The Development of Quantum Mechanics

A Story of People, Places and Philosophies

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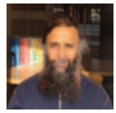
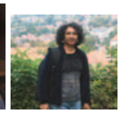
A historical account of the development of quantum mechanics, and the roles played by many outstanding physicists, their views and philosophical attitudes is presented. Ingenious and path-breaking experiments that helped this development along are highlighted. Ideas and notions that initially arose in the course of discussions on foundations of quantum mechanics and later paved the way for the emergence of the field of Quantum Information Science and Technology are briefly touched upon.

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1. Introduction

It is generally agreed that the major advances in physics during the 20th century are special relativity in 1905, general relativity in 1915, and the quantum theory and quantum mechanics over the years 1900 to 1927. The relativity theories, especially the general theory, are essentially the work of Albert Einstein. In contrast, in the cases of the old quantum theory and quantum mechanics, about a dozen exceptionally gifted physicists from many countries were involved in an effort lasting practically a quarter of a century.

The arrival of the new quantum mechanics was so keenly awaited that almost immediately a host of applications to many areas of physics, and chemistry too, followed. These include Slater's work on multielectron systems laying the foundations of solid state physics; Landau's work on magnetism showing that it is a quantum phenomenon; Pauling's theory of the chemical bond and the inauguration of quantum chemistry; Van Vleck's work on electric and magnetic susceptibilities of matter; and some decades later, the field of quantum optics, the basic understanding of Bose–

Keywords
Quantum mechanics, history of science, quantum information science and technology, quantum theory.

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Einstein condensation, the quantum Hall effect, and so many others. One can see that quantum mechanics is indeed a framework theory, capable of handling a vast variety of physical phenomena. The spin of the electron postulated by Goudsmit and Uhlenbeck just prior to quantum mechanics found its proper formulation in the new framework in Pauli's hands.

In addition to quantum mechanics making it possible to create so many new theories and concepts of direct use in understanding diverse phenomena, there were developments within quantum mechanics itself with regard to the interpretation of the theory. These were largely concerned with understanding and exploiting the features of quantum mechanics that set it apart from classical mechanics with seminal contributions coming from Schrödinger, Einstein Podolsky and Rosen, Bohm, Bell, and Feynman among others. This pursuit has played a significant role in shaping the currently much-discussed subject of 'Quantum Information Theory'.

Our aim in this article is to focus on these latter developments alone, as trying to cover the enormous number of successful applications would be much beyond its scope.

2. Before the Quantum-physics of Temperature Radiation

The story begins with Gustav Kirchoff, the teacher of Max Planck and often called 'the grandfather of quantum theory'. In 1859, the year of publication of Charles Darwin's *The Origin of Species*, he posed the problem of measuring and explaining 'temperature radiation'. This is radiation in equilibrium with matter at a common temperature T. Kirchoff himself had shown that the energy density of such radiation per unit frequency interval, $U(\nu, T)$, is a universal function of frequency ν and temperature T, independent of the nature of the surrounding matter. So the problem he posed was the measurement of $U(\nu, T)$, and the explanation of the results obtained.

In 1893 Wilhelm Wien showed by a beautiful argument that this

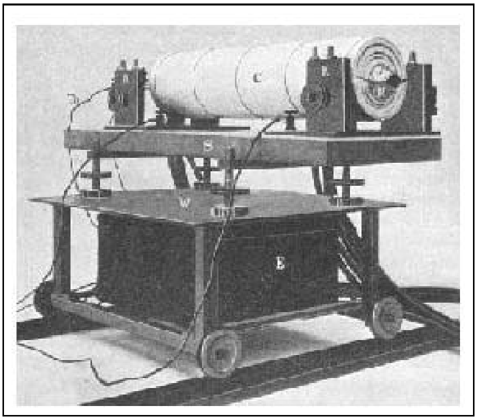

Figure 1. Otto Lummer's and Ferdinand Kurlbaum's 1898 black body radiation experimental setup.

function obeyed a 'scaling law' and depended essentially only on the ratio ν/T:

$$U(\nu, T) = \nu^3 f(\nu/T) \ . \tag{1}$$

This result, known as Wien's theorem, meant that one had to measure and understand one function of one variable, rather than a function of two independent variables. This immediately leads to the Stefan–Boltzmann law of 1879, 1884 – the total energy density, integrating over all frequencies, is proportional to T^4.

Soon after, in 1896, Wien proposed a form for $U(\nu, T)$ based on ideas connected with the Maxwell velocity distribution for an ideal gas. This Wien's law is:

$$U(\nu, T) = a\, \nu^3\, e^{-b\nu/T}. \tag{2}$$

Then in 1900, Lord Rayleigh and James Jeans showed that classical electrodynamics and statistical thermodynamics uniquely led to the form:

$$U(\nu, T) = \frac{8\pi\nu^2}{\mathrm{c}^3}\, \mathrm{k}T, \tag{3}$$

with k the Boltzmann constant and c the speed of light. This is consistent with Wien's theorem (1).

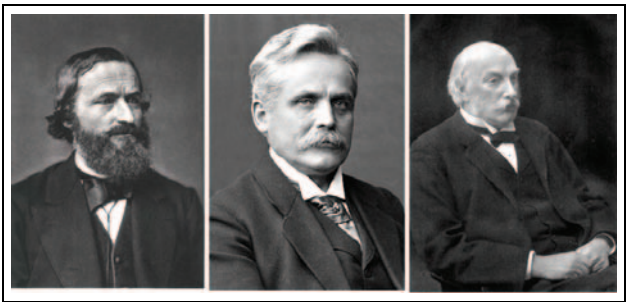

Figure 2. (from left) Kirchoff (1824–1887), Wilhelm Wien (1864–1928), Lord Rayleigh (1842–1919).

3. Planck's Discovery and Einstein's Contributions

Now we come to a momentous event – the discovery made by Planck on Sunday, October 7, 1900, incidentally the 15th birthday of Niels Bohr. Initial experiments had agreed with Wien's law (2) for high frequencies and Planck had believed this law to be true for all frequencies. His efforts had, therefore, been to find an explanation or derivation of this law. On the Sunday in question, Planck's experimental colleague Heinrich Rubens (and Mrs Rubens) visited the Plancks for tea. During the visit Rubens told Planck about the most recent low-frequency measurements by himself and Ferdinand Kurlbaum, which agreed with the Rayleigh–Jeans law (3). After the Rubens left, Planck realized that he had to reconcile two limiting forms for $U(\nu, T)$ – the high-frequency Wien form (2), and the low-frequency Rayleigh–Jeans form (3). Over a few hours of inspired work, involving the properties of the entropy of radiation, he succeeded in finding an expression with the desired limiting forms, the Planck's law as it became known:

$$U(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{\mathrm{h}\nu}{e^{\mathrm{h}\nu/kT} - 1} . \tag{4}$$

A new fundamental constant of Nature, Planck's constant h, thus made its appearance.

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Box 1. Experiments that Proved Classical Physics was Wrong

By the late 1800s, several physicists were beginning to realize that there were a lot of things that were logically inconsistent with classical theories of physics. The most pernickety mysteries were those surrounding the theories of blackbody radiation. Several radiation 'laws' were already known such as those formulated by Kirchhoff, Stefan and Boltzmann, Lord Rayleigh and by Wien. But they were as yet mere theoretical constructions, still to be experimentally verified. Blackbody radiation refers to radiation surrounding a body in thermodynamic equilibrium with its environment or to radiation emitted by an opaque, non-reflective body at constant uniform temperature. In 1895 Wilhelm Wien and Otto Lummer constructed an experimental apparatus to study blackbody radiation. Wien's experimental black body radiation emitter was a small hole punched into a completely closed oven (a platinum cylinder sheet within a ceramic tube). The temperature of the oven could go up to approximately $+1600^{\circ}$C.

The emitted infrared radiation was measured using a bolometer. A bolometer was a curious device invented by American astronomer Samuel Langley and consisted of two platinum strips covered with lampblack – one strip exposed to radiation and the other protected from it. These two strips formed two branches of a Wheatstone bridge, fitted with a galvanometer and connected to a battery. The entire setup was enclosed in a non-conducting cylinder. When radiation fell on the exposed strip, it would heat the strip which would then emit radiation. According to estimates made by physicists in 1881, the setup could detect radiation from a cow standing 400 meters away and was sensitive to temperature differences of 0.001 K. Later other physicists such as Lummer and Pringsheim and Kurlbaum designed other apparatuses which proved the validity of the Stefan–Boltzmann law (*Figure* 1). Lummer and Pringsheim extended their designs to study blackbody radiation at much higher temperatures and they soon found significant deviations from Wien's law. They reported in November 1899 that there were "systematic discrepancies between theory and experiment". But no one could resolve these issues. The last straw that broke the proverbial camel's back came from the experiments of Rubens and Kurlbaum in the summer of 1900 to test the validity of radiation law at long wavelengths and high temperatures. On October 25, 1900, Rubens and Kurlbaum compared their experimental measurements of the blackbody radiation spectrum with five different theoretical predictions: Planck's formula, Wien's formula, Rayleigh's law and two other formulae. Planck's theory provided the best fit to their experimental results – this was indeed the first triumph of quantum mechanics!

19, 1900, and then set about trying to find a derivation of (4). This was presented by him at a meeting on December 14, 1900. It was in this derivation that he introduced the revolutionary idea that the energy of a material oscillator could not have any continuous value but is quantized, that is, restricted to a discrete set of allowed values.

Now Einstein enters the story. On three occasions he worked around or on Planck's law, and each time extracted profound re-

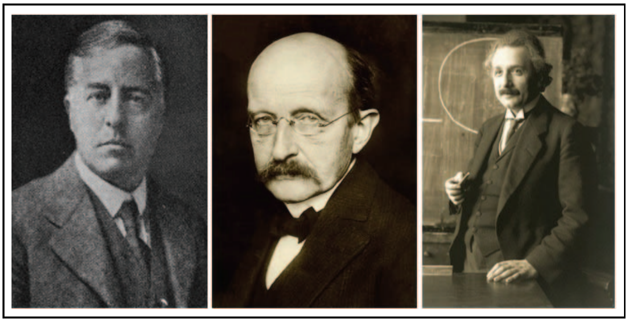

Figure 3. (from left) Sir James Jeans (1877–1946), Max Planck (1858–1947), Albert Einstein (1879–1955).

sults. In 1905 he analyzed the volume dependence of the entropy of radiation in the high-frequency Wien limit and was led to the corpuscular nature of light. His original name for these corpuscles was 'needle quanta' (in German), and he mentioned the photoelectric effect as one of three phenomena where they could be 'seen'. The name 'photon' came from the chemist G N Lewis in 1926. Next, in 1909 he turned to the complete Planck's law to calculate the energy fluctuations of radiation, and found that it was a sum of two terms – one, coming from the Wien law alone, was particle-like in nature; the other, from the Rayleigh–Jeans result alone, was wave-like in nature. This result led to the idea of wave-particle duality for light. Radiation described by Planck's law, and so more generally, had both a classical wave-like nature and a quantum particle-like nature. The third occasion was in 1916 when he presented a new derivation of Planck's law based on Niels Bohr's theory of atomic structure with discrete energy levels and transitions between them. It was here that his famous *A* and *B* coefficients, describing spontaneous emission, and absorption and stimulated emission, of radiation by matter, were introduced.

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4. The Bohr Atom and the Old Quantum Theory

So far, the emphasis had been on the thermodynamical and statistical properties of radiation. Meanwhile, ideas on applying Planck's discovery to the atomic structure of matter were also being developed. After Ernest Rutherford's discovery of the nu-

Box 2. Experimental Evidence for Photons

The origins of the discovery of the photoelectric effect lie in the experiments of the German physicist Heinrich Hertz who had demonstrated the existence of electromagnetic waves at the University of Karlsruhe, Germany in the late 1880s. Hertz's set up consisted of an antenna (with an oscillating high-voltage coil) and a copper wire loop with a gap, placed at the other end of the room. The rapidly switching voltage in the antenna caused sparks to jump across the gap in the copper wire loop, even though there was no physical connection between them. In 1887, Hertz carried forward his experiments on the production of sparks using ultraviolet light. The sparks were caused by the process of electron emission due to electromagnetic radiation. Hertz observed that when he placed a glass shield over the detector the size of the spark decreased; however when he used a quartz shield (which was known to transmit ultraviolet light) instead of glass, the size of the spark did not decrease! This experiment showed the capability of ultraviolet light to cause emission of electrons when incident upon a metallic surface, i.e., the 'photoelectric effect'.

Wilhelm Hallwachs, Sir J J Thomson and Phillip von Lenard also noticed such effects during their experiments with electrometers and cathode ray tubes. Lenard performed more detailed experiments and by 1902, he had observed that surprisingly, the energy of the emitted electrons depended on the frequency of the incident light and using light of shorter wavelengths led to the emission of faster electrons. Also, different metals required different minimum light frequencies for electron emission to occur. This puzzling phenomenon was explained in 1905 by Albert Einstein, using quantum ideas, in his extraordinary paper 'On a Heuristic Point of View Concerning the Production and Transformation of Light'. Einstein wrote, "the energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which ... can only be produced and absorbed as complete units" (these units, later on, came to be known as photons). Einstein used his theory to successfully explain the mysteries of the photoelectric effect. He posited that a single photon could be absorbed by a single electron which gains all the photon energy. If the electron is close to the metal surface, some of its newly gained energy will be used to overcome surface electrical forces, with the amount of energy required (denoted by ϕ) being a property of the metal itself. The emitted electrons will have a range of kinetic energies, as some electrons may be dislodged from beneath the surface while others may originate exactly at the surface (and would thus have maximum kinetic energy).

clear atom in 1911, the key step was taken by Bohr in 1913 – he found a way to use Planck's constant to quantize the motion of electrons within atoms, using the concepts of allowed discrete stationary states and transitions between them. This allowed him to explain the discrete lines of the Balmer spectrum of hydrogen.

This work initiated by Bohr led to what was later called the period of the Old Quantum Theory. It lasted till about 1923–24. There were initial successes, with important contributions from

Figure 4. (from left) Niels Bohr (1885–1962), Arnold Sommerfeld (1868–1951), Max Born (1882–1970).

Arnold Sommerfeld, Paul Epstein, Max Born and Einstein. But soon difficulties surfaced, the years 1923–24 becoming the 'period of crisis' of the old quantum theory. In 1924 Born used the name 'quantum mechanics' for a theory yet to be developed, and made an amazingly prophetic statement – in this theory, the basic quantities would refer to pairs of states, rather than to one state of motion at a time.

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Also in mid-1924 Satyendra Nath Bose found a new derivation of Planck's law using a new statistical method for photons. Einstein immediately saw the importance of Bose's work, generalized it to matter, and showed that matter too was subject to wave-particle duality. Now, however, it was a combination of classical particle nature and quantum wave nature.

5. The Coming of Quantum Mechanics

Now we come to the major step – the discovery of quantum mechanics. This is a story of great drama, starting in May–June 1925 and essentially concluding by mid-1926. The conventional interpretation was in hand by late 1927.

The first step was taken by Werner Heisenberg, earlier a student of Sommerfeld in Munich, then working with Max Born in Göttingen. His basic new insight was that the orbit of an electron in an atom is not directly observable, so one should not talk about it. A new theory should be based only on observable quantities, and these are the rates or strengths of transitions between pairs of stationary states. Recall the prediction Born had made in 1924.

Figure 5. Photo of Davisson (left) and Germer (right) with the vacuum tube used in their famous electron diffraction experiment. Image from Emilio Segrë Visual Archives – American Institute of Physics.

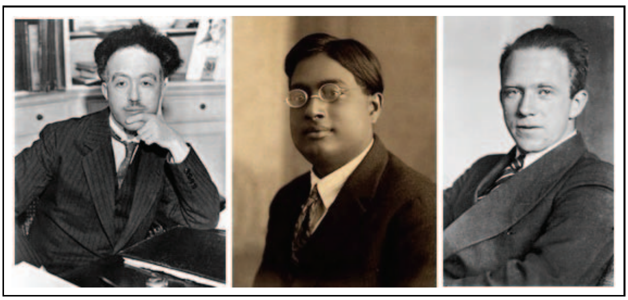

Figure 6. (from left) Louis de Broglie (1892–1987), Satyendranath Bose (1894–1974), Werner Karl Heisenberg (1901–1976).

In May–June 1925, Heisenberg had a severe attack of hay fever, so he went away for a few weeks to a grassless island – Heligoland – to recover. It was in this period that he formulated what is called the 'matrix mechanics'. The story is told in breathtaking fashion in one of the essays in his book *Physics and Beyond* published in English in 1971.

Heisenberg's inspired idea was that electron position and electron momentum, classically treated as real numerical quantities – should be represented by *arrays*, one entry for each possible transition between a pair of states. So position q is an array, momen-

Box 3. Experiment that Proved the de Broglie Hypothesis

The observation of the diffraction of electron waves by C J Davisson and L H Germer in 1927 was the first measurement of the wavelength of the electron. Their experiment was also the first to directly verify the de Broglie hypothesis that quantum mechanical particles can exhibit wave-like properties. Davisson got the Nobel Prize in Physics in 1937 for this work, along with George P Thomson from the University of Aberdeen, Scotland, who independently found evidence for electron diffraction. What is remarkable is that Davisson and Germer were working at the Bell Research Laboratories, doing materials science research on vacuum tube filaments, without any inputs from theoretical physicists. They were looking at the reflection of electron beams from nickel crystals. When the electron beam hits the crystal, nickel atoms scatter electrons in all directions. A detector measured the intensity of the scattered electrons with respect to the incident electron beam. The detector could be rotated to detect electrons being scattered at different angles. For usual polycrystalline samples, they got a smooth angular distribution of scattered electrons. In early 1925, one of their samples was accidentally re-crystallized in a lab accident, which made its structure monocrystalline. They found that for this sample, there was a peak in intensity of the scattered electron beam at a certain angle.

In 1926, Davisson attended the lectures by Max Born in the meeting of the British Association for the Advancement of Science at Oxford, England, and grasped concepts of de Broglie's wave-particle duality hypothesis and the Schroedinger wave mechanics. Davisson and Germer used Bragg's law and the known spacing of atoms in the single crystal of nickel to calculate the electron wavelength that would have produced this maximum scattering angle and found that this measured wavelength agreed very well within experimental error with the de Broglie wavelength of the electron. They varied the energy of the electrons (and hence the electron wavelength) and produced maxima at different angles and showed that in all these cases the experimentally determined wavelengths agreed with the de Broglie wavelength of the electron. For other monocrystalline samples, they found similar patterns, depending on the angle of incidence, sample orientation and chemical constitution.

tum p is another array. Now how do we compose or multiply such arrays? Classically, the square of a number or the product of two numbers is again a number. For an array q, what do we mean by its square, q^2? Heisenberg found a rule or procedure to 'multiply' two arrays to produce a third array. This was a new rule of multiplication, based on the Ritz combination law of spectroscopy. At that time, Heisenberg was ignorant of matrices; it was Born who later recognized and told him his arrays were matrices, and his rule of multiplication was matrix multiplication. Fortunately, Born had learnt these things in his student days.

Figure 7. (from left)Paul Dirac (1902–1984), Erwin Schrödinger (1887–1961), Wolfgang Pauli (1900–1958).

Heisenberg then showed that the Bohr quantum conditions led to the diagonal elements of the commutation relation $qp - pq = i\hbar I$. This noncommutativity, $qp \neq pq$, was new in physics, and a cause of great concern to Heisenberg.

Soon after, in summer 1925, Heisenberg gave a seminar in Berlin on his work. Einstein was present and asked searching questions. Their discussions continued at Einstein's home. Heisenberg said that his insistence on using only observable quantities was inspired by Einstein's own special relativity, to which Einstein responded – that may well be so, but it is the theory that decides which quantities are observable.

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In July 1925, Heisenberg visited Cambridge and gave a seminar with the title 'Term Zoology and Zeeman Botany'. His host was R H Fowler, and at that time Paul Adrian Maurice Dirac was his student. Dirac seems to have missed Heisenberg's seminar. Later, Heisenberg described his work on quantum mechanics to Fowler, and in September sent him the proof sheets of his paper. Fowler gave them to Dirac to have a look. At first, Dirac was not impressed, as his own ideas at the time were very different from Heisenberg's. But soon, after a week, he was suddenly inspired and realized that Heisenberg had made a great breakthrough.

Many years later, in 1972, while introducing Heisenberg at an evening lecture at the International Centre for Theoretical Physics in Trieste, Dirac said: "...we were students working on the same problem at the same time; he succeeded where I failed".

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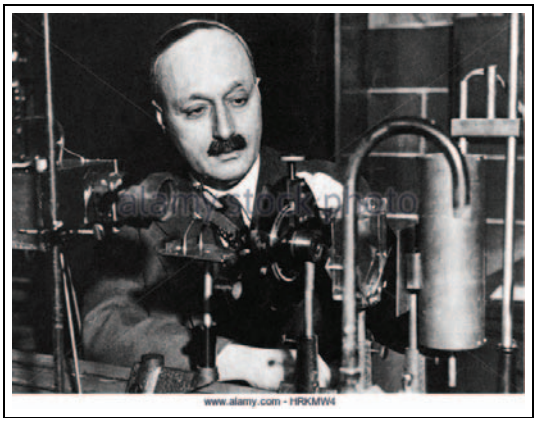

Figure 8. Franck working in his lab on the Franck–Hertz experiment. Alamy Stock Photo (www.alamy.com).

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as a cause for concern but as a basic feature of a new mechanics. He went on to develop his own form of quantum mechanics, more flexible than what Heisenberg had done. He showed that the classical counterpart or limiting form of the commutator of two quantities was their Poisson Bracket which appears in the Hamiltonian form of classical mechanics. Thanks to Fowler, Dirac's papers were published very quickly before the end of 1925, to signify a major contribution from England. Unlike Heisenberg, who had strong collaborations with Born and Pascual Jordan, all of Dirac's work was done in isolation.

The third important step was the creation of wave mechanics by Erwin Schrödinger in Zurich. His sources of inspiration were Louis de Broglie's concept of waves associated with matter (*Figure* 5); Einstein's remarks on de Broglie's thesis and his own discovery of wave-particle duality for matter; and a remark by Peter Debye in a seminar that all these should be related to eigenvalue problems of some differential equations. Over the 1925–26 Christmas vacation Schrödinger secluded himself (with a friend) in a Swiss chalet, and began the development of his wave mechanics. It resulted in a series of six landmark papers concluding in mid-1926.

Many fundamental features emerged from Schrödinger's work: The concept of a general state of a quantum system described by a complex wave function ψ; the time dependent equation of

Box 4. Experimental Validation of Bohr's Atomic Model

In 1914 James Franck and Gustav Hertz (nephew of Heinrich Hertz) performed an experiment which is now widely regarded as an experimental validation of Bohr's quantum theory for atoms and for which they received the Nobel Prize in 1925 (*Figure* 8). However, they were not looking for an experiment to test Bohr's atomic model and in fact they were not even aware of Bohr's theory at the time they performed their experiments!

Their setup consists of a cathode which can be heated to emit electrons; the cathode tube is filled with mercury vapor maintained at a temperature of 120°C and a vapor pressure of one mm. The emitted electrons are collected at a platinum anode after passing through a grid (a mesh screen). A voltage V is applied between the cathode and the grid to accelerate the electrons toward the grid, while a small retarding voltage is applied between the grid and anode to prevent electrons with very low energy from reaching the anode. Franck and Hertz observed that increasing the voltage V, led to an increase in the anode current, implying that the electrons passed through the mercury vapor without any loss in energy (due to elastic collisions between the electrons and the mercury atoms). However, they noticed that when the voltage V reached $\approx$ 4.9 V, the current suddenly decreased to nearly zero! This meant that at this particular value, an electron has lost all of its energy due to an inelastic collision with a mercury atom and can no longer overcome the retarding voltage and reach the anode.

Franck and Hertz wrote a paper titled 'Collisions between electrons and mercury vapor molecules and the ionization potential of such molecules' where they did not correctly interpret their experimental results – they believed that collisions between electrons and mercury atoms in their setup was ionizing the mercury atoms. It was left to other physicists to realize that the collisions in their setup were exciting the mercury atoms from the ground (n = 1) to the first excited (n = 2) quantum state, and that their experiment actually confirmed Bohr's model of the atom having allowed transitions only between discrete energy levels. The Franck–Hertz experiment was so breathtaking that Einstein after listening to a lecture by Franck, turned to Lise Meitner who was sitting alongside and said, "It's so lovely it makes you cry".

motion for ψ, as a replacement for the Newtonian equations of motion in classical mechanics; and the superposition principle. In all these respects, it went beyond what Heisenberg and Dirac had accomplished. Schrödinger also showed the equivalence of matrix and wave mechanics in a paper titled 'On the Relation of the Heisenberg–Born–Jordan Quantum Mechanics to Mine'!

It is interesting to read what Heisenberg and Schrödinger initially thought about each other's work. Schrödinger said he felt 'repelled' by the fact that matrix mechanics was so abstract and did not lead to any mental picture or visualization of what was going

Figure 9. (from left) John von Neumann (1903–1957), David Bohm (1917–1992).

on. And in a letter to Wolfgang Pauli, Heisenberg described wave mechanics as ‘bull shit’.

6. The Interpretation of Quantum Mechanics: 1927–1930

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So it was that after Heisenberg’s initial breakthrough, over just about a year the mathematical structure of quantum mechanics was worked out. Then came the problem of understanding what it all meant. This required intense efforts, in which Bohr played the role of an elder. Here is a sketch of the main steps more or less in sequence.

Schrödinger initially thought his complex wave function for a particle was one more field in physics, like the Maxwell field. However, this did not last, and in June 1926 came the Born interpretation: ψ is a probability amplitude, its squared modulus is a probability density.

There followed intense and exhausting discussions among Bohr, Heisenberg and Pauli at Bohr’s Institute in Copenhagen. Schrödinger also visited Bohr in this period. He disagreed with Bohr on the need for quantum jumps or the feature of discontinuity in quantum phenomena. His hope was that with his wave mechanics all discontinuities could be avoided. Bohr would not accept this, and finally, Schrödinger said: “If these quantum jumps remain, I am sorry I had anything to do with quantum mechanics.”

To which Bohr responded: 'But we are thankful to you all the same."

Later in 1927, after a deadlock in their discussions, Bohr and Heisenberg decided to have a break. Bohr went skiing in Norway, while Heisenberg remained in Copenhagen. Separately they respectively formulated the complementarity and the uncertainty principles. When Bohr returned from Norway, initially each was unhappy with the other's work, then came a reconciliation. All this is very well recounted by Heisenberg in *Physics and Beyond*.

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According to Bohr's Complementarity Principle, all the quantitative physical properties of a quantum system are not simultaneously accessible for measurement. Results of experiments cannot be viewed on their own detached from the corresponding experiments but are tied to them. Many mutually exclusive views are needed to build up a whole picture of the state of a quantum system. In later years, Bohr tried to extend such ideas to disciplines outside of physics, such as biology and psychology.

As for Heisenberg's uncertainty principle, as is generally quite well known, so-called mutually incompatible or conjugate quantities like particle position q and momentum p cannot be simultaneously measured with unlimited accuracies. Greater precision in one leads to less precision in the other. Heisenberg's original intuitive derivation was soon improved by Weyl, Robertson and Schrödinger. Somewhat later came Bohr's time-energy uncertainty principle, both physically and mathematically on a different footing than the one for position-momentum.

Out of all this emerged the so-called standard or Copenhagen interpretation of quantum mechanics, which is generally accepted by most working physicists.

About the measurement process, Bohr's view was that even if the system is quantum mechanical, the apparatus has to be treated as classical. The reason was that the only language available to describe and communicate the results of the experiments is that of classical physics. Later came the von Neumann point of view: both system and apparatus are quantum mechanical, parts of a

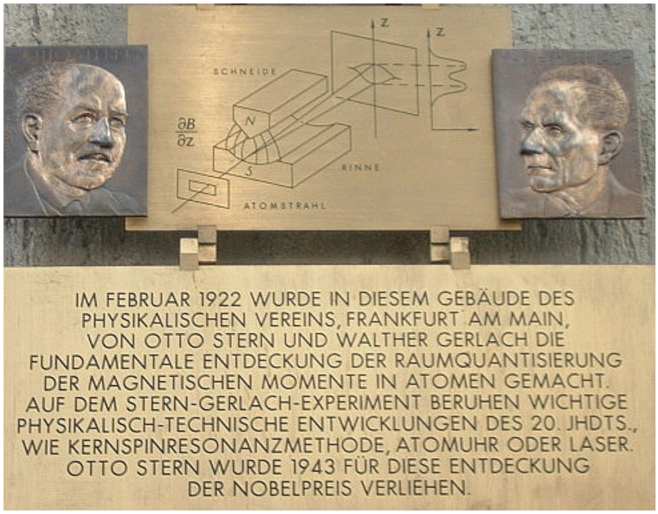

Figure 10. Plaque at the University of Frankfurt, Germany, commemorating the Stern–Gerlach experiment.

composite system obeying the overall Schrödinger equation including interaction. But then the idea of collapse is needed to arrive at the Born probability interpretation.

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Two important 'Solvay Conferences' followed in 1927 and 1930. In the first, Einstein suggested an experiment to violate the position-momentum uncertainty principle. In the second, he suggested an experiment to 'beat' the other time-energy uncertainty principle. On both occasions, Bohr succeeded in answering him and showing the internal consistency of quantum mechanics with its standard interpretation.

7. Developments After 1930

After these exchanges, Einstein came to the view that while quantum mechanics was *correct*, it was *incomplete* – something about Nature was missing and not captured by quantum mechanics. All his life, he remained unhappy with the standard interpretation, as were de Broglie and Schrödinger. Then in 1930, von Neumann proved a theorem on the basis of certain assumptions which seemed reasonable to him: there is no way to extend quantum mechanics to a larger or more complete theory by including new 'hidden variables' while retaining consistency.

In 1935, Einstein with Boris Podolsky and Nathan Rosen came back to his criticisms of quantum mechanics and proposed an experiment to show that it is incomplete. This is the well-known 'EPR paradox' paper, and it drew an immediate criticism and response from Bohr. EPR argued that action-at-a-distance does not exist and stated that for a theory of physics to be complete, it should include the concepts of local causality and physical reality. They devised a thought experiment and claimed that quantum mechanics is an incomplete description of physical reality, since it allows for a 'spooky action-at-a-distance' wherein making a measurement on one system immediately influences the other system which is physically at a distance. It is interesting that EPR did not refer at all to von Neumann's theorem. What they did, in fact, was to highlight a feature of nonlocality in space implicit in the standard interpretation. Bohr in his rebuttal to the EPR paradox defended quantum mechanics with his principle of complementarity and argued that the requirement of realism is not applicable at the microscopic level. Soon Schrödinger joined the fray with papers describing what he called 'entanglement' – a property of general states of composite quantum systems which leads to nonlocality. In fact, he declared that this was the most important difference between the classical and quantum situations.

Then in 1952, apparently unaware of von Neumann's theorem but pursuing the early ideas of de Broglie, David Bohm constructed a classical looking hidden variables interpretation of nonrelativistic quantum mechanics. This brought back Einstein's realist point of view, but at the cost of extreme nonlocality. In 1964 John Bell examined the whole problem very critically. He was able to pinpoint the von Neumann assumption which was not compelling and could be dispensed with. He then derived a set of inequalities which could discriminate between quantum mechanical predictions and those of any local realist hidden variable alternative of the kind Einstein was hoping for.

On the experimental side, tests of the Bell inequalities have invariably favoured standard quantum mechanics. While the first experimental tests using entangled photons were performed by Freedman and Clauser, probably the most famous experiments are those of Aspect. The net result is that a 'completion' of quantum mechanics of the type hoped for by Einstein is not possible.

This work of Bohm followed by Bell has been a very major development. Over the past half-century or so it has inspired an enormous amount of work on the foundations of quantum mechanics. The French experimental physicist Alain Aspect has suggested that quantum mechanics consists of two revolutions – the first, from 1909 to 1930, involving wave-particle duality; and the second, especially since the 1960's, involving entanglement.

On the experimental side, tests of the Bell inequalities have invariably favoured standard quantum mechanics. While the first experimental tests using entangled photons were performed by Freedman and Clauser, probably the most famous experiments are those of Aspect. The net result is that a 'completion' of quantum mechanics of the type hoped for by Einstein is not possible. Each attempt to do so leads to fresh undesirable features, the 'cure' being always worse than the 'disease'. All attempts to force quantum mechanics into something like the framework of the earlier classical physics end up being unreasonable in some way. It is in these discussions that the ideas of realism, locality, entanglement, contextuality, etc., come up prominently. However, in the currently very active field of 'Quantum Information Science and Technology', all these 'problems' are made use of and treated as 'resources'.

8. Later Work of Heisenberg, Dirac and Schrödinger

The three principal creators of quantum mechanics all received their Nobel Prizes in 1933 – Heisenberg for 1932, Dirac and Schrödinger jointly for 1933.

In later years, Heisenberg worked in nuclear physics, showing the validity of quantum mechanics at distances and energy scales differing by about six orders of magnitude from the atomic domain. The concepts of the nucleon, isotopic spin, internal symmetry and the S-matrix, are all due to him. He had deep knowledge of Greek science, classical music and Western philosophy, and great pride

Box 5. Experiment that Proved Angular Momentum is Quantized

The Stern–Gerlach (SG) experiment is considered a landmark experiment in quantum mechanics as it provided the first direct evidence that angular momentum is quantized in quantum systems in units proportional to Planck's constant (*Figure* 10). To recapitulate some of the historical developments leading to the SG experiment, Otto Stern was the first postdoctoral student of Einstein, first at Prague, then Zurich and later at Frankfurt. He served in the German army during World War I and returned to Frankfurt to become an assistant to Max Born at the Institute for Theoretical Physics and began working on molecular beams. Walther Gerlach received his PhD in 1912 for work on blackbody radiation and the photoelectric effect. In 1920 he was appointed as assistant in the Institute of Experimental Physics at Frankfurt, to work on the deflection of a beam of bismuth atoms in a strongly inhomogeneous magnetic field. His motto was "No experiment is so dumb, that it should not be tried"! In 1921 the problem of the 'anomalous' Zeeman effect (dealing with the splitting patterns of spectral lines in a magnetic field) still perplexed quantum physicists. Stern wanted to experimentally observe the property of space quantization of the Bohr model which predicted that a gas of hydrogenic atoms would be magnetically birefringent, i.e., that space quantization would be only twofold and the atomic beam would split into two, as the projection of the orbital angular momentum would be limited to $\pm h/2\pi$. Classical mechanics, on the other hand, predicted that the beam of atoms, when deflected by a magnetic field, would broaden but would not split. The experimental setup took over a year to finalize, and in 1922 Stern and Gerlach embarked upon their experiments. The setup consisted of a collimated beam of silver atoms produced from an oven and that passed through an inhomogeneous magnetic field of field strength 0.1 tesla and gradient 10 tesla/cm. The resultant splitting of the atomic beam was around 0.2 mm, and a very thin film of silver atoms would be deposited on a collector plate. In a further twist to the tale, Stern and Gerlach did not see any trace of the silver atom beam when they first did the experiment – then with Stern puffing a cigar and looking over Gerlach's shoulder, they gradually saw traces of the beam emerge! They realized that Stern was puffing a cheap cigar with a lot of sulphur content (since he could not afford good cigars on an assistant professor's salary!) and his exhaled breath on the plate turned the silver into silver sulfide which is black and hence easily visible! The SG experiment showed a doublet splitting for silver atoms which is due to the spin magnetic moment of the electron. This result seems almost a routine outcome of quantum mechanics today, but at that time it was both puzzling and exciting. Quantum physicists soon realized that when a system's angular momentum becomes so small that it becomes comparable to Planck's constant, only discrete states of directional quantization can exist.

in German culture. In the early years, he was very close to Bohr. During the tragic World War II period, when many European scientists emigrated to the USA and elsewhere, Heisenberg – like the much older Planck – stayed on in Germany. He seems to have felt that he should be on the scene when the period of reconstruction would come. He visited Bohr in Copenhagen in 1941, by which time their relationship had been broken. A fictional recreation of this visit was attempted in Michael Frayn's play 'Copenhagen' written in 1998. Heisenberg's role and relations with the Nazis have been the matter of much speculation.

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Schrödinger worked mainly on general relativity and classical unified field theories. This was of course in addition to his 1935 work on entanglement. He was about the same age as Bohr, and some fifteen years older than Heisenberg and Dirac. Like Heisenberg, Schrödinger too had a deep appreciation of Greek science, Western philosophy as well as of Vedanta. He wrote most eloquently in English. Some of his best-known works are *Mind and Matter*, *What is life?* and *My View of the World*.

Dirac, the youngest of the three, did an enormous amount of work of very great significance after quantum mechanics. In 1927 came the quantization of the Maxwell field and the inauguration of quantum field theory. Next, in 1928 he discovered the relativistic electron wave equation, which 'explained' electron spin, anomalous magnetic moment, and the fine structure of hydrogen. Through this wave equation, spinors entered physics in a fundamental way. In 1931, along with the idea of magnetic monopoles, he predicted the positron and the general conception of antimatter. And in 1934 he found a role for the classical Lagrangian in quantum mechanics, which then led in Richard Feynman's hands to the path integral form of quantum mechanics.

In contrast to Heisenberg and Schrödinger, Dirac had very little interest in philosophical matters. In a recent piece, Freeman Dyson wrote that "Dirac refused to engage in philosophical arguments about the interpretation of quantum mechanics... Dirac took no part in these debates and considered them to be meaningless... Human language describes the world of everyday life, and lacks the concepts that could describe quantum processes accurately. Dirac said we should stop arguing about words, stay with mathematics, and allow the philosophical fog to blow away."

Dirac was an exceptionally fine human being. Of him, Bohr said "he had among all physicists, the purest soul."

Figure 11. (from left) John Bell (1928–1990), Richard Feynman (1918–1988).

9. A New World Opens Up

Quantum mechanics has been applied to physical phenomena at different scales and has led us to discover the world in the domain of fundamental particles on the one hand, and collective quantum behaviour on the other. Quantum field theory and related developments have kept physicists busy during much of the 20th century, discovering fundamental particles and their interactions, which is often called the 'standard model' of particle physics. The universe which is made of quarks and leptons and their interactions is governed by quantum field theories of the appropriate kind.

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Without quantum mechanics, we cannot understand the properties of the material world: why some materials are conductors, while others are insulators, and how the properties of materials change with temperature and so on. When applied to a collection of particles, quantum theory successfully explains exotic phenomena such as superconductivity and superfluidity, wherein the first case electrons and in the second case atoms, can 'flow' around obstacles without getting scattered. Even now major research efforts are on to be able to understand many-body quantum phenomena. The entire world of electronic devices has opened up due to our ability to understand the properties of materials using the principles of quantum mechanics.

Quantum mechanics has been successful in explaining objects

Box 6. Experimental Tests of Bell's Inequalities

One of the first experimental tests of Bell's theorem was performed by Stuart J Freedman and John F Clauser in 1972, using entangled photons. They measured two photons (of 551 nm and 422 nm) emitted during the cascade decay of calcium atoms in an excited state. Photons were detected using photon-multiplier detectors, and their coincidence rate was measured electronically. The measurement for each photon was like a binary selection, i.e., transmission or no-transmission. They assumed that the two photons propagate as separated localized particles and that the probability of detecting a photon is independent of the transmission of the polarizers. Their experiment showed a clear violation of Bell's inequality, and their results hence showed a good agreement with the predictions of quantum mechanics. Their experiment confirms that a pair of entangled photons separated in space must still be considered to be a single entity since it is impossible to assign a local physical reality to each of the photons in this entangled pair. Later, the famous French physicist Alain Aspect built upon these experiments and in 1982, along with Philippe Grangier, Gerard Roger, and Jean Dalibard, performed a key experiment that showed a clear violation of Bell's inequalities and vindicated quantum theory.

from the scale of neutron stars down to quarks, spanning many orders of length and time scales. So far quantum mechanics has not been contradicted by any experimental findings. Serious efforts are on to understand more and more about the world around us using quantum mechanics and quantum field theory. With the advent of modern computers, very complex quantum systems can be simulated leading to predictions about their properties and behaviour.

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While the twentieth century can be called the century of quantum physics, parallelly, the science of information and computing has developed enormously. The end of the twentieth and the beginning of the twenty-first centuries have seen the emergence of a new research area, where quantum ideas are being applied to computing and information science. A new paradigm of computing has emerged called 'quantum computing', in which properties of quantum superposition, quantum entanglement and unitary quantum evolution have been exploited to show that quantum computers once built, can solve problems that are impossible to solve on the most powerful classical computers. In the domain of secure communication, it has been shown that communication of data can be made fundamentally secure against attack only when carried out via quantum channels. Serious efforts are on to build quantum devices for quantum communication and quantum computing. This area is expected to occupy the centrestage of scientific research in the twenty-first century.

10. Concluding Remarks

Now for some concluding remarks. Quantum mechanics grew out of an essentially European effort, centred in a few major places. In Munich, the work was led by Sommerfeld, who had a very physical approach to problems. Both Heisenberg and Pauli were his students. In Göttingen the leader was Max Born. Probably influenced by the presence of David Hilbert, the overall style was more formal and mathematical. Jordan was his student and Heisenberg his assistant. In Copenhagen, the leader was Bohr, who had many visitors and close collaborators. The approach was more philosophical, with concern for interpretation, the role and limitations of language, and such matters.

There was a great deal of interaction among these centres. And then there was Schrödinger in Zurich, in frequent contact with the other European centres; and Dirac rather isolated in Cambridge in England. But after his initial work, Dirac too visited Göttingen and Copenhagen and became a member of the group around Bohr.

Quantum mechanics is a very rich story, the work of many hands touching deep philosophical issues. In the words of Abraham Pais, the acclaimed biographer of both Einstein and Bohr, it is "a uniquely 20th century mode of thought". There is a very large number of excellent books that trace the growth of quantum mechanics in rich historical detail. Hopefully, this brief account successfully conveys the important features of the subject and prepares the reader for more exhaustive accounts.

Acknowledgement

All images unless otherwise stated are taken from Wikimedia Commons. N Mukunda thanks the Indian National Science Academy for enabling this work through the INSA Distinguished Professorship.

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