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α'-corrections to heterotic superstring effective action revisited

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Abstract: In this letter we establish that the supersymmetric R^2 effective action for the heterotic string, obtained from the supersymmetrisation of the Lorentz Chern-Simons term, is to order α' equivalent modulo field redefinitions to heterotic string effective actions computed by different methods.

Keywords: Supersymmetric Effective Theories, Superstrings and Heterotic Strings.

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1. Introduction

The possibility to compare calculations of the entropy of certain black holes by microscopic string methods and by direct methods of general relativity[1] has caused renewed interest in the structure of higher derivative contributions to the string effective action. In this paper we clarify the relation between two formulations of the order α' heterotic string effective action. One formulation follows from string amplitudes calculations [2, 3] and from the requirement of conformal symmetry of the corresponding sigma model to the appropriate order [3, 4], the other formulation [5, 6] is based on the supersymmetrisation of Lorentz-Chern-Simons (LCS) forms. Our interest in the relation between these results was triggered by a remark in a recent paper of Sahoo and Sen [7]. In that paper the entropy of a supersymmetric black hole was obtained using the method of [8], with [3] for the derivative corrections to the action. The result was found to agree with that obtained by several other methods, which was taken by [7] as an indirect indication that the bosonic expression for the order α' corrections given in [3] must be part of a supersymmetric invariant.

The result of [5] is supersymmetric to order α', in [6] results to order α'^2 and α'^3 are obtained as well. We will show in this paper that to order α' [5] agrees with [3], proving directly that the action of [3] is indeed part of a supersymmetric invariant. The field redefinitions required to establish this correspondence generate additional terms at higher orders in α'.

In section 2 we establish the equivalence between the two effective actions. The necessary background material and conventions can be found in the appendices. We discuss terms of order α'^2 and α'^3 in section 3. Conclusions are in section 4.

[1]For an extensive introduction to this field see [1].

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2. The heterotic string effective action

The heterotic string effective action to order α', as given in [3], reads

$$\mathcal{L}_{\rm MT} = -\frac{2}{\kappa^2} e\, e^{-2\Phi} \Big(R(\Gamma) - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + 4\partial_\mu \Phi \partial^\mu \Phi \tag{2.1}$$

$$+\frac{1}{8}\alpha' \Big\{ R_{\mu\nu ab}(\Gamma) R^{\mu\nu\, ab}(\Gamma) - \frac{1}{2} R_{\mu\nu\, ab}(\Gamma) H^{\mu\nu\, c} H^{abc}$$

$$-\frac{1}{8}(H^2)_{ab}(H^2)^{ab} + \frac{1}{24} H^4 \Big\} \Big) . \tag{2.2}$$

Here

$$H_{\mu\nu\rho} = 3\partial_{[\mu} B_{\nu\rho]} \,,$$

$$H^2 = H_{abc} H^{abc} \,, \quad (H^2)_{ab} = H_{acd} H_b{}^{cd} \,, \quad H^4 = H^{abc} H_a{}^{df} H_b{}^{ef} H_c{}^{de} \,, \tag{2.3}$$

normalisations are as in [3].

On the other hand there is the result of supersymmetrising the LCS form of [5, 6]. The bosonic terms[2] take on the form

$$\mathcal{L}_{\rm BR} = \frac{1}{2} e\, e^{-2\Phi} \Big[\Big\{ - R(\omega) - \frac{1}{12} \widetilde{H}_{\mu\nu\rho} \widetilde{H}^{\mu\nu\rho} + 4\partial_\mu \Phi \partial^\mu \Phi \Big\} \tag{2.4}$$

$$-\frac{1}{2} \alpha R_{\mu\nu ab}(\Omega_-) R^{\mu\nu\, ab}(\Omega_-) \Big] . \tag{2.5}$$

With respect to [6] we have redefined the dilaton and the normalisation of $B_{\mu\nu}$ (see appendix A.1). In (2.4) $\widetilde{H}$ contains the LCS term with H-torsion:

$$\widetilde{H}_{\mu\nu\rho} = H_{\mu\nu\rho} - 6\alpha\, \mathcal{O}_{3\,\mu\nu\rho}(\Omega_-) \,, \tag{2.6}$$

$$\mathcal{O}_{3\,\mu\nu\rho}(\Omega_-) = \Omega_{-[\mu}{}^{ab} \partial_\nu \Omega_{-\rho]}{}^{ab} - \frac{2}{3} \Omega_{-[\mu}{}^{ab} \Omega_{-\nu}{}^{ac} \Omega_{-\rho]}{}^{cb} \,, \tag{2.7}$$

$$\Omega_{-\mu}{}^{ab} = \omega_\mu{}^{ab} - \frac{1}{2} \widetilde{H}_\mu{}^{ab} \,. \tag{2.8}$$

The coefficient α is proportional to α', note that the relative normalisation between the LCS term and the R^2 action is fixed.

To establish the equivalence between (2.1)–(2.2) and (2.4)–(2.5) we expand $R(\Omega_-)$ in (2.5), perform the required field redefinitions and fix the normalisations.

To start with, we have

$$R_{\mu\nu}{}^{ab}(\Omega_-) = R_{\mu\nu}{}^{ab}(\omega) - \frac{1}{2}\Big(\mathcal{D}_\mu \widetilde{H}_\nu{}^{ab} - \mathcal{D}_\nu \widetilde{H}_\mu{}^{ab} \Big) - \frac{1}{8}\Big(\widetilde{H}_\mu{}^{ac} \widetilde{H}_\nu{}^{cb} - \widetilde{H}_\nu{}^{ac} \widetilde{H}_\mu{}^{cb} \Big) \,, \tag{2.9}$$

where the derivatives $\mathcal{D}$ are covariant with respect to local Lorentz transformations. Clearly the substitution of (2.9) in (2.5) gives terms similar to those in (2.2), additional terms come

[2]Throughout this paper we will only discuss the bosonic contributions to the effective action. Fermionic contributions can be found in [6].

from expanding $\widetilde{H}$ (see appendix A.3) in (2.4). The effect of these substitutions is, to order α:

$$\begin{aligned}
\mathcal{L}_{\mathrm{BR}} = \frac{1}{2} e\, e^{-2\Phi} \Big[& - R(\omega) - \frac{1}{12}\bar{H}_{\mu\nu\rho}\bar{H}^{\mu\nu\rho} + 4\partial_\mu\Phi\partial^\mu\Phi \\
& +\alpha \left\{ \frac{1}{2} H^{\mu\nu\rho}\partial_\mu(\omega_\nu{}^{ab}H_\rho{}^{ab}) - \frac{1}{2}R_{\mu\nu}{}^{ab}(\omega)\, H_\rho{}^{ab}\, H^{\mu\nu\rho} + \frac{1}{4}H^{\mu\nu\rho}H_\mu{}^{ab}\mathcal{D}_\nu H_\rho{}^{ab} - \frac{1}{12}H^4 \right\} \\
& -\frac{1}{2}\,\alpha \Big\{ R_{\mu\nu}{}^{ab}(\omega)R^{\mu\nu ab}(\omega) && (2.10) \\
& \qquad -2R^{\mu\nu ab}(\omega)\mathcal{D}_\mu H_{\nu ab} && (2.11) \\
& \qquad +\frac{1}{2}\Big(\mathcal{D}_\mu H_\nu{}^{ab} - \mathcal{D}_\nu H_\mu{}^{ab}\Big)\mathcal{D}^\mu H^{\nu ab} && (2.12) \\
& \qquad -R_{\mu\nu}{}^{ab}(\omega)H^{\mu ac}H^{\nu cb} && (2.13) \\
& \qquad +\frac{1}{2}\Big(\mathcal{D}_\mu H_\nu{}^{ab} - \mathcal{D}_\nu H_\mu{}^{ab}\Big)H^{\mu ac}H^{\nu cb} && (2.14) \\
& \qquad\qquad +\frac{1}{8}\Big((H^2)_{ab}(H^2)^{ab} - H^4\Big)\Big\}\Big]\,. && (2.15)
\end{aligned}$$

Here $\bar{H}$ contains the LCS term without H-torsion:

$$\bar{H}_{\mu\nu\rho} = H_{\mu\nu\rho} - 6\alpha\, \mathcal{O}_{3\,\mu\nu\rho}(w)\,. \tag{2.16}$$

We now rewrite the terms (2.10)–(2.15) in $\mathcal{L}_{\mathrm{BR}}$, see appendix A.4 for details. The result, keeping only contributions to order α, is

$$\begin{aligned}
\mathcal{L}_{\mathrm{BR}} = \frac{1}{2} e\, e^{-2\Phi} \Big[& - R(\omega) - \frac{1}{12}\bar{H}_{\mu\nu\rho}\bar{H}^{\mu\nu\rho} + 4\partial_\mu\Phi\partial^\mu\Phi \\
& -\frac{1}{2}\alpha \Big\{ R_{\mu\nu}{}^{ab}(\omega)R^{\mu\nu ab}(\omega) + \frac{1}{2}R_{\mu\nu}{}^{ab}(\omega)\, H_\rho{}^{ab}\, H^{\mu\nu\rho} \\
& \qquad + \frac{1}{8}(H^2)_{ab}(H^2)^{ab} + \frac{1}{24}H^4 \Big\} && (2.17) \\
& -\frac{1}{2}\alpha \Big\{ R_\mu{}^c(\omega)H^{\mu ab}H_{abc} + e^\mu{}_c e^\nu{}_d \mathcal{D}_\nu H_{abd}\mathcal{D}_\mu H_{abc} \\
& \qquad +2\partial_c\Phi\, H_{abd}\mathcal{D}_d H_{abc} - 2\partial_d\Phi\, H_{abd}\mathcal{D}_c H_{abc} \Big\}\Big]\,. && (2.18)
\end{aligned}$$

The term proportional to the Ricci tensor in (2.18) then contributes through a field redefinition to the terms quartic in H, and gives an additional contribution involving derivatives of Φ (see (A.15)). Using (A.13) and partial integrations all remaining terms can be made to cancel.

The final result is then

$$\begin{aligned}
\mathcal{L}_{\mathrm{BR}} = \frac{1}{2} e\, e^{-2\Phi} \Big[& - R(\omega) - \frac{1}{12}\bar{H}_{\mu\nu\rho}\bar{H}^{\mu\nu\rho} + 4\partial_\mu\Phi\partial^\mu\Phi && (2.19) \\
& -\frac{1}{2}\alpha \Big\{ R_{\mu\nu}{}^{ab}(\omega)R^{\mu\nu ab}(\omega) + \frac{1}{2}R_{\mu\nu}{}^{ab}(\omega)\, H_\rho{}^{ab}\, H^{\mu\nu\rho} - \frac{1}{8}(H^2)_{ab}(H^2)^{ab} + \frac{1}{24}H^4 \Big\}\Big]\,,
\end{aligned}$$

in agreement with [3] if we set $R(\Gamma) = -R(\omega)$ and $\alpha = -\frac{1}{4}\alpha'$, and adjust the overall normalisation. Of course [3] also includes the LCS term in H^2 for the heterotic string effective action, see the footnote in [3], page 400.

3. Higher orders and field redefinitions

In [6] it was shown that the effective action to order α^2 consists of terms which are bilinear in the fermions (2.4)–(2.5). This is no longer true when the effective action at order α is in the form (2.19).

Since the steps to go from (2.4)–(2.5) to (2.19) have all been explicitly determined, the effective action at order α^2 can in principle be constructed. Let us identify the sources of bosonic $\mathcal{O}(\alpha^2)$-terms that we have encountered:

(i) From the action (2.4) there are contributions outlined in appendix A.3. We should now expand $\widetilde{H}$ to order α^2, which means that in $\mathcal{A}$ (A.17) also terms of order α should be considered. Then one should calculate $\widetilde{H}^2$.

(ii) $\bar{H}$ contains the LCS term of order α. These should now also be kept in the higher order contributions.

(iii) In a number of places we have used the identity (A.19), the resulting R^2 terms contribute to order α^2.

(iv) We have used field redefinitions to modify the effective action at order α. A field redefinition is of the form
$$e_\mu{}^a \to e_\mu{}^a + \alpha\Delta_\mu^a\,, \tag{3.1}$$
and is applied to the order α^0 action. This has the effect of giving an extra contribution
$$\alpha\Delta_\mu^a\mathcal{E}^\mu{}_a \tag{3.2}$$
to the action, where $\mathcal{E}^\mu{}_a$ is the Einstein equation at order α^0. Thus one can eliminate a term
$$-\alpha\Delta_\mu^a\mathcal{E}^\mu{}_a\,. \tag{3.3}$$
at order α. Contributions of order α^2 arise because the transformation should also be applied to the order α action.

In this way the bosonic part of six-derivative terms in the effective action at order α^2, corresponding to the order α action (2.2), can be obtained, including the complete dependence on H. It would be interesting to extend the calculation of black hole entropy of [7] to this sector.

At order α^3 the situation is different. In [6] an invariant related to the supersymmetrisation of the LCS terms was constructed. The status of R^4 invariants was discussed in [9], with extensive reference to the earlier literature.

4. Conclusions

We have established the equivalence between the effective actions of [6] and [3] to order α. This indicates that the result of [7] might indeed be a consequence of supersymmetry.

In principle the method of [8] can be extended to corrections with any number of derivatives. Supersymmetry provides the derivative contributions at order α'^2, at α'^3 only partial results are known. It would be interesting to extend the analysis of [7] to include the next order.

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A. Calculational details

A.1 Lagrangian density and redefinitions

In [6] the Lagrangian density takes on the form

$$\mathcal{L}_R = \frac{1}{2} e\phi^{-3} \left(-R(\omega) - \frac{3}{2} \widetilde{H}_{\mu\nu\rho} \widetilde{H}^{\mu\nu\rho} + 9(\phi^{-1}\partial_\mu \phi)^2 \right) . \tag{A.1}$$

with the following definitions:[3]

$$\widetilde{H}_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} - \alpha\sqrt{2}\, \mathcal{O}_{3\,\mu\nu\rho}(\Omega_-) \,, \tag{A.2}$$

$$\mathcal{O}_{3\,\mu\nu\rho}(\Omega_-) = \Omega_{-[\mu}{}^{ab} \partial_\nu \Omega_{-\rho]}{}^{ab} - \frac{2}{3} \Omega_{-[\mu}{}^{ab} \Omega_{-\nu}{}^{ac} \Omega_{-\rho]}{}^{cb} \,, \tag{A.3}$$

$$\Omega_{-\mu}{}^{ab} = \omega_\mu{}^{ab} - \frac{3}{2}\sqrt{2} \widetilde{H}_\mu{}^{ab} \,. \tag{A.4}$$

Antisymmetrisation brackets are with weight 1.

First we redefine the fields to obtain agreement with the conventions in [7]. The redefinitions are

(i) The dilaton: the change is:

$$\phi^{-3} \to e^{-2\Phi} \,, \quad (\phi^{-1}\partial\phi) \to \frac{2}{3}\partial\Phi \,. \tag{A.5}$$

(ii) The 2- and 3-form fields: we set

$$\widetilde{H} \to \frac{1}{3\sqrt{2}} \widetilde{H} \,, \quad B \to \frac{1}{\sqrt{2}} B \,. \tag{A.6}$$

[3] In this letter we use the the notation and conventions of [6] and α is a free parameter proportional to α', the inverse of the string tension.

$\mathcal{L}_R$ now becomes

$$\mathcal{L}_R = \frac{1}{2} e\, e^{-2\Phi} \left(-R(\omega) - \frac{1}{12} \widetilde{H}_{\mu\nu\rho} \widetilde{H}^{\mu\nu\rho} + 4 \partial_\mu \Phi \partial^\mu \Phi \right) , \tag{A.7}$$

as in (2.4).

The spin connection $\omega(e)$ is the solution of

$$\mathcal{D}_\mu e_\nu{}^a - \mathcal{D}_\nu e_\mu{}^a = 0 , \qquad \text{with} \quad \mathcal{D}_\mu e_\nu{}^a \equiv \partial_\mu e_\nu{}^a - \omega_\mu{}^{ac} e_{\nu\, c} . \tag{A.8}$$

The Riemann tensor and related quantities are defined as

$$\begin{aligned} R_{\mu\nu}{}^{ab}(\omega) &= \partial_\mu \omega_\nu{}^{ab} - \partial_\nu \omega_\mu{}^{ab} - \omega_\mu{}^{ac} \omega_{\nu\, c}{}^b + \omega_\nu{}^{ac} \omega_{\mu\, c}{}^b , & \text{(A.9)} \\ R_\mu{}^a(\omega) &= e^\nu{}_b R_{\mu\nu}{}^{ab}(\omega) , & \text{(A.10)} \\ R(\omega) &= e^\mu{}_a R_\mu{}^a(\omega) . & \text{(A.11)} \end{aligned}$$

A.2 Equations of motion

The equations of motion at order $\alpha'^{\,0}$ are:

$$\begin{aligned} \mathcal{S} &= e e^{-2\Phi} \left\{ R(\omega) - 4 \mathcal{D}_a \partial^a \Phi + 4 (\partial_a \phi)^2 + \frac{1}{12} H^{abc} H_{abc} \right\} , & \text{(A.12)} \\ \mathcal{B}^{\nu\rho} &= \frac{1}{4} \partial_\mu \left(e e^{-2\Phi} H^{\mu\nu\rho} \right) = 0 , & \text{(A.13)} \\ \mathcal{E}^\lambda{}_c &= -\frac{1}{2} e^\lambda{}_c \mathcal{S} + e e^{-2\Phi} \left(R_c{}^\lambda(\omega) + \frac{1}{4} (H^2)_\lambda{}^c - 2 e_d^\lambda \mathcal{D}_c \Phi \partial^d \Phi \right) = 0 . & \text{(A.14)} \end{aligned}$$

In the main text we use a field redefinition to eliminate a contribution proportional to the Ricci tensor. The required equation is, modulo $\mathcal{E}$ and $\mathcal{S}$:

$$R_\mu{}^a(\omega) = 2 \mathcal{D}_\mu \partial^a \Phi - \frac{1}{4} (H^2)_\mu{}^a . \tag{A.15}$$

A.3 Expanding $\mathcal{L}_R$ in powers of α

The 3-form field $\widetilde{H}$ is defined recursively by (2.6), (2.7) and (2.8). We find

$$\widetilde{H}_{\mu\nu\rho} = H_{\mu\nu\rho} - 6\alpha \left(\mathcal{O}_{3\, \mu\nu\rho}(w) + \mathcal{A}_{\mu\nu\rho} \right) = \bar{H}_{\mu\nu\rho} - 6\alpha \mathcal{A}_{\mu\nu\rho} , \tag{A.16}$$

where $\mathcal{O}_{3\, \mu\nu\rho}(w)$ is the gravitational contribution (order α^0) of the Lorentz Chern-Simons term, and

$$\begin{aligned} \mathcal{A}_{\mu\nu\rho} = {} & \frac{1}{2} \partial_{[\mu} (\omega_\nu{}^{ab} \widetilde{H}_{\rho]}{}^{ab}) - \frac{1}{2} R_{[\mu\nu}{}^{ab}(\omega) \widetilde{H}_{\rho]}{}^{ab} + \frac{1}{4} \widetilde{H}_{[\mu}{}^{ab} \mathcal{D}_\nu \widetilde{H}_{\rho]}{}^{ab} \\ & + \frac{1}{12} \widetilde{H}_{[\mu}{}^{ab}\, \widetilde{H}_\nu{}^{ac}\, \widetilde{H}_{\rho]}{}^{cb} . \end{aligned} \tag{A.17}$$

To order α $\mathcal{L}_R$ (A.7) can be written as

$$\begin{aligned} \mathcal{L}_R = \frac{1}{2} e\, e^{-2\Phi} \Bigg[& - R(\omega) - \frac{1}{12} \bar{H}_{\mu\nu\rho} \bar{H}^{\mu\nu\rho} + 4 \partial_\mu \Phi \partial^\mu \Phi \\ & + \alpha \left\{ \frac{1}{2} H^{\mu\nu\rho} \partial_\mu (\omega_\nu{}^{ab} H_\rho{}^{ab}) - \frac{1}{2} R_{\mu\nu}{}^{ab}(\omega)\, H_\rho{}^{ab}\, H^{\mu\nu\rho} \right. \\ & \left. + \frac{1}{4} H^{\mu\nu\rho} H_\mu{}^{ab} \mathcal{D}_\nu H_\rho{}^{ab} + \frac{1}{12} H^{\mu\nu\rho}\, H_\mu{}^{ab}\, H_\nu{}^{ac} H_\rho{}^{cb} \right\} \Bigg] . \end{aligned} \tag{A.18}$$

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The term with the $H\partial(\omega H)$ is, after partial integration, proportional to (A.13) and can be eliminated by a field redefinition.

A.4 Simplification of $\mathcal{L}_{R^2}$ terms

We often use the identity

$$\mathcal{D}_{[a}(\Omega_-)\widetilde{H}_{bcd]} = -\frac{3}{2}\alpha\, R_{[ab}{}^{ef}(\Omega_-)R_{cd]}{}^{ef}(\Omega_-)\,, \tag{A.19}$$

to isolate terms that are of higher order in α. The term (2.13) can be simplified by using the cyclic identity for the Riemann tensor:

$$R_{\mu\nu}{}^{ab}(\omega)H^{\mu ac}H^{\nu cb} = -\frac{1}{2}R_{\mu\nu}{}^{ab}H^{\mu\nu c}H^{abc}\,. \tag{A.20}$$

Now we consider (2.14). Note that the two terms written in (2.14) are in fact the same. Then we have

$$\begin{aligned}\frac{1}{2}\left(\mathcal{D}_\mu H_\nu{}^{ab} - \mathcal{D}_\nu H_\mu{}^{ab}\right)H^{\mu ac}H^{\nu cb} &= -\mathcal{D}_\mu H_\nu{}^{ab}H^{\mu ac}H^{\nu bc}\\ &= -\mathcal{D}_{[e}H_{fab]}\,H^{eac}H^{fbc}\,.\end{aligned} \tag{A.21}$$

This term is completely of order $\alpha'^{\,2}$. Finally we consider (2.12). This can be expressed as

$$\begin{aligned}\frac{1}{2}ee^{-2\Phi}\left(\mathcal{D}_\mu H_\nu{}^{ab} - \mathcal{D}_\nu H_\mu{}^{ab}\right)\mathcal{D}^\mu H^{\nu ab} = ee^{-2\Phi}\Big(&2R_{\mu\nu}{}^{ab}H^{\mu ac}H^{\nu\, cb} + R_\mu{}^c H^{\mu ab}H_{abc}\\ &+e^\mu{}_c e^\nu{}_d \mathcal{D}_\nu H_{abd}\mathcal{D}_\mu H_{abc} + 2\partial_c\Phi\, H_{abd}\mathcal{D}_d H_{abc}\\ &-2\partial_d\Phi\, H_{abd}\mathcal{D}_c H_{abc} + 2\mathcal{D}_c H_{abd}\mathcal{D}_{[c}H_{abd]}\Big)\,.\end{aligned} \tag{A.22}$$

The last term is of order α'^2.

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