Pramāna – J. Phys., Vol. 32, No. 1, January 1989, pp. L77–L81. © Printed in India.

On the perfect penrose tiling and its basic building blocks

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MS received 12 September 1988; revised 10 November 1988

Abstract. We show that the basic building blocks of a perfect Penrose pattern (PPT) in two dimensions can be established by adding another condition to the Penrose's original edge rules. The implications of this result are discussed in the context of recent papers by Onada, Jaric, Ronchetti and others concerning growth algorithm for PPT's.

Keywords. Quasicrystals; Penrose-tiling.

PACS Nos 61·50; 61·55; 64·70

1. Introduction

Since the discovery of quasicrystals (Shechtman *et al* 1984; Levine and Steinhardt 1986), there has been a discussion on whether or not purely "local" growth rules can lead to perfect quasi-periodic structures or certain "global" rules are essential (Henley 1987). This issue is of importance since questions about the physical realization of such structures could be raised if global rules were found essential. In a recent paper Onada *et al* (1988) have examined this issue in the context of (arrowed rhombus) Penrose pattern (Penrose 1974) in two dimensions and claimed to have found a growth algorithm "for aggregation of Penrose tiles to form an infinite defect-free perfect Penrose tiling (PPT)" by use of *local rules alone*. However, their claim has been contested by Jaric and Ronchetti (1988) who have pointed out that in the growth rules of Onada *et al* *nonlocality is certainly implied*. Specifically, since in the procedure of Onada *et al* attaching a tile at a site requires an *examination of the entire surface* Jaric and Ronchetti assert that their growth algorithm in ultimately (and inevitably) non-local.

Interestingly, in their algorithm, Onada *et al* (1988) have imposed the condition that in the growth of any cluster only a restricted set of "eight vertex configurations" are allowed to occur. There is, however, no discussion regarding why only these eight configurations arise.

Here we will show that if on placement of Penrose's rhombi we add another condition to the Penrose's original "matching edge rules" which ensures indefinite continuity, then we naturally end up with the eight configurations of Onada *et al* (1988). The added condition on the placement of Penrose's rhombi around a point is as follows. When we fit all the rhombi around a point O, none of the terminal points P_1, $P_2,\ldots$ etc. of all the bonds OP_1, $OP_2,\ldots$ etc. (connected to O) should end up as a "dead end". (By a "dead end" we mean a site where one cannot affix a rhombus in a defect-free manner so further growth at that site is arrested.) In other words, we exclude configurations involving even a single "dead end" amongst P_1, $P_2,\ldots$. The proof of our proposition is presented in § 2 and the conclusions in § 3.