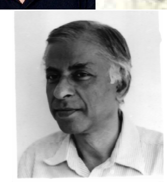
Space and Time in Life and Science

Vasant Natarajan, V Balakrishnan and N Mukunda

Space and time are concepts that seem to be embedded in our very consciousness. As we grow up, our 'intuitive understanding' of these concepts seems to grow as well. And yet the fact is that our understanding of space-time in the deepest scientific sense is far from complete, although we have covered a considerable distance along the route. There may still be many surprises awaiting us on the road ahead.





(left) Vasant Natarajan is at the Department of Physics, IISc. His research interests are in high-resolution laser spectroscopy, optical frequency measurement, and use of laser-cooled atoms to search for time-reversal violation in the laws of physics.

(right) V Balakrishnan is at the Department of Physics, IIT- Madras, Chennai. His research interests are in dynamical systems, stochastics and statistical physics.

(centre) N Mukunda is at the Centre for High Energy Physics, IISc, Bangalore. His interests are classical and quantum mechanics, theoretical optics and mathematical physics.

1. It Began with Geometry

All of us have some intuitive ideas about the natures of space and time in which we are embedded. Space appears to be the stage on which all events, experiences and phenomena take place, while time is like a background against which this happens. All objects, including ourselves, exist in space and change with time. In this article, we shall describe in simple and qualitative terms how our understanding of space and time has developed over the centuries, and what we have learnt about their properties. We will see that many strands come together in this story – not only from mathematics and physics, but also, in some important respects, from biology.

In one of his essays, Schrödinger[1] says this about the eminent philosopher Immanuel Kant (1724–1804): "*Kant … termed space and time, as he knew them, the forms of our mental intuition – space being the form of external, time that of internal, intuition.*" We begin with this quotation both because it is so profound and because some of Kant's other ideas will appear later on. All of us have at least a preliminary or common-sense understanding of the nature of space through our senses, mainly sight and touch.[2] We see objects arranged in various ways relative to one another; we touch them if they are nearby; we develop a feeling for shape, distance, perspective; and so on. Thus, through direct sensory experiences, space seems to have a definite character, even though the idea of space by itself is quite abstract. Time, on the other hand, is more subtle. We cannot reach it through our senses directly; we can experience it and be aware of its passage only internally through the mind in a non-sensory way. Because we cannot see or touch time, extension in space seems a little easier to grasp than duration in time. Thus, space has to be abstracted from external experience, time from internal experience. Over and above this subtlety, space and time are really not simple concepts at all. An obvious fundamental difference between the two is that space appears to be 'controllable' – in the sense that we can move from one point to another, with seemingly arbitrary freedom. On the other hand, time seems to be something over which we have no control – it just marches inexorably on. We are caught up in this march with no apparent choice. The old adage – 'time and tide wait for no man' – captures this notion perfectly. This also means that we have memories of what has happened up till now, but no idea of what the future holds (contrary to what soothsayers and horoscope-writers might tell you!)

Let us begin with space. The word *geometry*, as you know, means 'measuring the earth (or land)'. The origins of geometry go back several millennia to the great Egyptian civilization (see also *Box* 1). Geometry arose in ancient Egypt from the repeated need to survey and re-establish boundaries of land holdings after the annual floods of the river Nile. Here is a passage from the Greek traveller and historian Herodotus (482?–434? BC):

"*The King moreover (so they said) divided the country among all the Egyptians by giving each an equal square*

[1] The great physicist Erwin Schrödinger (1887–1961; Nobel Prize in Physics, 1933), who discovered wave mechanics and was one of the founders of quantum mechanics, was also a wonderfully lucid writer on many profound subjects. His famous books include *Science and Humanism*, *Expanding Universes*, *Statistical Thermodynamics*, *What is Life*?, *Mind and Matter*, *My View of the World*, *Space-Time Structure*, and *Nature and the Greeks*.

[2] It is an interesting fact that about 80% of our sensory input comes from our eyes, and about 40% of our brain capacity is devoted to processing visual information – we are very 'visual' creatures.

Keywords
Space, time, geometry, relativity.

Box 1. The Origins of Geometry

Ancient Babylon and Egypt are well known as the civilizations in which the subject of geometry originated. However, there were other nodal centres as well. You are undoubtedly familiar with some of the great and unique achievements of ancient Indian mathematics, such as the invention of the decimal place-value system of numbers, negative numbers, and zero. It is important to note that these remarkable mathematical advances (which took place from about 500 AD onwards) were preceded by equally significant achievements in the Indian sub-continent in even earlier times. In recent years, much new light has been shed on the scope and advanced nature of truly ancient Indian mathematics. It is being gradually recognized that ancient India (especially the Indus Valley Civilization) was in many respects (including mathematics) fully the peer of the ancient Egyptian and Babylonian civilizations. The historians of science J J O'Connor and E F Robertson (see http://www-history.mcs.st-andrews.ac.uk/HistTopics/Indian_mathematics.html) write, "... the study of mathematical astronomy in India goes back to at least the third millennium BC and mathematics and geometry must have existed to support this study in ancient times." They go on to quote V G Childe in *New Light on the Most Ancient East* (Routledge and Kegan Paul Ltd., London, 1952): "India confronts Egypt and Babylonia by the 3rd millennium with a thoroughly individual and independent civilization of her own, technically the peer of the rest." What is clear is that astronomy was a prime motivating factor behind the development of geometry, trigonometry and related topics in ancient India.

parcel of land, and made this his source of revenue, appointing the payment of a yearly tax. And any man who was robbed by the river of a part of his land would come to Sesostris and declare what had befallen him; then the King would send men to look into it and measure the space by which the land was diminished, so that thereafter he should pay the appointed tax in proportion to the loss. From this, to my thinking, the Greeks learned the art of measuring land."

So geometry was the product of practical human needs to measure space, much as trade and commerce led to arithmetic. Similarly, the practical need to measure time (for example, to keep track of daily sunrises or the annual floods) gave rise to clocks and calendars. Human scientific and technological progress has required evermore accurate clocks. A brief history of timekeeping over the centuries is presented in *Box* 2.

Geometry was the product of practical human needs to measure space, much as trade and commerce led to arithmetic.

Box 2. A Brief History of Timekeeping

Timekeeping on earth by nature is as old as the planet itself, since all it requires is a periodic process. The periodic rotation and revolution of the earth give rise to daily and yearly cycles, which have been used by nature as a clock much before human beings evolved. Many living organisms including humans show daily (or diurnal) rhythms regulated by the sun, and seasonal patterns that repeat every year. It is therefore natural that the earliest man-made clocks also relied on the sun. One example was the sundial, which consisted of a pointer and a calibrated plate on which the pointer cast a moving shadow. Of course this worked only when the sun was shining. The need to tell the time even when the sun was not out, such as on an overcast day or at night, made it necessary to invent other kinds of clocks.

The earliest all-weather clocks were water clocks, which were stone vessels with sloping sides that allowed water to drip at a constant rate from a small hole near the bottom. Markings on the inside surface indicated the passage of time as the water level reached them. Other clocks were used for measuring small intervals of time. Examples included candles marked in increments, oil lamps with marked reservoirs, hourglasses filled with sand, and small stone or metal mazes filled with incense that would burn at a constant rate.

Time measurements became significantly more accurate with the advent of the pendulum clock in the 17th century. Galileo had studied the motion of the pendulum as early as 1582, but the first pendulum clock was built by Christiaan Huygens (1629–1695) only in 1656. As we know from high-school physics, the time period of a pendulum executing small-amplitude oscillations depends only on its length and the acceleration due to gravity. Huygens' clock had an unprecedentedly small error of less than 1 minute per day. Later refinements allowed him to reduce it to less than 10 seconds a day. While very accurate compared to previous clocks, pendulum clocks still showed significant variations, because a change of just a few degrees in the ambient temperature could change the length of the pendulum (owing to thermal expansion). Many clever schemes were therefore devised in the 18th and 19th centuries to compensate for such changes in length.

In the history of clockmakers, the name of the great horologist John Harrison (1693–1776) stands out. He constructed many 'marine chronometers'– highly accurate clocks that were used on ships to tell the time from the start of the voyage. A comparison of *local noon* (that is, the time at which the sun was at its highest point) with the time on the clock (which would give the time of the noon at the starting point) could be used to determine with precision the longitude of the ship's current position. The British government had instituted the Longitude Prize so that ships could navigate on transatlantic voyages without getting lost. Harrison designed and built several prize-winning clocks based on the oscillations of a balance wheel. These maritime clocks had to maintain their accuracy over lengthy and rough sea voyages with widely-varying

Box 2. continued...

Box 2. continued...

conditions of temperature, pressure and humidity. The amazing features of Harrison's chronometer are exemplified by the fact that, on a voyage from London to Jamaica, the clock only lost 5 seconds, corresponding to an error in distance of 1 mile! Harrison's heroic story is brilliantly narrated by Dava Sobel in her book, *Longitude: The True Story of a Lone Genius Who Solved the Greatest Scientific Problem of His Time* (Penguin Reprint Edition, 1996).

However, all such mechanical clocks suffered from unpredictable changes in timekeeping accuracy owing to wear and tear of the moving parts. Clock accuracies improved dramatically in the first half of the 20th century with the development of the quartz crystal oscillator. These clocks rely on a property of quartz called piezoelectricity, wherein a mechanical deformation produces an electrical signal (voltage), and vice versa. A vibrating crystal therefore produces a periodic electrical signal which acts as a precise clock. (For practical reasons, the frequency of the crystal or 'resonator' is usually adjusted to be as close to 32768 or 2^{15} Hz as possible. This is then stepped down by successive digital dividers to 1 Hz.) Today, we have the ubiquitous and inexpensive quartz wristwatch that is accurate to within a few seconds over a month, and one does not have to worry about winding the clock every day or replacing the battery more than once a year or so.

Life in our technology-driven society needs ever more precise timekeeping. Computers, manufacturing plants, electric power grids, satellite communication – all these depend on ultra-precise timing. The Global Positioning System (GPS), which determines the position of a receiver by triangulating with respect to the three nearest satellites, requires timing accuracy of 1 part in 10^{12}. Modern atomic clocks, based on the oscillation frequency within an atom, have this kind of accuracy. But scientific and technological needs keep pushing the requirement ever higher. Today's best atomic clocks have an accuracy of 1 part in 10^{14}, or an error of 1 second in 3 million years. We have indeed come a long way from the pendulum clock of just a few centuries ago.

The science of geometry must have developed gradually. From the Egyptians this knowledge of length, direction, area and shape passed into the hands of the Greeks, who seem to have had a special gift for this subject and for abstract thinking in general. Around 300 BC, this mathematical knowledge was codified and presented in a beautifully organized form by Euclid of Alexandria (*circa* 325–265 BC). His *Elements* became the supreme pattern – or paradigm – for mathematical precision and rigour, indeed for all intellectual work, for many centuries. Starting from a few definitions and axioms (or postulates), which seemed self-evident and consistent, and proceeding by a series of logical steps, many

Results which were implicit in the starting axioms were made explicit, or brought out into the open, by logical arguments.

theorems were derived as consequences. Thus, results which were implicit in the starting axioms were made explicit, or brought out into the open, by logical arguments. From the economy of the axioms one could pass to the wealth of derived consequences. Achieving this style of thinking was a great intellectual step forward.

2. The Newtonian Worldview

We now pass over several centuries and come to the birth of modern science. Galileo Galilei (1564–1642) is generally regarded as the first modern scientist. In his writings he says clearly that mathematics is the language of Nature: "*It is written in mathematical language.*" Soon after, in the latter part of the 17th century, the first clear statements about space and time were given by Isaac Newton (1643–1727) in his monumental three-volume work on the mathematical principles of natural philosophy, usually referred to as the *Principia* for short. The style of this work is also 'Euclidean' – a small number of definitions and laws are given at the beginning, and many theorems are then proved as consequences. In fact, Newton went so far as to present all his derivations using geometrical arguments ("*con more geometrico*").

About space and time, he says that although they are commonly thought of in relation to material objects placed in them, he would define them on their own. Here are his famous statements from the opening pages of the *Principia*:

"*Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration.*"

Newton presented all his derivations using geometrical arguments.

"*Absolute space, in its own nature, without relation to anything external, remains always similar and immovable.*"

Newton assumed, of course, that space obeyed the laws of Euclidean geometry.

Newton assumed that space obeyed the laws of Euclidean geometry.

It must be emphasized that, for the further development of science, it was very important that Newton explained so clearly his views about space and time. They could be examined in a careful way, and made the basis for further progress. Newton's views were already criticized in his own lifetime by the philosopher George (Bishop) Berkeley (1685–1753) and the great mathematician, physicist and philosopher Gottfried Wilhelm von Leibnitz (1646–1716). The latter felt that there was no need to think of space apart from the relation of material objects to one another and declared: "*I hold space to be something purely relative as time is.*" Much later, in the late 1800's, the Austrian physicist-philosopher Ernst Mach (1838–1916) also critically examined Newton's views, not only about space and time, but about mechanics as well. All this only emphasizes the crucial role and importance of Newton's unambiguous statements. Albert Einstein (1879–1955; Nobel Prize, 1921) himself acknowledged this fact when he stated in one of his essays, "*... what we have gained up till now would have been impossible without Newton's clear system.*"

This Newtonian picture of space and time served physics extremely well for over two centuries. The word 'absolute' in his statements is important: it strongly suggests that space and time themselves are unaffected by all the physical processes that occur in them.[3] Once again, Einstein conveys the essence of this idea very well: he says 'absolute' means "*... physically real ... independent in its physical properties, having a physical effect, but not itself influenced by physical conditions.*" Following his statements on the absolute natures of space and time, Newton gives his three laws of motion, which we learn at school. Much later, towards the end of the *Principia*, he introduces his Law of Universal Gravitation. We do not go into Newtonian mechanics or the

[3] Thus, Newton treated space and time as quantities that appeared in his laws of motion as independent variables, while other physical quantities were functions of these. We have already noted the 'controllability' of space, in contrast to the 'inevitability' of time. These ideas are embodied in Newton's conception by supposing that objects exist in space, but evolve in time. The evolution in time was determined by his theory of *fluxions*, which we study today as differential calculus.

General relativity changed precisely the Newtonian assumption of the absoluteness of space and time.

inverse square law of gravitation, as our focus is on space and time. But we mention here that general relativity (Einstein's theory of gravitation, unveiled by him in 1915–16) changed precisely the Newtonian assumption of the absoluteness of space and time. We will return to this point later on.

3. Space, Time and Human Intuition

Building on the foundations provided by Galileo and Newton, both mechanics and astronomy made remarkable advances during the 18th century. Towards the end, the laws of electricity and magnetism also seemed to follow the general Galilean–Newtonian pattern. Kant was so impressed by these successes that he tried to present a philosophical justification for them. His main idea was that of the *synthetic a priori* principles. These are non-trivial statements about the properties of nature which are necessarily true or binding, but they are regarded as not derived from experience. That is the meaning of the phrase '*a priori*', namely, something known even before experience. As far as we are concerned here, Kant regarded the concepts of absolute and separate space and time, as Newton had viewed them, as synthetic *a priori* principles; so also the uniform flow of time and the Euclidean geometry of space.[4]

[4] As it happens, he was wrong on both counts (for reasons he could not have foreseen). A profound lesson about nature is to be learnt here. See *Box* 3.

Kant claimed that all these ideas are already present in our minds before we have contact with, and experience of, nature; and that they must be necessarily valid because science would be impossible without them. The modern understanding of this matter comes, surprisingly enough, from evolutionary biology. It is largely due to the work of the zoologist and ethologist Konrad Lorenz (1903–89; Nobel Prize, 1973) around 1940, further developed and described by the molecular biologist Max Delbrück (1906–81; Nobel Prize, 1969). The basic idea is that as species evolve biologically over long periods of time, governed by natural selection, they acquire

Box 3. Caution: Nature Cannot be Second-Guessed!

If a single loud message comes across from the developments in the physical sciences and the life sciences during the last one hundred years or so, it is surely this: *Do not try to second-guess nature*! As Sherlock Holmes put it so pithily, in *A Study in Scarlet*, "*It is a capital mistake to theorize before you have all the evidence. It biases the judgment.*" This dictum can hardly be over-emphasized when it comes to unlocking the secrets of nature. Time and again, nature has corroborated the great geneticist and biologist J B S Haldane (1892–1964) who said, "*Now my own suspicion is that the Universe is not only queerer than we suppose, but queerer than we* **can** *suppose*". Both relativity and quantum mechanics, the two cornerstones of our current understanding of the physical universe, show us in dramatic fashion how misleading our 'intuition' can be, with regard to the most fundamental aspects of nature. We know now that (i) space is *not* Euclidean, (ii) the Newtonian assumptions of space and time as separate absolutes are not strictly correct, and (iii) space-time is quite complex even at the level up to which we understand it currently (and is likely to turn out to be even more so at the fundamental level). Now, philosophy may lead to some insights into, and systematization of, scientific knowledge. *But it is not, in itself, a reliable way to discover scientific facts.* This is why it is not appropriate to begin scientific enquiry with a pre-conceived set of notions that are postulated to be 'absolute truths' simply because they appear to be self-evident or based on 'common sense'. As Einstein pointed out, "*Common sense* [in this context] *is the collection of prejudices acquired by age eighteen*".

As we have explained subsequently in the main text, what is meant by all this is the following: the 'intuition' we acquire with regard to the natural world around us is based on the behaviour of objects and phenomena we encounter in it in everyday life, and perceive with our senses. For both these reasons, the information thus gathered is restricted to a rather narrow range of variation (or window) of the values of physical quantities such as mass, length, time, velocity, momentum, force, and so on. This is the so-called 'world of middle dimensions'. It informs our beliefs and our expectations ("intuition") regarding the nature of the physical world. Broadly speaking, comprehension of the world of middle dimensions at a certain level, and the ability to deal with it, has been hard-wired into our brains for reasons based on survival in the evolutionary sense. However, there is absolutely no reason to expect that nature itself would be restricted to this narrow range of values of physical quantities, and indeed it is not – in a most spectacular way. It turns out that our own windows of mass, length and time only encompass about seven or eight orders of magnitude, while our present state of knowledge shows that nature and natural phenomena extend over at least *ten times that number of orders of magnitude.* It is hardly surprising, then, that our naive, essentially hard-wired, intuition and common sense are totally inadequate to comprehend the universe unaided. We need the assistance of better, finer and more powerful tools to do so. These are provided by our instruments (to probe nature) and by our mathematics (to analyze our findings).

Our intuition does not apply either to the sub-microscopic world of atoms or, at the other end, to the cosmos of galaxies, since these remote parts of nature do not appear to be immediately relevant to our day-to-day existence.

and retain those capabilities which are most useful for survival. Among these are the abilities to recognize the most important physical features of the world around us, corresponding to our own scales of size and time. These include the properties of space and time as stated by Newton, features of nature that are valid (to a high degree of approximation) at our own level. Thus, these abilities are the result of slow experience of nature by the species over immense stretches of time, but to the individual member of the species they are available at and soon after birth. This is why they seem to be given *a priori*, available *before* any actual experiences during life. From the present perspective, the main point is that all this relates only to our world of normal everyday experiences in life. It does not apply either to the sub-microscopic world of atoms or, at the other end, to the cosmos of galaxies, since these remote parts of nature do not appear to be immediately relevant to our day-to-day existence.

We referred earlier to our intuitive ideas about space and time. Now this intuition, and the way it is built up, also has a fascinating explanation. It is not something available ready-made at birth. Rather, it is acquired in the early years of infancy out of experience, using the abilities and apparatus of sensory perception. This is what biological evolution gives to each of us at birth – not *knowledge* about the important features of nature around us, but the *capacity* to learn about them.

The experiments of the child psychologist Jean Piaget in the 1950's led to remarkable insights into the learning process in infancy and childhood.

The experiments of the child psychologist Jean Piaget (1896–1980) in the 1950's led to remarkable insights into the learning process in infancy and childhood, in particular into learning about space and time. It seems that the regularity of events occurring soon after one another in time is recognized and remembered very early on; while the idea of space comes a little later. Several years later come the concepts of universal space and time common to, and containing, all things and events. Piaget suggested that the order in which children acquire concepts of space is opposite to what is taught at school and college. Broad concepts that are topological in character – the idea of nearness, of the deformability of shapes into one another, and so on – are acquired first; next, concepts of projective geometry such as direction and perspective; and only at the end, the underlying concepts of distance, shape, size, congruence, etc., on which Euclidean geometry is based. But in formal teaching the sequence followed is exactly the opposite. Euclidean geometry is always taught first, at an elementary level. This is followed, at a more mature level of instruction in mathematics (or applications of mathematics), by the concepts of projective geometry. Topology is introduced at an even more advanced level of instruction!

We are born with the capacity to acquire knowledge about nature, not the knowledge itself.

To summarize: We are born with the capacity to acquire knowledge about nature, not the knowledge itself. Natural selection has tuned these capacities to the actual properties of the world of those dimensions (or sizes, or magnitudes) that are directly and immediately relevant to our survival in the natural environment. But we retain no later memories of this learning process that we go through in our infancy; and when we reach adulthood we imagine that we were born with an 'intuitive' knowledge of nature in this domain. Returning to Lorenz, his ideas are well captured in these remarks by Delbrück:

"*... there can be little doubt that our spatial concepts develop in childhood by way of an adaptation to the world in which we live ... the cognitive capacity that permits man to analyze space in geometric terms must, to a large extent, be evolutionarily derived.*"

In an appendix titled 'Physics and Perception' in his book *Special Relativity*, the physicist-philosopher David Bohm (1917–94) describes how a child gradually builds concepts of space and time outside of herself, and the role of memory in the construction of the ideas of past, present and future time. Only by the age of ten or more does a child develop the mental capacity to conceive of a universal space and a universal time common to all objects and events that are perceived. This just goes to show how deep and genuinely difficult these concepts are, and that we are not born with them.

4. Beyond Euclidean Geometry

After this brief excursion into biology, let us come back to mathematics. Euclid's geometry has been admired and studied for over two thousand years. As we have already said, the approach starts with a very small number of fundamental postulates, and builds upon them by logical argument. But from very early times people were puzzled by the status of the fifth postulate of Euclid, also known as the 'parallel postulate'.[5] Was it independent of the first four postulates, or could it be derived from them (in which case there would be no need for an additional fifth postulate)? Eminent thinkers over the centuries pondered over this problem, from the astronomer and geographer Claudius Ptolemy (*circa* 85–165 AD) to the famous mathematician Adrian-Marie Legendre (1752–1833). Many equivalent reformulations of this postulate were found, but the unchanging belief was that the geometry of space had to be Euclidean, for the sake of consistency. Along the way, René Descartes (1596–1650) invented the method of coordinates to represent points in space, so that the powerful mathematics of algebra became available for tackling problems in geometry.[6] As we have seen, Kant subsequently expressed the view that physical space had necessarily to obey the laws of Euclidean geometry, and that there was no other option.

A few decades after Kant, in the early 1800's, the final resolution of this long-standing problem occurred. The fifth postulate was indeed independent of the others, since it could be replaced in a self-consistent man-

[5] Let *L* be a straight line of infinite extent at both ends, and let *P* be a point that does not lie on *L*. The parallel postulate asserts that there is one – and only one – straight line passing through *P* that is parallel to *L*.
See C R Pranesachar, Euclid and 'The Elements', *Resonance*, Vol.12, No.4, 2007.

[6] Like so many fundamental advances, the idea of coordinate geometry first appeared in the appendix to a book! The title of Descartes' book, published in 1637, translates to *Discourse on the Method of Properly Conducting One's Reason and of Seeking the Truth in the Sciences*. Nearly four hundred years later, the titles of scientific books and papers today are somewhat more specialized, if less imposing!

ner by other assumptions, and one could think of alternatives to Euclid's geometry. This breakthrough was achieved independently by three mathematicians–the incomparable Carl Friedrich Gauss (1777–1855), around 1824, building on his theory of curved surfaces; Nikolai Ivanovich Lobachevsky (1792–1856), in 1829; and János Bolyai (1802–60) in 1832. Thus was born the subject of non-Euclidean geometry.[7]

Around the middle of the 19th century, Gauss' gifted student and one of the greatest of mathematicians, Bernhard Riemann (1826–66), took the next big step: the creation of differential geometry. He presented his ideas in his famous probationary lecture titled *On the hypotheses that lie at the foundations of geometry*, given at Göttingen on June 10, 1854. The fundamental insight was to determine the geometry of a space starting from the definition of (the square of) the interval or distance between nearby points in the space. From this, the concepts of tensors, parallel transport, intrinsic differentiation, connection, curvature, etc., could all be developed. This was a stupendous achievement, and many great contributions from a galaxy of geometers followed.[8] In particular, Riemann's methods were powerful enough to deal with spaces of any number of dimensions.

5. Maxwell and the Road to Special Relativity

We now come back to physics. In 1865, not long after Riemann developed differential geometry, one of the greatest scientists of all time, the brilliant physicist James Clerk Maxwell (1831–79), put together his system of coupled partial differential equations for time-dependent electric and magnetic fields–the basis for *all* electromagnetic phenomena. He then discovered the possibility of freely propagating electromagnetic waves, identified them with light, and thus unified three fields of classical physics–electricity, magnetism and optics. Maxwell assumed, tacitly, the Newtonian picture of separate and

[7] There is evidence that Gauss had conceived the idea of non-Euclidean geometry already in 1792, as a precocious teenager. But his first mention of the subject is in a letter written in 1824. Lobachevsky made his discovery in 1826, and published his work in 1829. This was the first formal publication of non-Euclidean geometry. Bolyai arrived at the idea in 1823, but the first publication of his results was dated 1832. All three mathematicians dealt with what we now call hyperbolic geometry, applicable to spaces of negative curvature. Subsequently, Riemann completed the picture with spherical geometry, applicable to spaces of positive curvature.

[8] The list includes Bruno Christoffel (1829–1900), Gregorio Ricci-Curbastro (1853–1925), his son-in-law Tullio Levi-Civita (1873–1941), and Luigi Bianchi (1856–1928), among others. Like the Soviet tradition in the theory of probability, and the later Indian tradition in statistics, Italy has had a fine tradition in geometry.

absolute space and time, and also believed in the existence of a medium, the 'ether', to carry electromagnetic waves.[9]

[9] For a brief but excellent overview of Maxwell's ideas and discoveries, see the article titled *James Clerk Maxwell: a force for physics* by F Everitt, at the website http://physicsworld.com/cws/article/print/26527. It is interesting to note that the terms *field* and *relativity*, both of which are so basic to contemporary physics, were first introduced in their current senses by Maxwell.

However, it soon became clear that there was a deep-seated contradiction between Newton's mechanics and the set of equations encapsulating this new understanding of electromagnetism. The equations of mechanics were unchanged in form (or 'form-invariant') under a set of transformations of the space and time coordinates called Galilean transformations. These transformations allow for the arbitrariness in the origin of the coordinate axes and of time, in the alignment or orientation of the coordinate axes, and in the choice of the inertial frame of reference used to analyze the mechanical system. Time, of course, is assumed to 'flow inexorably' at exactly the same rate in all mutually inertial frames of reference. Astonishingly enough, Maxwell's equations did not share the property of form-invariance–they did *not* remain unchanged in form under Galilean transformations! This, in turn, implied that the ether had a special feature that could be established experimentally: Maxwell proposed electromagnetic experiments which could detect the motion of the earth through the ether, the medium that he believed to be at, and in fact defined, *absolute rest*. (Remember that, in Galilean–Newtonian mechanics, there is no such thing as absolute rest.) However, the famous and crucial experiments of Michelson and Morley,[10] done in 1887, failed to detect the expected effects, and this led to a crisis. Many great figures in physics and mathematics, including Hendrik Antoon Lorentz (1853–1928; Nobel Prize, 1902) and Henri Poincaré (1854–1912) worked on attempts to settle this serious issue.

[10] Albert Abraham Michelson (1852–1931; Nobel Prize, 1907) and Edward Morley (1838–1923).

The final resolution of the problem came in 1905, with Einstein's Special Theory of Relativity. It turned out that Newton's mechanics had to be modified to fall in line with Maxwell's electromagnetism! It was the latter that led to a new and more correct view of space and time, and mechanics had to be made consistent with this view. It became clear that the equations of both mechanics and electromagnetism would have to be form-invariant under a common set of transformations of the space and time coordinates, comprising the so-called Poincaré group (also known as the inhomogeneous Lorentz group). Newton's separate and individually absolute space and time were replaced by a new unified space-time which alone was absolute, the same for all observers. But each observer separates space-time into her own space and her own time in her own way. The earlier Galilean transformations of space and time gave way to new transformations named after Lorentz, and these form the basis of special relativity. Newton had posited that time was universal, the same for everyone, flowing inexorably from the past into the future. If two events occurred simultaneously, all observers would agree on it. But now, with the special theory of relativity, that was shown to be untrue. Simultaneity is not absolute, but depends on the frame of reference. Two events that occur simultaneously according to one observer need not do so for another observer in a different frame of reference. According to the Lorentz transformations, not only the coordinates of points in space, but also the times of events change in a specific manner from one reference frame to another. As a consequence, the distances between objects or events, as well as the time intervals between events, also vary from one frame of reference to another. Even though space and time remain profoundly different in their physical characteristics, the new rules of transformation unite them much more closely than is the case in the Galilean–Newtonian worldview.

The earlier Galilean transformations of space and time gave way to new transformations named after Lorentz, and these form the basis of special relativity.

The distances between objects or events, as well as the time intervals between events, also vary from one frame of reference to another.

Even in special relativity, though, combined space-time remains absolute. Its quantitative characterization by different observers (using metre rods, clocks and their equivalents) is encoded in the Lorentz transformation rules. It serves as the stage for all physical phenomena, *but is unaffected by them.*

6. General Relativity: Dynamical Space-Time

The essentially final step (within the framework of classical physics) in the development of our understanding of space and time occurred with Einstein's General Theory of Relativity, which he completed in 1915. This is a theory of superlative beauty.[11] In it, finally, space and time no longer remain a stage on which physical processes take place, but themselves become actors or active participants in the proceedings. The spirit behind this idea is beautifully expressed by Einstein:

[11] "A field theory *par excellence*" – L D Landau and E M Lifshitz, *The Classical Theory of Fields*, 4th edition, Butterworth–Heinemann, 1980.

"*It is contrary to the mode of thinking in science to conceive of a thing ... which acts itself, but which cannot be acted upon.*"

The very geometry of space-time, which Einstein identified with gravitation, now becomes changeable and dynamical. Geometry acts upon matter and electromagnetism, and in turn is acted upon by them. In the expressive words of Wheeler:[12] "*Space-time tells matter how to move, matter tells space-time how to curve.*" In general relativity, Einstein used extensively the mathematics initiated by Riemann and developed further by Christoffel, Ricci, Levi-Civita and others. But the fundamental physical sense in which he went beyond Riemann was that, from the geometrical point of view, time was treated on the same footing as space, in spite of the profound differences between them. This was something Riemann had not foreseen. It became possible only because special relativity had already been developed (by Einstein himself). A more detailed discussion of general relativity requires mathematics, and we do not go into this here. However, we describe some of the relevant salient features of general relativity in *Box* 4.

[12] The eminent physicist John Archibald Wheeler (1907–2008) was one of the world's leading experts on gravitation. As you may know, he coined the term 'black hole'.

The very geometry of space-time, which Einstein identified with gravitation, now becomes changeable and dynamical.

Box 4. A Paean to General Relativity

There were three unique features about general relativity that Einstein found appealing. They also represented part of the reason for his lifelong opposition to quantum mechanics as an approach to understanding nature. He believed that quantum mechanics was indeed successful in describing nature within its domain of applicability, and that a future unified field theory would have to reproduce the results of quantum mechanics, perhaps as a linear approximation to a deeper nonlinear theory. If true, this would be similar to how the relativistic gravitational field of general relativity (with a finite propagation speed of the gravitational force) led to Newton's law of gravitation (with its action-at-a-distance force) in the non-relativistic limit. But Einstein was convinced that quantum mechanics was not the correct approach to deducing the fundamental laws of physics. The three important features of general relativity were as follows.

• With the equations of general relativity, Einstein found that space-time was no longer just a passive stage on which particles and fields performed their acts, but was an active participant in the performance. Thus, the geometric structure of space-time was determined by the matter in it, and of course the matter responded to this geometry and was constrained by its structure.

• General relativity was the first theory in physics that was inherently nonlinear. An important consequence of this was that the equation of motion of an infinitesimal test particle in the gravitational field of a massive object was contained in the field equations themselves. This equation specified the geodesic path followed by the particle in the curved space-time in which it moved. In other words, one did not have separate equations for the gravitational interactions between the matter present, and for the response of the matter to these interactions. Everything was contained in one set of field equations. In contrast, a linear theory like Maxwell's equations for the electromagnetic field could only describe the (electromagnetic) field manifestation of (charged) matter, while the response of the matter (or inertial manifestation) was contained in separate 'equations of motion'. Indeed, this is true of any linear theory – hence Einstein's opposition to regarding the linear formalism of quantum mechanics as the final or underlying theory.

• For the first time in physics, a theory predicted that the inertia of a system depended on its surroundings. In keeping with Mach's idea that inertia is a consequence of a body's interactions with the rest of the universe, the equations of general relativity showed that the inertia of a system increases when it is placed in the vicinity of other ponderable masses. Inertia was no longer some inherent, given property of a system, but was at least partly determined by the environment. Einstein's unfulfilled dream was to find a fully unified field theory which would show that all of the inertia (and not just part of it) was due to the interactions with the environment.

The well-known physicist Chen-Ning Yang (1922– ; Nobel Prize, 1957) was once asked by Dirac[13] what he (Yang) regarded as Einstein's greatest discovery. When Yang chose general relativity, Dirac disagreed and opined that it had to be special relativity, because it showed for the first time that space and time had to be treated and transformed together.

[13] Paul Adrien Maurice Dirac (1902–84; Nobel Prize, 1933), one of the founders of quantum mechanics.

General relativity provided for the first time a dependable language for discussing problems of cosmology – the behaviour of the universe as a whole. It also led to the study of exotic objects like black holes. On the practical side, the Global Positioning System in current use takes into account the effects of both special and general relativity. If this is not done, the GPS would go haywire within about an hour. In the last few years, a spectacular confirmation of general relativistic effects has been found in a double-pulsar system about 2000 light years away from us. The pulsars, separated by a distance of a million kilometres, orbit around each other with a time period of 2.4 hours. This time period is affected by the strong gravitational fields present (about 100,000 times stronger than that due to the sun). A comparison of the observed variations in the interval between the pulses arriving here from the system with the calculated values shows that the predictions of general relativity are borne out to an astonishing accuracy of 99.95%.

Soon after Einstein presented general relativity, Hermann Weyl[14] in 1917–18 suggested an extension of Riemannian geometry for space-time, involving what he called the gauge principle, which he thought would unify gravitation and Maxwell's classical electromagnetism. A few years later, Theodore Kaluza (1885–1954) in 1921 and Oskar Klein (1894–1977) in 1926 independently published their ideas on another way to unify the two forces, based on increasing the number of dimensions of space-time from four to five – namely, four spatial dimensions and one time dimension. Klein, in fact, pioneered the

[14] Hermann Weyl (1885–1955) was one of the great mathematicians – today, we would also rank him as a pre-eminent mathematical physicist – of the 20th century. His book *Raum-Zeit-Materie* (*Space-Time-Matter*), first published in 1918, was highly influential for several decades in spreading the ideas of relativity among physicists and mathematicians. Weyl also played a prominent role in applying the ideas of symmetry and group theory to physics, including quantum mechanics.

idea that the extra space dimension was actually 'curled up' into a tiny circle of radius of the order of the Planck length, $l_P = \sqrt{Gh/c^3} \sim 10^{-35}$ metres. He also advanced the idea that the 'quantization' of electric charge could be related to a topological feature such as the curling up of a dimension into a circle. These ideas were extremely original and beautiful, but did not succeed or lead to any progress at that time. Many decades later, however, they have become important, in a modified and extended form, in the context of string theory.

7. Space and Time in Quantum Physics

All that we have discussed so far has been in the realm of classical physics. We must now give at least a brief account of the surprising results that emerge in quantum theory *vis-à-vis* space and time.

The initial form of quantum mechanics, completed during the period 1925–27, assumed that space and time were Newtonian, i.e., non-relativistic, as we would say today. This suffices for most of chemistry, atomic, molecular and even nuclear physics.[15] Soon after, in 1928, Dirac found a description of the electron which combined the (then) new quantum mechanics with special relativity and was spectacularly successful. From the 1930's to the 1970's and beyond, this led to the impressive development of quantum field theory in which, too, space-time is treated according to special relativity–so that once again it (space-time) remains a stage for phenomena. The theories of the strong, electromagnetic and weak interactions are all of this form. Along the way, in 1956, Tsung-Dao Lee (1926– ; Nobel Prize, 1957) and Yang found that in the weak processes, such as those responsible for radioactive decays, space reflection or parity is not a valid symmetry. That is, nature does distinguish between left-handedness and right-handedness at a fundamental level, contrary to what one might expect 'intuitively'.

[15] With the important proviso that the effects of *spin* are to be incorporated. The *origin* of spin is often believed to lie in relativistic quantum physics, but is actually a logical and rigorous consequence of non-relativistic quantum mechanics.

Nature does distinguish between left-handedness and right-handedness at a fundamental level.

The inexorable march of time gives us the notion of an *arrow of time* that always points to the future.

Analogous to space reflection, we have time reflection or time reversal. The situation here is quite subtle. The inexorable march of time gives us the notion of an *arrow of time* that always points to the future. A drop of ink that falls into a glass of water disperses spontaneously in the water, till it becomes virtually invisible. It does not re-assemble into the original drop, no matter how long we wait. All of us age and grow older; we never evolve 'backwards' from old age to infancy. And yet, the laws governing almost all kinds of time evolution, both classical and quantum mechanical, are – *at the fundamental level* – almost always time-reversal invariant. That is, they remain unchanged when the sign of the time variable is changed. The deep question, then, is how this *microscopic reversibility* in time leads, nevertheless, to *macroscopic irreversibility*. The answer is a subject in itself, and we shall not go into it here. It is related, as you might guess, to the statistical nature of the second law of thermodynamics and the mechanism by which entropy increases spontaneously. It is also related to dynamical chaos (or exponential sensitivity to initial conditions), and the impossibility of specifying these conditions with infinite precision. We must also mention that a complete answer that is fully within the purview of quantum mechanics is not yet known, and that the matter is the subject of on-going research.

But there is yet another twist to the tale. There is now indirect evidence that there is a certain violation of time-reversal symmetry in the fundamental physical laws themselves. This is based on the discovery in 1964 of what is called CP-violation in certain elementary particle decays, for which James W Cronin (1931–) and Val L Fitch (1923–) were awarded the Nobel Prize in 1980. A consequence of the violation of time-reversal invariance is that a particle such as an electron, or a neutron, or a free atom will have an intrinsic *electric* dipole moment.[16] The search for such an electric dipole mo-

[16] As you may be aware, protons and neutrons (via the quarks of which they are composed), as well as electrons, have intrinsic magnetic dipole moments related to the spins of these particles. This does not violate time-reversal invariance. We are now speaking of intrinsic electric dipole moments, whose existence would indicate such a violation.

ment is an important endeavour in experimental physics, as it amounts to a test of the validity of a fundamental principle of nature. Several laboratories around the world are currently engaged in this pursuit (e.g. at Harvard, Yale, Colorado, Washington, and Princeton in the US, and Imperial College and KVI Groningen in Europe), including that of one of the authors (VN) at the Indian Institute of Science. The violations of parity invariance and time-reversal invariance mentioned above suggest that nature has some asymmetry built into it: elementary particle interactions do not exhibit the degree of symmetry with regard to space-time itself that one might expect *a priori*. These asymmetries, in turn, have implications in cosmology and the manner in which the universe has evolved from the Big Bang. As you can see, all these aspects go to show how deeply intertwined are the apparently diverse parts of the subject of physics–clearly, nature is above these artificial distinctions.

The violations of parity invariance and time-reversal invariance suggest that nature has some asymmetry built into it.

8. What Does the Future Hold?

Finally, we come to the problem of combining general relativity and quantum mechanics. This problem has been attacked since the early 1930's, but it has proved to be uncommonly difficult. In a very real sense, it may be regarded as an 'ultimate problem' of sorts, at least as far as the physics built up over the last four centuries or so is concerned. As the jury is still out on this question what we go on to say now must be regarded as tentative and speculative.

In recent decades, some promising lines of progress may have appeared in the attempts to tackle the problem. They involve quite novel ideas and, along with them, complicated mathematics to implement these ideas in a consistent manner. In essence, it appears that the ingredients or features necessary for a consistent and satisfactory amalgamation of gravity and quantum mechanics

The problem of combining general relativity and quantum mechanics is an 'ultimate problem' of sorts.

There could be an underlying symmetry between bosons and fermions called *supersymmetry* that is spontaneously broken in the present-day universe.

are likely to be, at the very least, the following: (i) the replacement of zero-dimensional point-like particles as the ultimate fundamental objects in the universe by extended objects, specifically, one-dimensional strings ; (ii) a space-time with a dimensionality higher than four– perhaps a space-time with nine spatial dimensions and one time dimension, six of the spatial dimensions being curled up (or 'compactified') to a size within the Planck length, but in a far more (topologically) intricate and complicated manner than Klein anticipated; and (iii) an underlying symmetry between bosons and fermions called *supersymmetry* that is spontaneously broken in the present-day universe. Taken together, they constitute the basic features of the so-called superstring theory.

What is slowly emerging is even more interesting, and cuts even deeper. Over and above the features listed above, there appear to be theoretical indications that the resolution of the problem will entail even more drastic revisions of our understanding of both quantum mechanics and space-time itself. For instance: this universe may be only one of a vast (or even infinite) array of ever-branching-out universes in a 'multiverse'; the dimensionality of space-time may emerge as a dynamical property of this universe; there may be an ultimate underlying granularity or discreteness in the very notions of length and time durations; and space-time coordinates (even including the possible higher-dimensional ones) may have to be augmented by other, 'internal' variables of a different kind in order to provide a complete description of nature at the most fundamental level.

These are but theoretical and mathematical speculations, as yet. Time will tell (!) whether they are correct, and to what extent. While there has been a great deal of progress on the theoretical side over the past twenty-five years or so, experimental advances in elementary particle physics have lagged behind. This phase lag is rather an anomaly, and the experience of the past four hundred years suggests strongly that true progress will most likely be made only when the gap between experiment and theory is reduced considerably. This is one of the leading reasons why physicists are eagerly awaiting the first results from the Large Hadron Collider (LHC), which is expected to go on stream at CERN, Geneva, in the very near future.

But it is good to conclude this account here, seeing how far we have come from the Egyptian farmers toiling on the banks of the Nile, from whose mud the subject of geometry was born – aided by the upward look into the sky, and the challenge posed by the splendour of the cosmos stretching out before the eyes of humankind.

Address for Correspondence
Vasant Natarajan
Indian Institute of Science
Bangalore 560 012, India.
Email:
vasant@physics.iisc.ernet.in

V Balakrishnan
Department of Physics
Indian Institute of Technology
Chennai 600 036, India.
Email:
vbalki@physics.iitm.ac.in

N Mukunda
103, 6th Main Road
Malleswaram
Bangalore 560 003, India.
Email:
nmukunda@gmail.com

Einstein, (shown here with Niels Bohr in a photo taken by Paul Ehrenfest in 1925) was opposed all his life to the "Copenhagen interpretation'' of quantum mechanics championed by Bohr. The result was several public debates with Bohr in the 1920s, and in 1926 Einstein wrote in a letter "I, at any rate, am convinced that He [God] does not throw dice.'' In 1935, he wrote a path-breaking paper on what is now called the EPR paradox, which brought out the inherent nonlocality in quantum mechanics.

From: *Wikipedia*