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THREE DIMENSIONAL ELASTICITY SOLUTIONS OF SANDWICH PANELS USING THE EXTENDED KANTOROVICH METHOD

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Summary

In year 1968, Kerr [1] proposed a very elegant method, called the extended Kantorovich method (EKM), to obtain semi-analytical solutions of two dimensional (2D) problems involving bivariate partial differential equations. In this method, approximate solution of the bivariate function w_m *is assumed in the form* $w_m(x,y)=\sum_{n=1}^{m} f_n(x)g_n(y)$ (1)

where $f_n(x)$ *are unknown functions of one of the independent variable, and* $g_n(y)$ *are a priori assumed functions of the other independent variable. Now substituting Eq.(1) into the variational equation and applying variational calculus yields m ordinary differential equations (ODEs) for the determination of m unknowns for the* $f_n(x)$. *In the next step, the obtained functions* $f_n(x)$ *are considered as a priori known functions and the* $g_n(y)$ *functions are considered unknown to be determined. This iterative process is repeated until the result converges to any desired degree. This method thus reduces the PDEs of the 2D problem to a double set of ODEs. This method by far outperform the other approximate methods such as the Ritz and Galerkin methods, since the initial assumed functions are not required to satisfy the geometric or natural boundary conditions and the solution based on the iterative process which converges very rapidly and the final solution is independent of the initial choice of function. Since then, it has been used to analyse many plates and shells problems based on the 2D theories. In these 2D theories, the 3D elasticity equations are reduced to 2D by making a priori assumptions of the distribution of the displacements and stresses across the thickness direction. These include the free vibration of rectangular clamped membrane [2], rectangular clamped thin isotropic [3], and orthotropic plates [4], bending of rectangular orthotropic plates [5], thin annular sector plates [6], and buckling of thin rectangular symmetric layered plates [7]. The method has also been employed for multi-variable problem involving the deflection and rotation variables for the bending of isotropic rectangular Reissner plates [8, 10]. Thus, all the problems solved so far using the EKM pertain to the 2D theory based solutions of plates and shells and concern with homogeneous boundary conditions. The 3D elasticity formulations of laminated plates and shells of infinite length also involve bivariate functions, but surprisingly no work has been reported so far on the use of* *the EKM to such problems. In the 3D elasticity solution of plates and shell problem, the distribution of displacement and stress fields are not assumed apriori but are obtained by solving the equation of 3D elasticity. Analytical 3D elasticity solutions of laminated infinite panels under arbitrary boundary conditions are of great importance not only in their own right, but also for assessing the accuracy of 2D theories. In this work, EKM is employed to obtain the 3D elasticity solution for the static response of transversely loaded angle-ply sandwich panels in cylindrical bending with arbitrary end condition. Unlike the 2D plate theories, solution needs to satisfy non-homogeneous boundary conditions in the thickness direction due to the applied pressure loading and also the boundary conditions involving both displacements and stresses. A single-term solution in the separable form is superimposed with a known solution which satisfies the non-homogenous boundary conditions. The Reissner-type variational principle is used to develop a mixed formulation in terms displacements as well as stress components as primary variables. This approach allows for exact satisfaction of boundary conditions at all points, and also ensures the same order of accuracy for all displacements and stress variables. Applying the variational process along the inplane (x) and the thickness (z) directions, yields a system of 6 differential and 2 algebraic equations in terms of x and a similar set of equations in z for each layer. Exact closed form solutions are obtained for each algebraic differential system of equations, satisfying exactly the interface continuity and/or the boundary conditions. In the first iteration, initial trial functions are assumed in x-direction and functions of z are solved exactly, which are taken as known apriori in the next step to determine the functions in x. This process is repeated till convergence is achieved. Numerical results are presented for thin and thick cross-ply and angle-ply sandwich panels under simply supported and non-simply supported boundary conditions, and the accuracy of the method are established by direct comparisons with the 3D exact elasticity solution and the 3D FE solution, respectively.*

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