Online detection of impending instability in a combustion system using tools from symbolic time series analysis

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ABSTRACT

In this paper, we introduce a novel technique (anomaly detection) for the online detection of impending instability in a combustion system based on symbolic time series analysis. The experimental results presented in this paper illustrate the application of anomaly detection to a combustor in which the flame is stabilized either by a bluff body or by a swirler. The detection unit works on the principle that in the transition region from combustion noise to thermoacoustic instability, combustion systems exhibit peculiar dynamics which results in the formation of specific patterns in the time series. Further, tools from symbolic time series analysis is used to recognize these patterns and then define an anomaly measure indicative of the proximity of system to regimes of thermoacoustic instability.

Keywords: Symbolic time series analysis; Anomaly detection; Instability detection; Probabilistic finite state automata.

1. INTRODUCTION

One of the techniques used in many combustors to reduce emission is to employ ultra-lean combustion which ensures low combustion temperature and consequently reduced NO_x emission. However, in a confined environment such as a combustion chamber, ultra lean combustion can lead to thermoacoustic instability characterized by high amplitude pressure fluctuations. In systems such as gas turbine engines and rocket motors, these high amplitude pressure fluctuations are a major concern considering the cyclic loads that the structural components of the engine or rocket motor has to support during the instability [1]. Often, instability can lead to a permanent failure of the structure of the engine or at least reduce its life time by a significant amount. Hence, a methodology to prevent the system from entering regimes of instability or forewarn the operator about an impending instability is essential in order to ensure improved performance of combustion systems.

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In the past, various techniques were introduced in order to detect and control instabilities in combustion systems. Poinsot et al. [2], introduced a technique to control instabilities in gas turbine engines. In this technique, the pressure fluctuation in the combustion chamber is measured and a delayed signal (control signal) is generated based on the pressure fluctuation signal, which in turn is used to modulate the fuel pressure inside the fuel line. By selecting an appropriate delay for the control signal, they were able to actively control the instability. Hobson et al. [3] analyzed the stability of industrial gas turbine engines by monitoring the casing vibration and the pressure fluctuations inside the combustion chamber. They analyzed the stability of the engine in terms of frequency and bandwidth of the principal peak in the vibration and pressure spectra. It was observed that as the system approached the stability limit, the bandwidth of the principal peak decreased. This was an indication for damping approaching zero near the stability limit. In a similar way, Lieuwen [4] used autocorrelation of the pressure signals inside the combustor to characterize the damping of the system and thereby predict the stability margin. The techniques used above rely on detecting the characteristics of the instability to detect it. In our work we were interested in understanding the dynamics of the transition regime from combustion noise to combustion instability in order to come up with a precursor which can forewarn the operators about an impending instability.

Previously, there have been a few studies that focused on the dynamics of the regime of transition from stable to unstable operation of a combustor. Chakravarthy et al. [5] in an experimental study suggested that a lock on mechanism between vortex shedding and duct acoustics was responsible for the transition. When the vortex shedding and the acoustic modes are not locked on, the system has low amplitude broadband noise and once the lock-on happens, the system has high amplitude tonal oscillations. Further, Gotoda et al. [6] reported that in a turbulent combustor, as the equivalence ratio is varied from stoichiometric to lean limits, transition from stochastic fluctuations(combustion noise) to periodic oscillation happened through a state of low dimensional chaos.

Recently, Nair et al. [7] studied the characteristics of the transition region using tools from dynamical system theory. They established that combustion noise resulting from combustion of a turbulent flow in a confined enviornment is not stochastic and is in fact chaos of moderately high dimension (of the order of 8 to 10). Further, it was shown that the onset of instability is essentially a transition from chaos to order. They used a test for chaos, known as 0–1 test for chaos [8], as a measure of the proximity of the combustor to an impending instability. Nair et al. [7] also indicated that the transition from chaos to order happened through intermittency [9]. In this context, they introduced recurrence parameters as precursors to the impending instability in a practical gas turbine combustor [10]. In this paper, we introduce a method based on symbolic time series analysis in order to detect the proximity of a particular dynamic state of the combustor to that of instability by analyzing the peculiar patterns in time series data of unsteady pressure fluctuations born out of the dynamics in the transition regime.

Symbolic time series analysis provides a simpler way to analyze the dynamics of nonlinear systems [11]. A symbolic time series can be generated from a time series obtained from experiments or simulations in various ways. The different approaches to construct a symbolic time series is described by Daw et al. [12]. References [13] and [14] describe different methods used to generate a symbolic time series. Further they also describe a procedure for early detection of anomalous behaviors in dynamical systems based on symbolic dynamic filtering (SDF). Application of anomaly detection using SDF was carried out successfully in various physical systems [15, 16]. Gupta et al. [17], Chakraborty et al. [18] and Datta et al. [19] used SDF to detect the extinction events in a mathematical model describing a pulse combustor. Dynamics of a spark ignition engine was characterized by Daw et al. [20] using symbolic time series analysis. Mukhopadhyay et al. [21] used symbolic time series analysis for prediction of lean blow out in laboratory-scale gas turbine combustors.

In this paper, we describe a patent pending methodology to detect the proximity of the dynamic state of a combustor to an impending instability [22]. This methodology uses symbolic time series analysis for identifying the precursor to the impending instability. A detailed description of symbolic time series analysis is given in the following section.

2. SYMBOLIC TIME SERIES ANALYSIS

Symbolic time series analysis is a technique used in order to encode the complex dynamics of a system embedded in a time series signal $\{T[k] | k = 0 \ldots N-1\}$ into a set of finite number of variables. The particular analysis technique used in this paper involves a three-step process. The first step is the generation of the symbolic time series $\{ST[k]\}$ from the actual time series. The second step is the construction of a state vector SV_p corresponding to the symbolic time series, representing the dynamics of the state that is responsible for the generation of the actual time series $\{T[k] | k = 0 \ldots N-1\}$. In the third step, an anomaly measure M is defined which serves as an indicator of the proximity of a dynamical state to the stability margin. Detailed explanations for these processes are given in the following subsections.

2.a. Construction of a symbolic time series

Consider the time series signal $\{T[k] | k = 0 \ldots N-1\}$. This signal is a discrete function in time represented by the N data points, N being the length of the time series signal. The data point at the k^{th} instant is $T[k]$. Each of the data point has a particular value $T[k]$ and a particular time stamp 'k' associated with it. Now in order to construct the symbolic time series from the time series signal $\{T[k] | k = 0 \ldots N-1\}$, the N data points constituting the time series signal are partitioned into a mutually exclusive and exhaustive set of finitely many segments. In this paper, the partitioning is performed by dividing the points into different sets based on the range of the instantaneous value ($T[k]$) in which they lie (illustrated in Fig. 1a, the details on how the upper and lower limit of value of $T[k]$ for each partition is selected and how the number of partitions is decided are described later). This technique of directly partitioning the time series is called 'simple partitioning'. Further descriptions of partitioning techniques are described by Ray [11, 24]. In this paper the number of segments for partition is fixed to be 10.

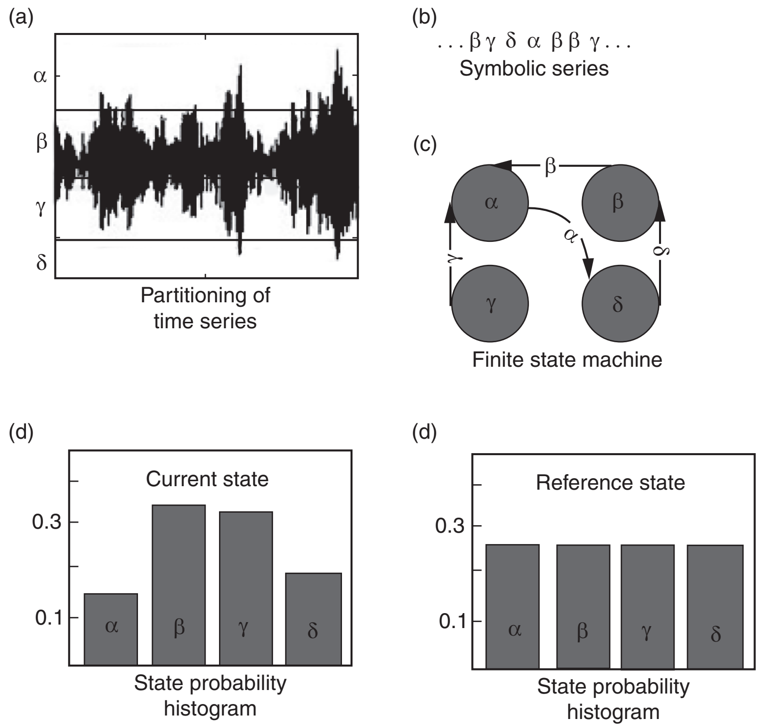

Figure 1: Construction of symbolic state probability histogram from a finite time-time series data (in this paper we use the unsteady pressure time series data from a combustor). Note that the different steps involved are a) Simple partitioning, b) generation of a 'symbolic sequence', c) construction of a 'finite state machine', and d) developing a 'state probability histogram'(representative of state probability vector).

Once the data points are partitioned, each partition is represented by a particular symbol (For the purpose of illustration, assume that we are partitioning the data points into 4 segments, α, β, γ and δ). Now, the value of the time series data at each instant k is replaced by the symbol corresponding to the partition to which that particular data point belongs. Thus a symbolic time series is generated (Fig. 1.b).

2.b. Construction of probabilistic finite state automata(PFSA)[23][24]

Once the symbolic time series $\{ST[k]\}$ is generated, a probabilistic finite time automata is constructed to represent the dynamic state that generated the time series $\{T[k]|k = 0 \ldots N - 1\}$. The main assumption in construction of PFSA is that the symbolic process (represented by the symbolic series) under all conditions can be approximated as a Markov chain of order D (D- Markov machine) representing a quasi-stationary stochastic process. For a D- Markov machine, probability of occurrence of a new symbol depends only on the last D symbols, implying that the memory of the system is only extended up to D past observational instances. With this assumption, the states of the D- Markov machine are essentially represented by a word of length D in the symbol string of the symbolic time series. Hence, for a symbolic series represented by P symbols, the number of possible states in a D- Markov machine is P^D. With increase in the word size D, the memory embedded in the Markov states of the PFSA increases. However, as D increases, the total number of possible states for PFSA increases and hence the computational expenses needed to construct the PFSA also increases [25]. Keeping this in mind, in this paper we have restricted the word size D to 1. Hence, the number of states possible for the PFSA constructed in this paper is P. It is seen from the experiments that with a word size of 1 itself, the predictability of the control system for the instability analysis is quite impressive although $D > 1$ is expected to produce more accurate results at the expense of significant increase in computational effort.

2.c. Construction of the anomaly measure

First step in constructing an anomaly measure is identifying a reference state. The anomaly measure in this particular work is expected to indicate the proximity of the current state to the onset of instability (here, the current state is the state for which the anomaly measure is estimated). Hence, it is only natural to select the dynamical state corresponding to the onset of instability as the reference state. Once the reference state is identified (in the context of this paper, a method for identifying the appropriate reference state, i.e. the state that is considered as the onset of instability/ stability margin is described in Section 4), then the data points in the pressure time series corresponding to the reference state is partitioned into P mutually exclusive and exhaustive segments in such a way that each of the segment contains approximately equal number of data points. The partition technique used here is equiprobable partitioning, which is based on maximization of information entropy, as seen in Fig. 1. The implication is that when PFSA is constructed using this partition, the reference state will have a uniform probability for all symbolic states (P^0 is the state probability vector for the reference state). Once the partitioning of reference state is performed, same partition is used in order to construct the symbolic time series corresponding to the other dynamical states. Hence, when PFSA is constructed for dynamical states other than the reference dynamical state, the probabilities associated with symbolic states have a non-uniform distribution (P^k is the state probability vector for the current state). Thus, an anomaly measure, which is an indicator of proximity of a dynamical state to the reference state, is defined as follows,

$$M = Cos^{-1}\left(\frac{< P^k P^0 >}{||P^k||\,||P^0||}\right)$$

Here, $< P^k P^0 >$ is the inner product of the state probability vectors P^k and P^0 and $||P||$ denotes the Euclidian norm of the vector P. More details of anomaly detection using symbolic logic have been described by Ray [11].

3. EXPERIMENTAL SETUP

The experiments discussed in this paper were performed on two burners, (1) a swirl stabilized burner with an axial swirl generator and (2) a bluff body stabilized burner. In both the configuration, the fuel used is LPG. The fuel and the air get partially mixed before entering into the combustion chamber. The flame is stabilized in the combustor either by the use of a swirler or by the use of a bluff body. Pressure transducers are mounted along the combustion chamber in order to measure the acoustic oscillations. The total length of the combustion chamber is 700 mm, and the cross sectional area is 90×90 sq. mm. The experiments are performed at Reynolds numbers(~20,000) where the flow is turbulent. A more detailed description of the experimental setup and the uncertainties in the measurements is available in [7].

4. RESULTS AND DISCUSSIONS

In this study, the experiments were conducted as follows. For a given value of fuel flow rate (for example, 28 SLPM), the air flow rate was varied in a quasi-steady manner. The initial airflow rate was chosen in such a way that it corresponded to an equivalence ratio of one. Subsequently air flow rate was increased, thereby reducing the equivalence ratio. At each equivalence ratio, the dynamics of the pressure fluctuations inside the combustor was recorded by acquiring the unsteady pressure time trace using a piezoelectric transducer. The time series data of these pressure fluctuations at various air flow rates are then used to perform symbolic time series analysis for anomaly detection. As described in Section 2c, the first step in the analysis is identifying an appropriate reference dynamical state. In order to identify that, we adopt the following procedure. Henceforth, we refer to this procedure as the training drill for online anomaly detection.

The aim of the training drill is to identify that particular pattern of oscillations in the combustor, which is to be considered as the appropriate margin for the operational regime of the combustor such that instability can be avoided. In order to identify such a margin, we apply anomaly detection technique to the normalized time series data

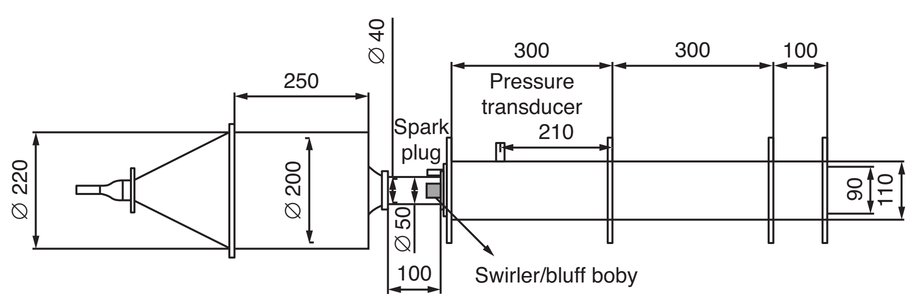

Figure 2: The schematic of experimental setup. Note that a pressure transducer is attached to the combustor to measure the unsteady pressure fluctuations in the combustor.

taking a unit amplitude sinusoidal wave with a frequency approximately equal to the instability frequency (in this context, the instability frequency corresponds to the natural frequency of the combustor at which an instability is expected) as the reference state. On doing so, we essentially are comparing the pattern of the normalized time series representing a particular dynamical state to that of a state of instability. Hence, an anomaly measure defined in this context as described in the Section 2 will approach zero as we approach a state of instability (Fig. 3a and Fig.4a). Due to the particular nature of the partitioning performed in this work (simple equiprobable partitioning), this decrease in anomaly measure is rather rapid and very close to the onset of instability (note that the anomaly ratio is almost constant prior to the first drop). Nevertheless, this decrease in anomaly measure is an indicator for the dynamics of the

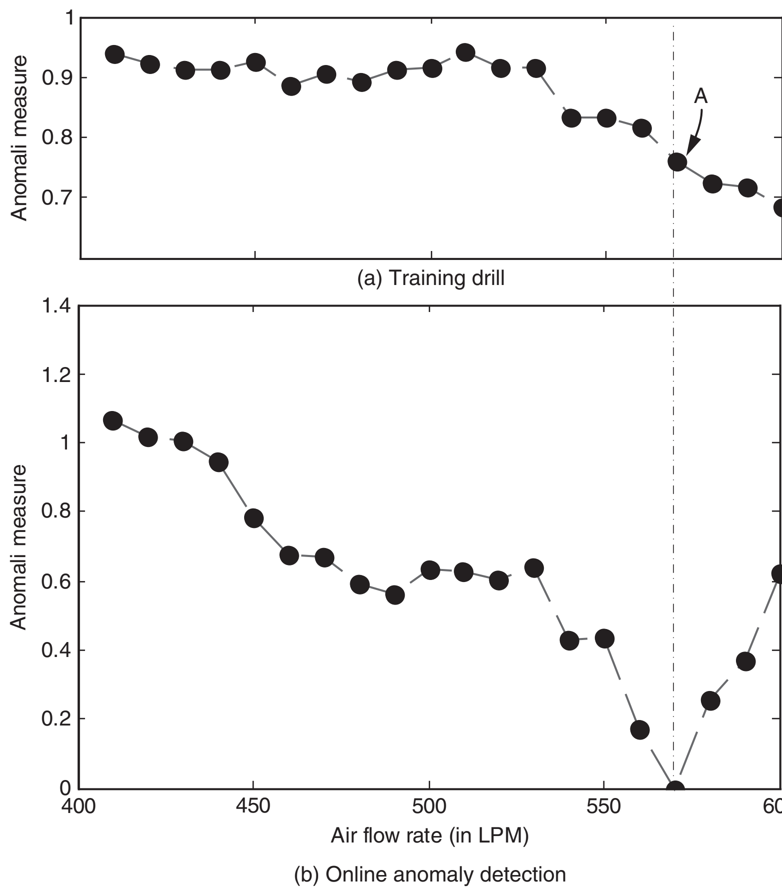

Figure 3: Variation of anomaly measure for combustor with a swirl stabilized flame. a) Represent the training drill and b) represent the online anomaly detection drill. The reduction of anomaly measure as we approach the reference state (*A*) is the precursor to instability.

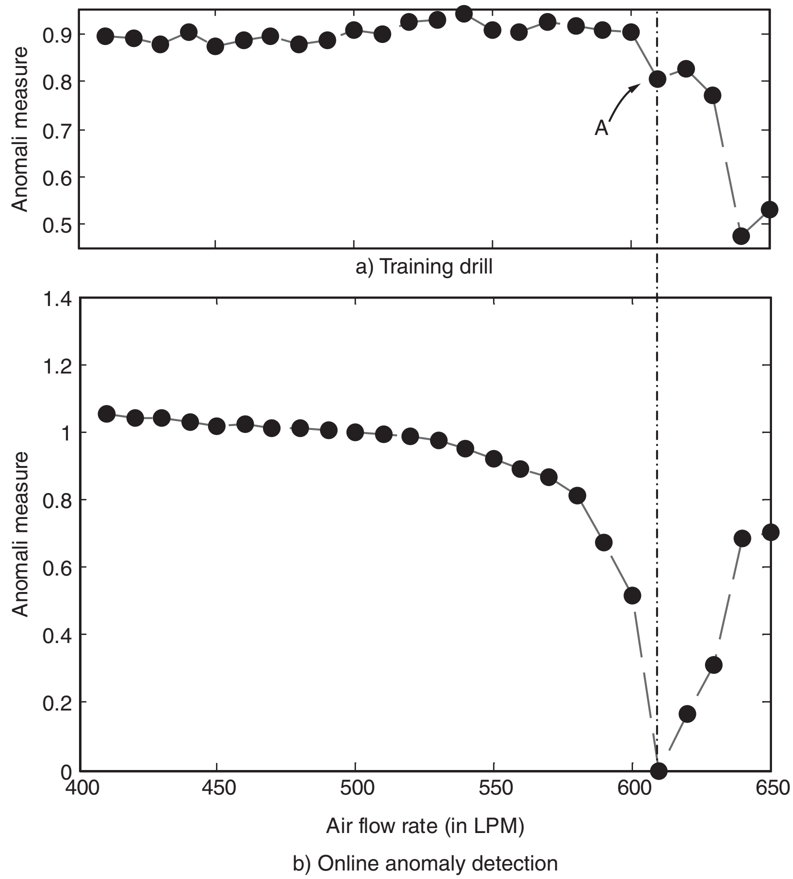

Figure 4: Application of anomaly detection technique to a combustor with bluff body stabilized flame. a) Represent the training drill and b) represent the online anomaly detection drill. The reduction of anomaly measure as we approach the reference state (*A*) is the precursor to instability.

state being close to that of a limit cycle behavior. Hence, from this training drill, we select the time series corresponding to point 'A' (marked in both Fig. 3a and Fig. 4a) as the representative time series for the reference dynamical state to be used in the online anomaly detection for the respective cases. However, depending up on the discretion of the operator and the design of a particular combustor, dynamic states further close to instability can also be used as the reference state. Further, it is to be noted that a rise is pressure amplitude is not necessarily suggestive of presence of instability. The rise in the amplitude of pressure signals could even correspond to an increase in the intensity of combustion noise. Whereas, with the aforementioned technique, we can clearly identify whether we are approaching an impending instability since a decrease in the anomaly measure is a sure indicator of the increased presence of sinusoidal component in the time series.

Once the reference state for online anomaly detection is fixed, the anomaly measure for online detection of an impending instability is defined as described in Section 2. Examining the variation of anomaly measure with airflow rate, we see that (Fig. 3b and Fig. 4b) as the system approaches the reference state, the anomaly measure starts to reduce; i.e., the angle between the PFSA of the reference state and the current state approaches zero as the current state approaches the reference state. Furthermore, unlike in the case of anomaly measure defined for the training drill, the anomaly measure for online detection starts reducing towards zero much before the reference dynamical state. These behaviors are explained in the following paragraphs.

At low airflow rates (i.e., near stoichiometric equivalence ratios), the pressure fluctuation inside the combustor is mainly due to combustion and flow noise. These oscillations have amplitudes of the order of 200 Pa (Fig. 5.a). Further, it has been already shown that these oscillations are chaos of moderately high dimension (8 to 10 dimensions) [7]. Hence when the PFSA is formed for the time series corresponding to combustion and flow noise, using the partition corresponding to the time series representing the reference state (A), most of the data points fall in those symbolic segments which correspond to low values of $|P|$. Hence, the corresponding PFSA has a probability distribution as seen in the Fig. 5.e.

Now as we increase the airflow rate (i.e., move towards lean equivalence ratios), the pressure oscillations start to become intermittent. This intermittent behavior of the pressure signals arise from the fact that the state point corresponding to the dynamical state responsible for these oscillations follow a homoclinic orbit in the phase space [9]. In this homoclinic orbit, the state point alternates between a chaotic attractor corresponding to the low amplitude fluctuations to an attractor corresponding to the high amplitude limit cycle oscillations in an apparently random fashion. As the equivalence ratio decreases away from the stoichiometric equivalence ratio, the time spent by the state point in the chaotic attractors reduces and the time spent in the attractor corresponding to the limit cycle increases. In the time series, the effect of this dynamical behavior is reflected.

During intermittency, the time series is characterized by alternating low amplitude chaotic oscillations and high amplitude limit cycle oscillations. As we approach closer to instability, in the time series, the duration for which the limit cycle oscillations are observed increases and the duration of chaotic oscillation decreases. This implies the probability associated with the symbols corresponding to the high amplitude oscillations increases as we approach the instability. Also, the symbol sequence increasingly matches to that of the reference state (a near limit cycle state).Ultimately at the reference state all the symbols have an equal probability associated with it (Fig. 5.g). If we move beyond reference state, the amplitude of the time series increases due to increased proximity to the instability and hence the symbols corresponding to the higher amplitudes have a higher occurrence probability (Fig. 5.h). This leads to an increase in the anomaly measure beyond the reference state. From Fig. 3b and Fig. 4b it can be observed that a state before and beyond the reference state can have the same value of anomaly measure.

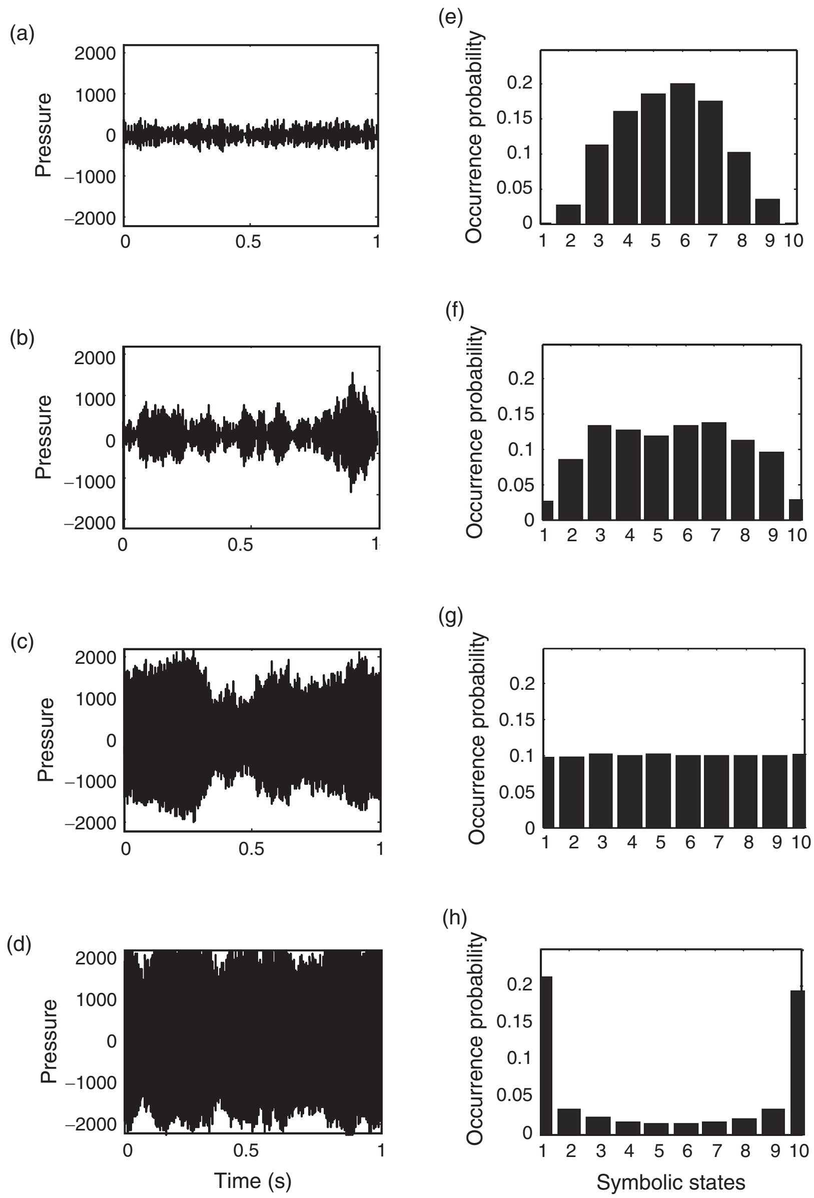

Figure 5: The variation of signal characteristics and corresponding PSFAs as the system approaches instability. a) and e) represent combustion noise at equivalence ratio 1. b) and f) represent an intermediate state, c) and g) represent the reference state and d) and h) represent a state beyond the reference state.

However, in such a case, they can be distinguished by comparing the probability distribution of the symbolic states. For example, Fig. 5b and Fig. 5d represent two time series with similar value for anomaly measure. However, comparing the corresponding occurrence probability distribution, it is clear that Fig. 5d represents a dynamical state beyond the reference state. From the above discussions, it is clear that the anomaly measure approaching zero is a precursor to the onset of instability.

In essence, using symbolic time series analysis, we are able to compare the patterns present in the time series corresponding to any dynamical state to that of the reference state by defining a vector measure (state probability vector, Fig. 4) corresponding to each pattern. We see that the occurrence probability distribution of symbolic state varies continuously as we approach the reference state (Fig. 4). We exploit this behavior of state probability vectors to define the anomaly measure which indicates the proximity of any state to the reference state. Since the particular patterns found in the pressure time series arises due to intermittency and intermittency is a dynamic behavior, time series corresponding to other measurements (such as heat release rate fluctuations, temperature fluctuations etc.) from the combustion system must also exhibit similar patterns close to instability. Hence, such measurements also could be used in a similar fashion to predict the onset of instability.

CONCLUSIONS

Anomaly detection technique, a novel strategy of online prediction of an impending instability is developed based on symbolic time series analysis. A precursor for an impending instability, i.e. the anomaly measure, was identified from unsteady pressure measurements for two types of combustors (a swirl stabilized combustor and a bluff body stabilized combustor). Using anomaly detection technique, we were able to compare the pattern of a time series at any instant to that of a reference pattern and thereby quantify the proximity of the instantaneous dynamical state to a reference dynamical state (a dynamical state close to onset of instability). In this context, a novel method of identifying an appropriate reference state for online instability detection, i.e. the training drill, was introduced. The current technique of anomaly detection uses a simple method for partition of time series data known as simple equiprobable partition. More research has to be performed in order to identify the optimal partitioning technique to be used for predicting an impending instability in a combustor. Further, the effects of increasing the word size D for the formation of PFSA on the predictive capabilities of anomaly measure also have to be studied. However, anomaly measure thus identified was able to indicate the proximity of the combustion system to regimes of instability for both the combustors.

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