 1971, 1973 *d*, Dar and Vary 1972, Subramanian 1972, Gottfried 1973, 1974). The fourth reason is the unique possibility that hadron-nucleus collisions offer for studying the space-time development of the particle production process (Gottfried 1973, 1974).

Nuclear emulsion is endowed with unique spatial and ionisation resolutions, which make it an excellent producer-detector for studying hadron-nucleus collisions. Nuclear emulsion ( $\langle A \rangle = 73$) is mainly composed of Ag, Br, C, N, O and H. Table 1 gives the composition of the Ilford G5 emulsion together with the probability of an inelastic collision occurring in each constituent. Approximately 71% of the collisions occur in the heavy nuclei, Ag and Br, 25% in the light nuclei, C, N and O, and only 4% in hydrogen.

We have carried out a study of proton-emulsion collisions at 200 GeV/$c$. The relevant experimental details as well as some of the results from this investigation have been described in an earlier publication (Gurtu *et al* 1974)—hereafter referred to as I. In this paper we present some additional experimental results on multiplicity; we also analyse the available data on proton-emulsion collisions in the range 7·1 to 8000 GeV/$c$ to draw conclusions on models of multiparticle production.

## 2. Multiplicity distributions at 200 GeV/$c$

Figure 1 shows the $n_s$ distributions of our events with $N_h \geq 2$ (1530 events) and $N_h \geq 9$ (745 events). Also shown is the distribution for $N_h = 0$ and 1 events recorded by Babecki *et al** (1973) in p-emulsion collisions at 200 GeV/$c$. The 242 events of Babecki *et al* have been normalised to 574 events using the fact that the percentages of events with $N_h \leq 1$ and $N_h \geq 2$ are $27 \cdot 4 \pm 1 \cdot 2$ and $72 \cdot 6 \pm 2 \cdot 2$ respectively; these numbers are the weighted averages of the values obtained by Cuer *et al* (1973) and Babecki *et al.* The overall histogram therefore refers to the complete set of p-emulsion collisions at 200 GeV/$c$.

## 3. Energy dependence of $\langle N_h \rangle$

The available data on $\langle N_h \rangle$ as a function of $p_{lab}$ is plotted in figure 2. The data have been taken from Lock and March (1955), Lorry *et al* (1958), Daniel *et al* (1960), Winzeler (1965), Bogachev *et al* (1958), Barashenkov *et al* (1959), Bricman *et al* (1961), Meyer *et al* (1963), Barbaro-Galtieri *et al* (1961), Babecki *et al* (1973), Cuer *et al* (1973), Lohrman *et al* (1961) and the present investigation (I). The value of Winzeler at 7·1 GeV/$c$ is shown as an upper limit in view of a 10% loss of events, the bulk of which are expected to have $N_h \leq 2$. The point at 8000 GeV/$c$ has been obtained from a compilation of all the world data where the primary events were located by following the cascades back to the origin and therefore with a minimum of bias (Malhotra 1972).

It is clear from figure 2 that $\langle N_h \rangle$ shows hardly any energy dependence beyond $p_{lab} \sim 20$ GeV/$c$. One may also mention here that even for $\alpha$-emulsion collisions at 165 GeV/nucleon, Lohrman *et al* (1961) find $\langle N_h \rangle = 8 \cdot 3 \pm 1 \cdot 8$ which is about the same as $\langle N_h \rangle = 8 \cdot 5 \pm 1 \cdot 2$ observed by them in $p$-emulsion collisions at 250 GeV/$c$.

This constancy of $\langle N_h \rangle$ beyond $p_{lab} \gtrsim 20$ GeV/$c$ has a strong bearing on the

* Since we have recorded only $N_h \geq 2$ events, we have used the data of Babecki *et al* (1973) for $N_h = 0$ and 1. We are grateful to J Gierula for providing us with $n_s$ distribution for these $N_h = 0$ and 1 events.

**Table 1.** Composition of Ilford G5 emulsion. The small amount of iodine has been included under Ag itself. $\sigma_{in}(A)$ is the p-nucleus inelastic cross section as calculated using Woods-Saxon density distribution (*see* Section 7). $F_A$ is the fraction of inelastic collisions occurring in nucleus $A$.

| Element | $A$ | Atoms/ $10^{-22}$ cm$^3$ | $\sigma_{in}(A)$ mb | $F_A$ |
|---|---|---|---|---|
| Ag | 107·9 | 1·020 | 1053 | 0·391 |
| Br | 79·9 | 1·008 | 869 | 0·319 |
| Si | 32·1 | 0·014 | 477 | 0·002 |
| $O_2$ | 16·0 | 0·938 | 293 | 0·100 |
| N | 14·0 | 0·318 | 266 | 0·031 |
| C | 12·0 | 1·391 | 238 | 0·120 |
| $H_2$ | 1·0 | 3·22 | 32·0 | 0·037 |

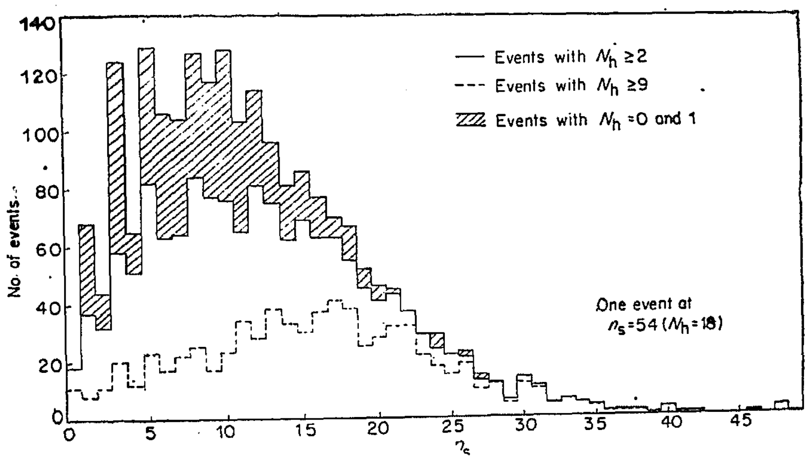


**Figure 1.** Distribution of $n_s$ in p-emulsion collisions at 200 GeV/$c$ for $N_h \geq 2$, $N_h > 9$, $N_h \leq 1$ and all $N_h$. The $N_h \leq 1$ data are those of Babecki *et al* (1973)—242 events normalised to 575 on the basis of the observed percentage of $N_h \leq 1$ events.

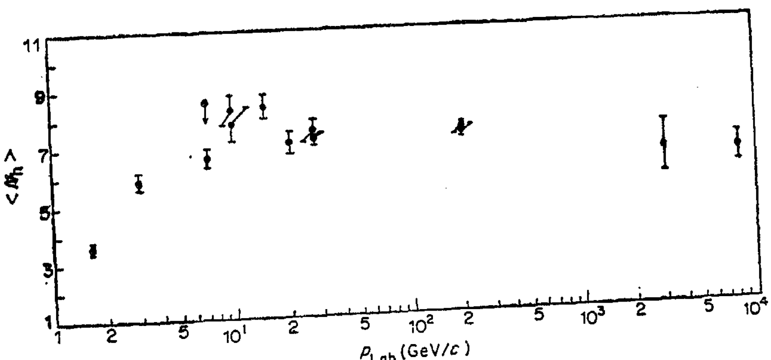


**Figure 2.** Plot of $\langle N_h \rangle$ as a function of $p_{lab}$ (GeV/$c$).

models of multiparticle production in hadron-nucleus collisions and in particular one can conclude that there cannot be any appreciable cascading inside the nucleus.

## 4. Energy dependence of $R_{em}$, the ratio of average multiplicity in p-emulsion collisions to that in pp collisions

In order to investigate the energy dependence of $R_{em}$ we have compiled the available data on $\langle n_s \rangle$ in the range 7·1 to 8000 GeV/$c$ in table 2. The values of $\langle n_{ch} \rangle$ for $p_{lab} \leq 27 \cdot 9$ GeV/$c$ have been obtained from the relation $\langle n_{ch} \rangle = 0 \cdot 348 + 1 \cdot 883\, E_{av}^{0 \cdot 461}$, $E_{av} = \sqrt{s} - 2m$, where $s$ is the square of the c.m. energy and $M$ is the nucleon mass; this relation has been found by Ganguli and Malhotra (1972 *a*) to fit the 4–69 GeV/$c$ data exceedingly well. The $\langle N_{ch} \rangle$ values at 69 GeV/$c$ and 205 GeV/$c$ are those of Soviet-French collaboration (1972) and Charlton *et al* (1972) respectively. The values of $\langle n_{ch} \rangle$ for $p_{lab} = 1000$ to 8000 GeV/$c$ have been calculated from $\langle n_{ch} \rangle = -3 \cdot 02 + 1 \cdot 81$ (ln $s$), which is the best fit to the accelerator and ISR data* in the range $p_{lab} = 69 \cdot 0$ GeV/$c$ to 1500 GeV/$c$.

The values of $R_{em}$ so obtained are presented in table 2 and figure 3.

The salient features of $R_{em}$ are: (*a*) it is small at all energies considered, and (*b*) it exhibits a slow increase in the region of 10 to 68 GeV/$c$ and attains an essentially *constant value of* $1 \cdot 71 \pm 0 \cdot 04$ *in* the region of 200 to 8000 GeV/$c$.

## 5. Dependence of $R_A$ on $A$

In order to abstract the dependence of $\langle n_s \rangle$ on the atomic weight $A$ of the nucleus, we may express it in the following model independent manner

$$\langle n_s (A) \rangle = \langle n_{ch} \rangle A^{\alpha}$$

$$R_A \equiv \langle n_s (A) \rangle / \langle n_{ch} \rangle = A^{\alpha} \tag{1}$$

where $\langle n_{ch} \rangle$ is the average charged particle multiplicity in pp-collisions at the same energy. For emulsion we then have

$$R_{em} = \frac{\sum_A (F_A A^{\alpha})}{\sum_A F_A} \tag{2}$$

where $F_A$ is the probability of an inelastic collision occurring in the nucleus $A$ of the emulsion; $F_A$'s are tabulated in table 1. Using eq. 2 we have evaluated the effective value of $\alpha$ at each energy and the values obtained are given in table 2 and plotted in figure 4. We find that $\alpha$ increases from $\sim 0$ at 10 GeV/$c$ to 0·12 at 68 GeV/$c$, and beyond 200 GeV/$c$ it attains essentially a constant value of

$$\alpha = 0 \cdot 131 \pm 0 \cdot 005 \tag{3}$$

Thus, not only is $R_{em}$ nearly constant at $p_{lab} \gtrsim 100$ GeV/$c$, its absolute value of $1 \cdot 71 \pm 0 \cdot 04$ implies an $A$ dependence of the type $A^{0 \cdot 13}$, which is very slow indeed.

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* The data used have been taken from Soviet French Collaboration (1972), Bromberg *et al* (1973), Charlton *et al* (1972), Dao *et al* (1972), Antinucci *et al* (1973) and Breidenbach *et al* (1972). The ISR data of the last two references have been appropriately corrected for the fact that they used a constant value of $\sigma_{in}$ (pp) = 32 mb; we have scaled $\langle n_{ch} \rangle$ by $32 \cdot 0/\sigma_{in}(s)$, where $\sigma_{in}(s) = 23 \cdot 9\, s^{0 \cdot 40}$ as given by Morrison (1973).

**Table 2.** Compilation of $\langle n_s \rangle$, $R_{em}$ and $\alpha$ for an average emulsion collision

| $p_{lab}$ GeV/$c$ | $\langle n_{ch} \rangle$ | $\langle n_s \rangle$ | $R_{em}$ | $\alpha$ | Reference for emulsion data |
|---|---|---|---|---|---|
| 7·1 | 2·94±0·03 | 2·80±0·04 | 0·95±0·02 | −0·013±0·006 | Winzeler (1965) |
| | | 2·62±0·05 | 0·89±0·02 | −0·030±0·006 | Daniel *et al* (1960) |
| 9·9 | 3·24±0·03 | 3·2 ±0·2 | 0·99±0·06 | −0·003±0·016 | Barashenkov *et al* (1959) |
| 20·5 | 4·10±0·04 | 5·29±0·13 | 1·29±0·03 | 0·064±0·006 | Meyer *et al* (1963) |
| 23·4 | 4·22±0·04 | 5·61±0·11 | 1·31±0·03 | 0·072±0·006 | Winzeler (1965) |
| 27·0 | 4·41±0·04 | 6·23±0·2 | 1·41±0·05 | 0·084±0·008 | Meyer *et al* (1963) |
| 27·9 | 4·46±0·04 | 6·6±0·1 | 1·48±0·04 | .. | Barbaro-Galtieri *et al* (1961) |
| 67·9 | 5·89±0·07 | 9·57±0·23 | 1·62±0·04 | 0·125±0·006 | Babecki *et al* (1973) |
| | | 9·73±0·23[a] | | | |
| 200 | 7·64±0·17 | 13·04±0·4 | 1·71±0·06 | 0·130±0·006 | Gurtu *et al* (1974) |
| | | 13·08±0·3 | 1·71±0·06 | | Babecki *et al* (1973) |
| | | 13·31±0·3[a] | | | |
| | | 12·9 ±0·4 | 1·69±0·06 | | Cuer *et al* (1973) |
| 1000 | 10·6 ±0·6 | 19·2 ±1·9 | 1·81±0·20 | 0·147±0·026 | Babecki *et al* (1973) |
| 3000 | 12·6 ±0·7 | 21·7 ±1·6[b] | 1·72±0·16 | 0·135±0·022 | Lohrman *et al* (1961) |
| 8000 | 14·4 ±0·8 | 23·3 ±2·0 | 1·62±0·16 | 0·120±0·024 | Malhotra (1972) |

*a* These numbers have been obtained by the authors after excluding the coherent events.

*b* A small correction has been applied to the data of Lohrman and Teucher (1962) for the fact that events with $n_s \leq 5$ had been excluded.

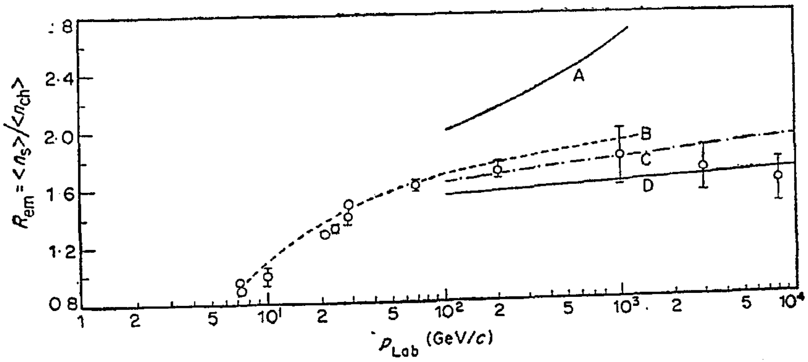


**Figure 3.** Dependence of $R_{em}$ on $p_{lab}$. The 200 GeV/$c$ point represents the value obtained by using $\langle n_s \rangle = 13·0±0·2$ which is the weighted mean of the value obtained in this, Cuer *et al* (1973) and Babecki *et al* (1973) investigations. *See* text for explanations of the curves A, B, C and D.

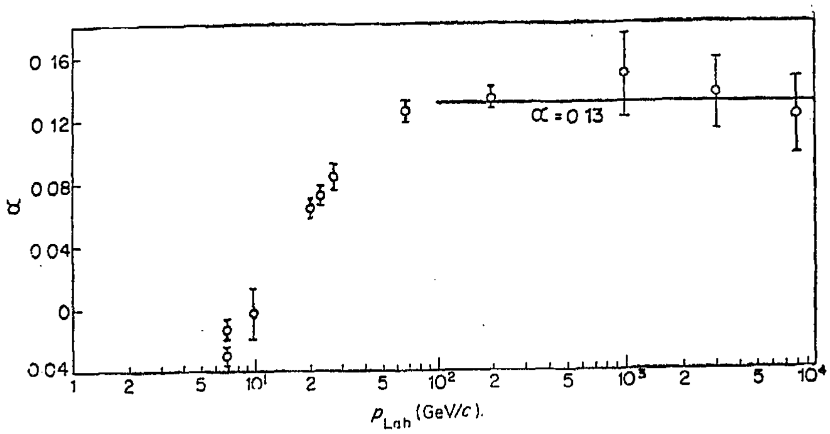


**Figure 4.** Dependence of $\alpha$, defined by eq. (1), on $p_{\text{lab}}$ (GeV/$c$).

It may be mentioned that although the negative values of $\alpha$ at 7·1 and 9·9 GeV/$c$ could in principle be interpreted as due to absorption in the nucleus, the real or more important reason for this is the fact that whereas $\langle n_{ch}\rangle$ includes slow protons ($\beta < 0{\cdot}7$), $\langle n_s\rangle$ does not. If protons are excluded from $\langle n_s\rangle$ and $\langle n_{ch}\rangle$, the effect of this would be to increase the value of $\alpha$, at high energies it would change from 0·13 to 0·15.

## 6. Comparison of data with models of proton-nucleus collisions

We now consider the predictions, particularly for $R_{em}$, of the different models of multiparticle production in proton-nucleus collisions. There are in general two ingredients which form the basis of these models, (i) a model for multiparticle production in proton-nucleon collisions, and (ii) nuclear effects relating to propagation inside the nucleus. It is expected that a study of proton-nucleus collisions would provide means of distinguishing between the various models of proton-nucleon collisions (Fishbane and Trefil 1971, 1973 *d*, Dar and Vary 1972, Subramanian 1972). One may in general divide models of proton-nucleon collisions into two classes. In the first class of models (SSM) the final state is formed instantaneously, or in other words in a *single-step*, *e.g.*, the multiperipheral model and its multi-Regge generalisations and the bremstrahlung model of Feynman. In the second class of models (DSM) production takes place in a *double-step*, *e.g.*, the fragmentation model, the diffractive excitation models (*e.g.*, the nova model), the fireball model and hydrodynamical model. In the SSM case one would in general expect cascading mechanism to dominate whereas in the DSM case, in the first instance one or more compound systems are produced and these decay subsequently into the final state particles.

Before we consider the predictions of the different models and confront the same with the experimental data presented above, it is necessary to calculate $\langle \nu_{em}\rangle$, the average number of proton-nucleon inelastic collisions in emulsion nuclei.

### 6.1. *Average number of collisions in a nucleus*

In order to calculate the average number of collisions suffered by the incident hadron inside the nucleus, we have used the Glauber theory (Glauber 1967). If $P_\nu$ is the probability for the incident proton to have suffered $\nu_A$ collisions in a nucleus with atomic weight $A$, then

$$\langle \nu_A \rangle = \frac{\sum_\nu \nu P_\nu}{\sum_\nu P_\nu} = \frac{2\pi \int \sigma_{in}\, T(b)\, b\mathrm{d}b}{\sigma_{in}(A)} = \frac{A\sigma_{in}}{\sigma_{in}(A)} \tag{4}$$

where,

$$\sigma_{in}(A) = 2\pi \int [1 - \exp(-\sigma_{in}\, T(b)]\, b\mathrm{d}b \tag{5}$$

$$T(b) = A \int \rho(r)\, \mathrm{d}z; \quad r^2 = z^2 + b^2. \tag{6}$$

Here, $\sigma_{in}$ and $\sigma_{in}(A)$ are the p-nucleon and p-nucleus inelastic cross sections respectively, and $T(b)$ is the number of nucleons per unit area in the path of the incident proton at impact parameter $b$. For the density distribution of the nucleus, $\rho(r)$ we have used Woods-Saxon form

$$\rho(r) = \rho_0 \left[\exp\left(\frac{r-c}{a}\right) + 1\right]^{-1} \tag{7}$$

with $a = 0{\cdot}545$ and $c = 1{\cdot}07\ A^{1/3}$ fm (Glauber 1967). In this way we have calculated $\sigma_{in}(A)$ and $\langle \nu_A \rangle$ as a function of $\sigma_{in}$. The values of $\sigma_{in}(A)$ so obtained for $\sigma_{in} = 32{\cdot}0$ mb, *i.e.*, at 200 GeV/$c$ are given in table 1. It is found that $\langle \nu_A \rangle$ can be well expressed as

$$\langle \nu_A \rangle = 0{\cdot}716\, A^{0.326} \tag{8}$$

Note that $\langle \nu_A \rangle$ has an energy dependence since $\sigma_{in}$ depends on $s$; we have used $\sigma_{in}(s)$ as given by Morrison (1973). In this way we obtain

$$\langle \nu_{em} \rangle = 2{\cdot}72 \tag{9}$$

at 200 GeV/$c$ ($\sigma_{in} = 32{\cdot}0$ mb), averaged over the emulsion composition. It may be pointed out that this value is significantly lower than 3·2 obtained by Gottfried (1973, 1974) using a uniform density distribution for the nucleus.

We shall now discuss some of the model calculations for proton-nucleus collisions.

### 6.2. *Intranuclear cascade calculations*

In the cascade model one assumes that the incident hadron collides successively with a number of nucleons inside the nucleus producing secondary particles at each collision. Each of the secondaries may in turn suffer further collisions leading to a build-up of an intranuclear cascade. It is rather well known that such a model explains adequately data on multiplicity and $\langle N_h \rangle$ up to primary energies of about 25 GeV but at higher energies it grossly overestimates these quantities (*see* e.g. Barashenkov *et al* 1964 and Artykov *et al* 1966).

One can criticize the simple cascade calculation in so far as high energies are concerned at least on one count. As pointed out by Fishbane *et al* (1972 *a*) it is important to use Glauber multiple scattering theory which takes into account appropriately the non-classical effects. These non-classical terms are associated with rescattering effects and shadowing of the propagating particle by each other. The importance of such effects can be appreciated when one realises that at 200 GeV/$c$ the forward cone hadrons are so highly collimated that for a nucleus of radius $r_A = 4$ fm their overall spatial opening before they leave the nucleus would be $\simeq r_A (2M/E_{Lab})^{1/2} = 0{\cdot}4$ fm only. It therefore seems unreasonable to assume that forward cone particles can be treated as independent entities while inside the nucleus. In this context one may mention the important observation made by

Bemporad *et al* (1971) who found that the effective interaction cross sections of systems of 3 and 5 hadrons produced inside a nucleus are not appreciably different from that of a single hadron.

Attempts have been made to refine the cascade calculation by taking account of the above mentioned high collimation of the secondary particles at high energies. Different recipes have been used. Artykov *et al* (1968) have assumed that because of the high collimation 'a large number of secondary particles interact with one and the same intranuclear nucleon'. Taking into account the effect of such many-particle interactions Artykov *et al* (1968), carried out a very elaborate Monte Carlo calculation. Curve B in figure 3 presents their results in the range 7 to 1000 GeV/$c$. As can be seen their predictions agree rather well with the data up to about 1000 GeV/$c$ beyond which there are indications of some disagreement. However, even this refined cascade calculation leads to far too high values of $\langle N_h \rangle$ for $p_{lab} > 50$ GeV/$c$, *e.g.*, at 200 GeV/$c$ the predicted value is $13 \cdot 0 \pm 0 \cdot 6$ whereas our experimental value is $7 \cdot 3 \pm 0 \cdot 2$.

Fishbane *et al* (1972 *b*) and Fishbane and Trefil (1973 *a*, *b*, *d*) in a series of papers have described their calculation for SSM case (which in their terminology is called IPM). An important feature of this calculation is that they use Glauber multiple scattering theory and therefore take into account the non-classical nuclear effects such as associated with rescattering and shadowing of propagating particles by each other. The essential effect of this is to considerably reduce the effective cross section of $n$ independent particles and therefore the nuclear multiplicity compared to simple cascade calculation. Curve A in figure 3 shows the results of Fishbane and Trefil. Clearly, even this refined SSM calculation is in gross disagreement with the experimental data and if we are to accept the validity of these calculations we are led to the conclusion that the class of models for hadron-nucleon collisions which invoke instantaneous or single-step production are not favoured by the data presented here.

One of the important and rather unexpected predictions of Fishbane and Trefil (1973 *b*, *d*) for SSM case is that the multiplicity is independent of the mass number of the nucleus for $A > 10$. It would be worthwhile to check this prediction by comparing the $\langle n_s \rangle$ obtained here in p-emulsion collisions with the corresponding value in p-neon collisions by carrying out a neon-filled bubble chamber exposure at the NAL.

### 6.3. *Diffractive excitation model*

We shall now consider the DSM case. There are essentially three models which come in this category, namely, the diffractive excitation model (Dar and Vary 1972, Fishbane and Trefil 1973 *c*), hydrodynamical model (Belenkij and Landau 1956) and the energy flux cascade model of Gottfried (1973, 1974). In the diffractive excitation model, the particle production takes place through the diffractive excitation of the nucleons (isobars or novas). In the first collision of the proton two novas are excited, a fast one and a slow one. At high enough energies, the life time of the fast nova exceeds that of the time of transit through the nucleus, because of time dilatation, and therefore particle production takes place only after the nova has left the nucleus. The slow nova does not appreciably cascade, except perhaps at very high energies, and the fast nova in its next collision just changes its own state of excitation and produces another slow nova. If $P_d$ and $P_s$ are probabilities of double and single excitation of novas respectively, such that $P_d + P_s = 1$, then it can be shown that

$$R_A = \frac{1}{2}\left(\langle \nu_A \rangle + \frac{2}{1+P_d}\right) - \frac{(1+P_d)\langle \nu_A \rangle}{4\langle n_{ch} \rangle} \tag{10}$$

where the second term is a correction to take account of the fact that in a nucleus nearly half of the nucleons are neutrons. Equation (10) implies that (i) $R_A$ has an $A$ dependence of the type $R_A \simeq aA^{1/3} + b$ and (ii) the value of $R_A$ is maximum for $P_d = 0$ and minimum for $P_d = 1$. There is a yet no definite information on the relative values of $P_s$ and $P_d$ but there are some indications, *e.g.*, the low value of the observed coherent cross section at 200 GeV (Anzon *et al* 1973), that the single diffractive excitation of the projectile or target hadron cannot account for any appreciable fraction of the inelastic hadron-nucleon cross section. If we assume $P_d = 1$ and use the value of $\langle \nu_{em} \rangle = 2{\cdot}72$ obtained above for emulsion, we find that $R_{em} = 1{\cdot}68$ at 200 GeV/$c$. Curve C in figure 3 represents the prediction of eq. (10); the energy dependence is due to the energy dependence of $\sigma_{in}$. As can be seen there is a very good agreement between the predictions and the data (provided $P_d \sim 1$). To give an idea regarding the sensitivity of $R_A$ on $P_d$, it may be mentioned that $R_A = 1{\cdot}81$ for $P_d = 2/3$.

In addition to the low value of $R_A$, the diffractive excitation model also predicts that for p-nucleus collisions, (i) the inelasticity (in the laboratory system) should not be much greater than that for pp-collisions, (ii) the log tan $\theta$ (which is a measure of the rapidity) distribution of the forward cone particles, after excluding coherent production, should be similar to that of the pp-collisions and (iii) since the additional multiplication results from the decay of target novas, the log tan $\theta$ distribution in the target fragmentation region would show an excess of events over that observed in pp-collisions. All of these features seem to be in agreement with observations (Lal *et al* 1965), Feinberg 1972, Barcelona *et al* 1974 and Gottfried 1973). A particularly attractive feature of the diffractive excitation model is that the above mentioned predictions for proton-nucleus collisions are a consequence of the very nature of the diffractive excitation model for pp-collisions and that the nuclear effects are relatively less important in this model. However, it should be pointed out that the simple diffractive excitation model fails (Ganguli and Malhotra 1972 *b*) to explain the charged particle multiplicity distribution and the observed energy dependence of $\langle n_{ch} \rangle/D$ in pp-collisions. Because of such reasons, models which envisage two components (independent or otherwise), *e.g.*, diffraction and pionisation, have received a great deal of attention (*see e.g.*, Wroblewski 1973). We would like to point out that the above discussion can provide a guideline for further attempts in this direction; in particular one may note that the data on $R_{em}$ cannot tolerate any appreciate cascading which is general would be associated with the pionisation component.

## 6.4. *Hydrodynamical model*

We next consider the hydrodynamical model due to Landau (1953). In this model multiparticle production in a collision between two hadrons at high energies is envisaged as follows. In the first stage, two hadronic discs meet each other and coalesce into a single body which expands until the volume is large enough so that interaction between the 'observed' particles becomes small. At that point the second stage begins whence the particles escape freely. The first stage which essentially determines the multiparticle production is governed by relativistic hydrodynamics. This model predicts energy dependence of multiplicity in pp-collisions as $\langle n_{ch} \rangle = as^{1/4}$ which is in good agreement with the data (Ganguli and Malhotra 1972 *a*). An extension of this model to a hadron-nucleus collision implies that at high enough energies the hadron will essentially collide with $\langle \nu_A \rangle$ nucleons at rest, where $\langle \nu_A \rangle$ is the mean number of nucleons contained in the 'tube' traversed by the incident hadron (Belenkij and Landau 1956). After carrying out a rather complex computation, they give following prediction

$$R_A = A^{0\cdot 19} \tag{11}$$

However, since in this model multiplicity grows as $s_A{}^{1/4}$ and for a hadron-nucleus collision $s_A = s \langle \nu_A \rangle$, if we ignore *transverse motion* we expect

$$R_A = \langle \nu_A \rangle^{1/4} = 0\cdot 92\, A^{0\cdot 08} \tag{12}$$

which may be compared with the experimental result $R_A = A^{0\cdot 13}$, given by eq. (3). It is not clear why (11) and (12) differ so much, even at high energies. It seems to us that there is a need for a better computation of $A$ dependence of $R_A$ on Landau's model. A special feature of this model is that the elasticity, defined as the average energy retained by the nucleon, is expected to be low since there are $\langle \nu_A \rangle$ nucleons in the final state, while at the same time the fraction of energy radiated in the form of created particles is not expected to be much greater than in a pp-collision. While the second point is in agreement with the experimental observations, experimental information on the inclusive proton spectrum in p-nucleus collisions is singularly lacking. Finally, we note that in agreement with observations, Landau's model predicts approximately the same linear relation between $D$ and $n_s$ for p-nucleus collisions as for pp-collisions.

### 6.5. *Energy flux cascade model*

Recently Gottfried has proposed an 'energy flux cascade' model (Gottfried 1973, 1974) for hadron-nucleus collisions. In common with Landau's model, this model assumes that the energy flux of hadronic matter is the essential variable that governs the early evolution of the system, and it is a cascade of this flux, and not of conventional hadrons, that occurs in a nucleon-nucleus collision. The essential difference between the two models lies in the temporal structure of the developing state *e.g.*, whereas in Landau's model the expansion phase is relatively slow, in Gottfried's model the expansion occurs with a rapidity close to that of the incident particle. An important prediction of Gottfried's model is that

$$R_A = \tfrac{1}{3}\,(\langle \nu_A \rangle + 2) + 0\,(\ln^{-1} s) \tag{13}$$

If we ignore the $0\,(\ln^{-1} s)$ term which is quite small even at 200 GeV/*c*, then it implies that $R_{em} = 1\cdot 57$ at 200 GeV/*c* since $\langle \nu_{em} \rangle = 2\cdot 72$. Curve D in figure 3 represents the prediction of eq. (13). We consider the agreement to be rather good; the slight deviation at 200 GeV/*c* may imply that transverse motion which is neglected in arriving at eq. (13), cannot be altogether ignored at energies as low as 200 GeV/*c*. This model also seems to explain, at least qualitatively, features such as low $\langle N_h \rangle$, near independence of inelasticity on $A$ and the observed nuclear ln (tan $\theta$) distribution in emulsion at 200 GeV/$c$ (Cuer *et al* 1973, Barcelona *et al* 1974, Gottfried 1973).

## 7. Conclusions

The conclusions arrived at in this investigation are summarised below:

(i) The ratio $R_{em} = \langle n_s \rangle / \langle n_{ch} \rangle$, where $\langle n_{ch} \rangle$ is the charged particle multiplicity in pp-collisions, increases slowly from about 1 at 10 GeV/$c$ to 1·6 at 68 GeV/$c$ and attains a constant value of $1 \cdot 71 \pm 0 \cdot 04$ in the region 200 to 8000 GeV/$c$. (ii) The above value of ratio $R_{em} = 1 \cdot 71 \pm 0 \cdot 04$ in the high energy region implies an effective $A$ dependence of $R_A = A^{0 \cdot 13}$, which is a very weak dependence indeed. (iii) The energy dependence of $R_{em}$ tends to favour models of hadron-nucleon collisions in which production of particles takes place through a *double-step* mechanism (DSM), *e.g.*, diffractive excitation, hydrodynamical and energy flux cascade as opposed to models which envisage instantaneous production, *e.g.*, the multiperipheral model. However, recalling the well-known difficulties of the diffractive excitation model to explain the multiplicity distribution in pp-collisions, it appears that the two component picture (DSM and SSM) with a dominant DSM deserves to be pursued.

It has been demonstrated here that study of hadron-nucleus collisions can lead to valuable information on models of particle production in hadron-hadron collisions. Such studies should be carried out with greater detail using homogeneous target such as neon in a bubble chamber. Evaluation of total coherent cross section can give valuable information on the relative importance of single and double excitation. Dependence of multiplicity on mass number of the target is important to check the predictions of the models, *e.g.*, Fishbane and Trefil's (1973 *b*, *d*) calculation for SSM case indicates that multiplicity is independent of $A$ for $A > 10$. The observation of Bemporad *et al* (1971) regarding the effective interaction cross section of systems of 3 and 5 hadrons propagating inside a nucleus needs to be extended to higher multiplicities. Detailed measurements of rapidity distributions as a function of multiplicity and $N_h$ in target as well as projectile fragmentation regions need to be carried out.

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