MiniBooNE results and neutrino schemes with 2 sterile neutrinos: possible mass orderings and observables related to neutrino masses

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MiniBooNE results and neutrino schemes with 2 sterile neutrinos: possible mass orderings and observables related to neutrino masses

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Abstract: The MiniBooNE and LSND experiments are compatible with each other when two sterile neutrinos are added to the three active ones. In this case there are eight possible mass orderings. In two of them both sterile neutrinos are heavier than the three active ones. In the next two scenarios both sterile neutrinos are lighter than the three active ones. The remaining four scenarios have one sterile neutrino heavier and another lighter than the three active ones. We analyze all scenarios with respect to their predictions for mass-related observables. These are the sum of neutrino masses as constrained by cosmological observations, the kinematic mass parameter as measurable in the KATRIN experiment, and the effective mass governing neutrinoless double beta decay. It is investigated how these non-oscillation probes can distinguish between the eight scenarios. Six of the eight possible mass orderings predict positive signals in the KATRIN and future neutrinoless double beta decay experiments. We also remark on scenarios with three sterile neutrinos. In addition we make some comments on the possibility of using decays of high energy astrophysical neutrinos to discriminate between the mass orderings in presence of two sterile neutrinos.

Keywords: Neutrino Physics, Beyond Standard Model.

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1. Introduction

The long awaited results of the MiniBooNE experiment [1] showed that scenarios in which one sterile neutrino is added to the three active ones are incompatible with data. Such schemes were motivated by the results from the LSND experiment [2], which observed flavor transitions interpreted as $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ neutrino oscillations. A number of authors [3–7] investigated the implications of such schemes. In particular, it was realized [8] that so-called 2+2 scenarios (two pairs of neutrinos close in mass separated by a large gap)

are ruled out and that only 3+1 scenarios (three mostly active neutrinos separated by a large gap from the mostly sterile one) are allowed, though only small part of the parameter space survived. The MiniBooNE results ruled out even this part [9] by excluding the LSND parameter space at the 98 % C.L. [1].

Allowing one more sterile neutrino to enter the stage improves the compatibility of LSND with other experiments [10, 11] and in particular renders the MiniBooNE and LSND experiments compatible [9]. Only comparably few models for neutrino schemes with two extra sterile neutrinos have been constructed [12], and the potentially rich phenomenology of such scenarios is hardly investigated [13, 14].

With two sterile neutrinos added to the usual three, one has eight possible mass orderings, which should be compared with the two schemes (normal and inverted ordering) in case of "only" three active neutrinos. We study in this paper the predictions of the eight cases for mass-related observables. We investigate the sum of neutrino masses as constrained by cosmological observations, the kinematic mass parameter as measurable in the KATRIN experiment (partly analyzed also in ref. [10]), and the effective mass controlling neutrinoless double beta decay.[1] We also investigate how and if mass-related observables can contribute to distinguish the possibilities. The mass patterns in the 3+2 scheme can be classified in three main classes:

- two 2+3 scenarios: the two sterile neutrinos are heavier than the three active ones;
- two 3+2 scenarios: the two sterile neutrinos are lighter than the three active ones;
- four 1+3+1 scenarios: one sterile neutrino is heavier than the three active ones which in turn are heavier than the second sterile neutrino.

The paper is build up as follows: first we summarize the required formalism in section 2 before outlining in section 3 the eight possible mass orderings for scenarios with two sterile neutrinos. In section 4 we study the mass-related observables for the two scenarios which have the sterile neutrinos heavier than the active ones, while in section 5 the two scenarios in which the sterile neutrinos are lighter than the active neutrinos are analyzed. Section 6 contains the four 1+3+1 scenarios and a short discussion on scenarios with three sterile neutrinos is delegated to appendix A. In appendix B we discuss another interesting possibility to distinguish between the different scenarios outlined above at neutrino telescopes, allowing high energy astrophysical neutrinos to decay. Finally, in section 7 we discuss and summarize our findings.

2. Formalism

2.1 Neutrino mixing

Neutrino mixing is described by the leptonic mixing, or Pontecorvo-Maki-Nakagawa-Sakata

[1] For related analyzes in different sterile neutrino scenarios, see, e.g., ref. [15].

(PMNS) matrix U:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & U_{e5} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & U_{\mu 5} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & U_{\tau 5} \\ U_{s_1 1} & U_{s_1 2} & U_{s_1 3} & U_{s_1 4} & U_{s_1 5} \\ U_{s_2 1} & U_{s_2 2} & U_{s_2 3} & U_{s_2 4} & U_{s_2 5} \end{pmatrix} . \tag{2.1}$$

It links the mass eigenstates $\nu_{1,2,3,4,5}$ having masses $m_{1,2,3,4,5}$ with the three active flavor states $\nu_{e,\mu,\tau}$ and the two new sterile states ν_{s_1} and ν_{s_2}. In what follows we denote the mostly active neutrinos with $\nu_{1,2,3}$. Regardless of their ordering (normal or inverted) we have

$$|U_{e1}|^2 \simeq \cos^2\theta_\odot\,,\ |U_{e2}|^2 \simeq \sin^2\theta_\odot \ \text{ and } \ |U_{e3}|^2 \simeq \sin^2\theta_{\text{CHOOZ}}\,, \tag{2.2}$$

where $\theta_\odot$ is the mixing angle for solar and KamLAND neutrinos and θ_{CHOOZ} the mixing angle for short baseline reactor neutrinos. We further have the mass-squared differences governing solar and atmospheric neutrino oscillations. In the following, we will use the following best-fit and 3σ ranges [16]:

$$\sin^2\theta_\odot = 0.30^{+0.08}_{-0.05} \quad \text{with} \quad \Delta m^2_\odot = \left(8.0^{+1.0}_{-1.0}\right)\cdot 10^{-5}\ \text{eV}^2\,, \tag{2.3}$$

$$\sin^2\theta_{\text{CHOOZ}} = 0.00^{+0.04}_{-0.0} \quad \text{with} \quad \Delta m^2_{\text{A}} = \left(2.6^{+0.6}_{-0.6}\right)\cdot 10^{-3}\ \text{eV}^2\ . \tag{2.4}$$

The related typical mass scales are therefore $\sqrt{\Delta m^2_\odot} \simeq 0.009$ eV and $\sqrt{\Delta m^2_{\text{A}}} \simeq 0.05$ eV, respectively. In what regards the two additional sterile neutrinos, the analysis in ref. [9] resulted in the following best-fit values[2]

$$\begin{aligned} \Delta m^2_{\text{s1}} &= 6.49^{+1.0}_{-1.0}\ \text{eV}^2 \quad \text{with} \quad |U_{e5}| = 0.12\,, \\ \Delta m^2_{\text{s2}} &= 0.89^{+0.1}_{-0.1}\ \text{eV}^2 \quad \text{with} \quad |U_{e4}| = 0.11\ . \end{aligned} \tag{2.5}$$

In what follows we will give for our observables explicit numerical values obtained with not only these best-fit values, but also for another typical illustrative point in the parameter space:

$$\begin{aligned} \Delta m^2_{\text{s1}} &= 1.90^{+0.60}_{-0.90}\ \text{eV}^2 \quad \text{with} \quad |U_{e5}| = 0.12\,, \\ \Delta m^2_{\text{s2}} &= 0.90^{+0.05}_{-0.20}\ \text{eV}^2 \quad \text{with} \quad |U_{e4}| = 0.11\ . \end{aligned} \tag{2.6}$$

The main feature of this point is of course the smaller overall neutrino mass it implies. It corresponds approximately to the center of another, isolated region allowed at 90 % C.L. of figure 6 from ref. [9]. The two central points from eqs. (2.5) and (2.6) are quite typical for the situation in the presence of two sterile neutrinos and we will make frequent use of them. With the values of the two new mass-squared differences we can estimate typical neutrino mass scales, which we will encounter frequently in the following: $\sqrt{\Delta m^2_{\text{s1}}} \simeq 2.55\ (1.38)$ eV, $\sqrt{\Delta m^2_{\text{s2}}} \simeq 0.94\ (0.95)$ eV, $\sqrt{\Delta m^2_{\text{s1}} - \Delta m^2_{\text{s2}}} \simeq 2.37\ (1.00)$ eV, and $\sqrt{\Delta m^2_{\text{s1}} + \Delta m^2_{\text{s2}}} \simeq 2.72\ (1.67)$ eV.

[2]We consider only the analysis which uses the MiniBooNE results above reconstructed neutrino energies of 475 MeV, because results from the lowest energy bin are not well understood [1].

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scheme	Σ	m_β	$\langle m \rangle$
NH	$\sqrt{\Delta m^2_{\rm A}}$	$\sqrt{\sin^2\theta_\odot\,\Delta m^2_\odot + \sin^2\theta_{\rm CHOOZ}\,\Delta m^2_{\rm A}}$	$\left\lvert \sin^2\theta_\odot \sqrt{\Delta m^2_\odot} + \sin^2\theta_{\rm CHOOZ}\sqrt{\Delta m^2_{\rm A}}\, e^{i\alpha_{32}} \right\rvert$
IH	$2\sqrt{\Delta m^2_{\rm A}}$	$\sqrt{\Delta m^2_{\rm A}}$	$\sqrt{\Delta m^2_{\rm A}}\sqrt{1-\sin^2 2\theta_\odot \sin^2\alpha_2/2}$
QD	$3\,m_0$	m_0	$m_0\sqrt{1-\sin^2 2\theta_\odot \sin^2\alpha_2/2}$

Table 1: Extreme limits of 3-flavor scenarios and the resulting mass-related observables. We have defined $\alpha_{32} = \alpha_3 - \alpha_2$.

In ref. [9] the allowed ranges of the mixing matrix elements $|U_{e4}|$ and $|U_{e5}|$ are not given. However, we will see in our discussions that especially for neutrinoless double beta decay it is sometimes important to analyze the impact of varying these parameters. In absence of any information we have varied $|U_{e4}|$ and $|U_{e5}|$ by 50% around the best-fit points in eqs. (2.5) and (2.6). Thus we consider the following ranges for the parameters $|U_{e4}|$ and $|U_{e5}|$:

$$|U_{e5}| = 0.12^{+0.06}_{-0.06} \quad \text{and} \quad |U_{e4}| = 0.11^{+0.05}_{-0.05}\,. \tag{2.7}$$

for both the best-fit point and the second illustrative point considered in eq. (2.6).

2.2 Neutrino masses

As neutrino oscillations are sensitive only to mass-squared differences, the neutrino mass scale is not known, but only limited from above by different experiments and observations. Typically, the mass scale is inversely proportional to the scale of the mechanism which is responsible for neutrino mass. Therefore, knowing the mass is a very important step towards the understanding of neutrino physics.

Mass-related observables are the sum of neutrino masses

$$\sum = \sum_i m_i\,, \tag{2.8}$$

which can be inferred from cosmological observations. Typical limits are smaller than about 1 eV [17 – 20], but they depend on the used data sets, the number of neutrino species and how the mass is distributed among the different neutrinos. We will discuss this in more detail in section 7. One also has the kinematic neutrino mass parameter measurable in β-decay experiments

$$m_\beta = \sqrt{\sum_i |U_{ei}|^2\, m_i^2}\,. \tag{2.9}$$

This quantity is measured when the electron energy interval around the endpoint of the investigated beta decay is much larger than the m_i, otherwise corrections to this formula are required [21].

This condition is fulfilled for the values of the neutrino masses we choose.[3] The current limit on m_β is 2.2 eV at 95% C.L. [23], and improvement by one order of magnitude is expected by the KATRIN experiment [22]. Finally, we have the effective mass in neutrinoless double beta decay:

$$\langle m \rangle = \left| \sum_i U_{ei}^2 \, m_i \right| = \left| \sum_i |U_{ei}|^2 \, e^{i\alpha_i} \, m_i \right| \; . \tag{2.10}$$

Here $\alpha_{2,3,4,5}$ are the four possible and unknown[4] Majorana phases (we can choose $\alpha_1 = 0$). Applying nuclear matrix element uncertainties on the current 90% C.L. limits on lifetimes [24, 25] gives limits on the effective mass in the range of 1 eV, and sizable improvement is expected also in this field [26].

Let us summarize for the sake of comparison the situation in 3-flavor scenarios (see for instance [27, 28]). We have three extreme cases of the mass ordering, the normal hierarchy (NH, $m_3^2 \simeq \Delta m^2_{\rm A} \gg m_2^2 \simeq \Delta m^2_\odot \gg m_1^2$), the inverted hierarchy (IH, $m_2^2 \simeq m_1^2 \simeq \Delta m^2_{\rm A} \gg m_3^2$) and quasi-degenerate neutrinos (QD, $m_3^2 \simeq m_2^2 \simeq m_1^2 \equiv m_0^2 \gg \Delta m^2_{\rm A}, \Delta m^2_\odot$). Table 1 shows the results for the mass-related observables. Obviously, m_β is unobservably low for NH and IH, while $\langle m \rangle$ and Σ are for NH. It may be possible to probe the inverted hierarchy regime through future cosmological observations [17].

Certain mass orderings to be discussed in the following will have problems with some of the three observables Σ, m_β or $\langle m \rangle$. Does this mean that they are ruled out? Not necessarily, because cosmological neutrino mass [29] (and number [30]) limits can in principle be evaded, or relaxed by a factor of a few, by means of unknown neutrino interactions or other cosmological features. Furthermore, confronting a mass ordering with the limit on the effective mass makes only sense when neutrinos are Majorana particles, which however is a very well justified assumption. Only the kinematic parameter m_β does not suffer from any underlying model assumption and provides an unambiguous test. We will leave aside discussions of the validity of the different limits in particular on Σ. Our aim is simply to study the predictions for the observables for all eight possible mass orderings, which we will outline in the next section.

3. Eight possible mass orderings in neutrino scenarios with two sterile neutrinos

As mentioned in the Introduction, analyzes of the LSND and MiniBooNE, as well as various other experiments, give a consistent picture only if two additional independent mass-squared differences $\Delta m^2_{\rm s1} > \Delta m^2_{\rm s2}$ are present. We assume here that the difference

[3] In principle, the analysis of ref. [9] allows sterile neutrino mass values of $\Delta m^2 \geq 10$ eV^2, which are indeed close to the energy interval used by the upcoming KATRIN experiment [22]. These values are however in very strong conflict with all mass-related observables and we therefore omit them. For the values used we estimate corrections to eq. (2.9) to be at most of order 10%.

[4] In fact, the analysis of neutrino scenarios with two sterile neutrinos gives some constraint on a CP phase [9] (see also [11]). Being a "Dirac-phase", however, it does not appear in $\langle m \rangle$.

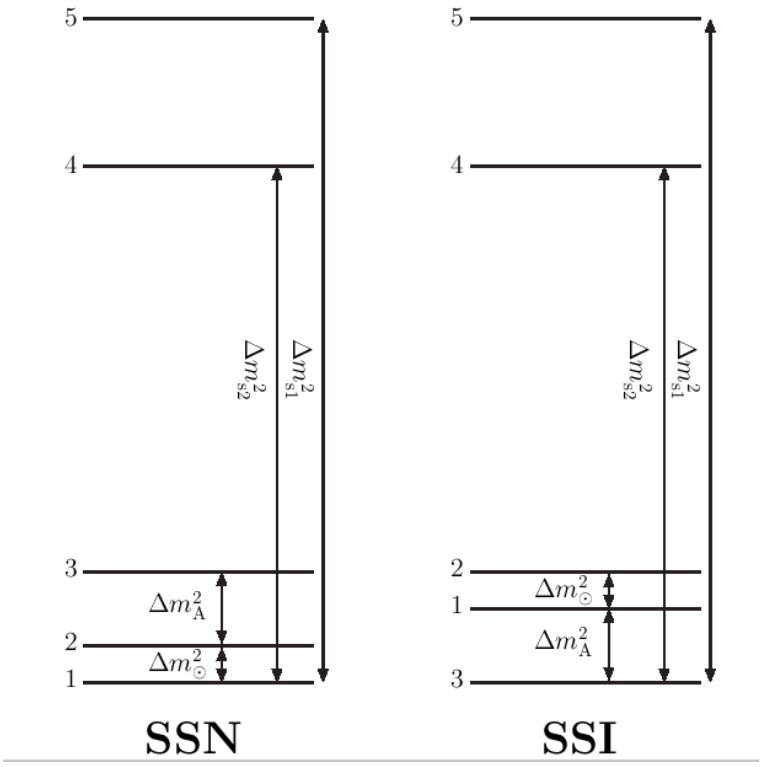

Figure 1: Allowed 2+3 mass orderings which are defined by having two sterile neutrinos heavier than the three active neutrinos. (Not to scale).

between the two, $\Delta m^2_{ss} \equiv \Delta m^2_{\rm s1} - \Delta m^2_{\rm s2}$, is much larger than $\Delta m^2_{\rm A}$, an assumption used in the 3+2 analyzes in [9] from where we take the values of the additional parameters.

Our convention is the following: we call the masses of the three predominantly active neutrinos m_3, m_2 and m_1. The mixing among them is responsible for the solar, atmospheric and short baseline reactor neutrino oscillation results. If they are normally ordered, then $m_3 > m_2 > m_1$ with

$$m^2_3 - m^2_2 = \Delta m^2_{\rm A} \quad \text{and} \quad m^2_2 - m^2_1 = \Delta m^2_{\odot} \; . \tag{3.1}$$

They can also be inversely ordered, in which case $m_2 > m_1 > m_3$ and

$$m^2_1 - m^2_3 = \Delta m^2_{\rm A} \quad \text{and} \quad m^2_2 - m^2_1 = \Delta m^2_{\odot} \; . \tag{3.2}$$

We have to add two predominately sterile neutrino states, whose masses we denote by m_4 and m_5. They can either be heavier or lighter than the three active neutrinos (in what follows, we will omit for simplicity the word "predominately" or "mostly" in front of "sterile neutrinos" and "active neutrinos"). Without loss of generality, we can choose that m_5 is either the largest or the smallest mass and associate it always with $\Delta m^2_{\rm s1}$, while m_4 is always associated with $\Delta m^2_{\rm s2}$. In this case, we do not have to rename the matrix elements U_{e4} and U_{e5}, which quantify the mixing of the two additional neutrinos with the

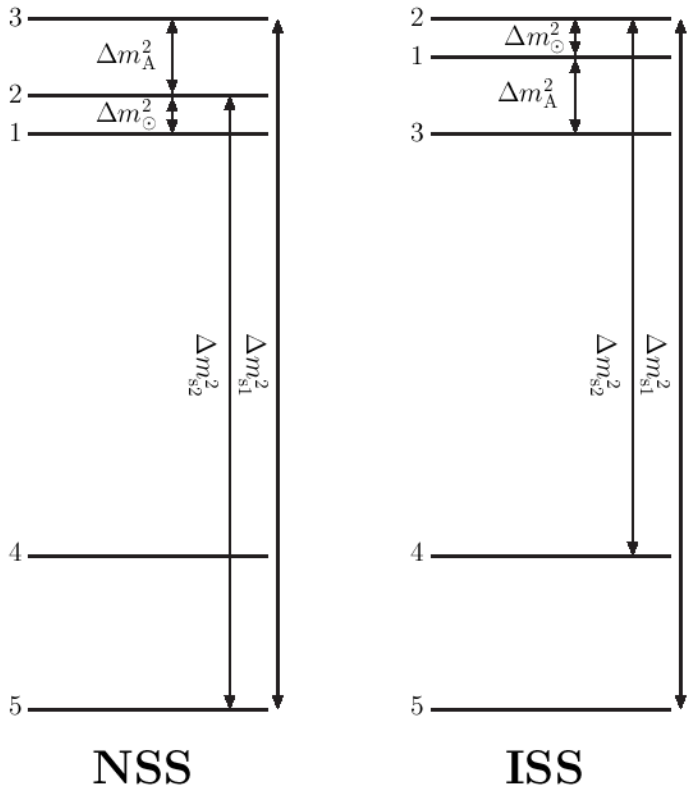

Figure 2: Allowed 3+2 mass orderings which are defined by having two sterile neutrinos lighter than the three active neutrinos. (Not to scale).

electron neutrino. In general, one could choose the labels of the masses such that always $m_5 > m_4 > m_3 > m_2 > m_1$ holds. In this case, however, the mixing matrix elements would be different for each of the possible mass orderings. With our convention, the values of $|U_{ei}|$ are fixed by eqs. (2.2) and (2.5), (2.6) and do not have to be relabeled.

Let us first discuss the case of *both* sterile neutrinos being either heavier or lighter than the three active ones: the largest of the two independent new mass-squared differences, $\Delta m^2_{\rm s1}$, is then always the largest possible mass-squared difference. If the two sterile states are above the three active ones ("2+3 scenarios"), then $\Delta m^2_{\rm s2}$ is the mass-squared difference between the lightest sterile state (which is also the second heaviest state) and the lightest available state (which is active). If the two sterile states are below the three active ones ("3+2 scenarios"), then $\Delta m^2_{\rm s2}$ is the mass-squared difference between the heaviest state (which is active) and the heaviest sterile state (which is the second lightest state). The four possible schemes are shown in figures 1 and 2. The names of the schemes are defined as follows: depending on whether the active neutrinos are normally or inversely ordered, the scheme has the capital letter "N" or "I" in its name. Depending on whether the two sterile neutrino masses m_4 and m_5 are lighter or heavier than the active ones, the capital letters "SS" appear after or in front of this capital letter. For instance, if $m_5 > m_4 > m_3 >$

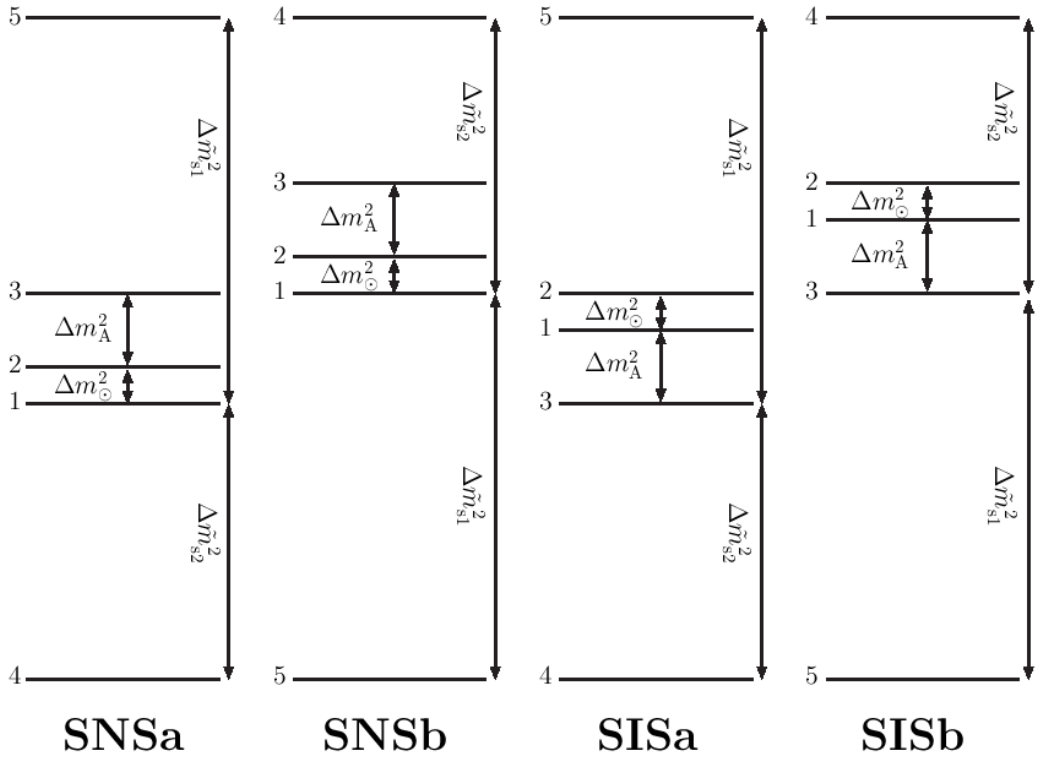

Figure 3: Allowed 1+3+1 mass orderings which are defined by having one sterile neutrino heavier than the three active ones which in turn are heavier than the second sterile neutrino. (Not to scale). Note that not necessarily $\Delta\tilde{m}^2_{s1} = \Delta m^2_{s1}$ and $\Delta\tilde{m}^2_{s2} = \Delta m^2_{s2}$ holds.

$m_2 > m_1$, then we call the scenario SSN, while for $m_2 > m_1 > m_3 > m_4 > m_5$ we call it ISS.

The last possible class of mass orderings is when one sterile neutrino is heavier then the three active ones, which in turn are heavier than the second sterile neutrino ("1+3+1 scenarios"). The heaviest neutrino can either be separated by Δm^2_{s1} or Δm^2_{s2} from the active neutrinos. If it is separated by Δm^2_{s1} (Δm^2_{s2}) then we call the scenario SNSa or SISa (SNSb or SISb). The possibilities are shown in figure 3. We note here that the fit of ref. [9], and also the analyzes of refs. [10, 11], do strictly speaking not apply to these schemes. The reason is that in the oscillation probabilities for $\nu_\mu \to \nu_e$ transitions there are not only terms proportional to $\sin^2 \Delta m^2_{s1} \frac{L}{4E}$ and to $\sin^2 \Delta m^2_{s2} \frac{L}{4E}$, but also an interference term proportional to $\cos(\Delta\tilde{m}^2 \frac{L}{4E} + \delta)$, where δ is a CP phase (which does not appear in survival probabilities). Refs. [9 – 11] make the implicit assumption $|\Delta\tilde{m}^2| = |\Delta m^2_{s1}| - |\Delta m^2_{s2}|$, which is not fulfilled for the 1+3+1 scenarios. In lack of any detailed fit of the data within these schemes we will assume for simplicity that the mass-squared differences are the same. The values of the mass-squared differences resulting from fits taking into account the 1+3+1 case will remain of course in the eV range.

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Summarizing we end up with eight different schemes.[5] We can already at the present stage make some general statements. First of all, the sum of neutrino masses depends basically only on the new mass-squared differences and typical values will be

$$\Sigma \geq \sqrt{\Delta m^2_{\rm s1}} + \sqrt{\Delta m^2_{\rm s2}} \quad \text{or} \quad \Sigma \geq 3\sqrt{\Delta m^2_{\rm s1}} \quad \text{or} \quad \Sigma \geq 3\sqrt{\Delta m^2_{\rm s2}}\,, \tag{3.3}$$

depending on the details of the mass ordering. In general, all of them are expected to face serious problems with cosmology, and the smaller the mass-squared differences $\Delta m^2_{\rm s1}$ and $\Delta m^2_{\rm s2}$ are, the smaller Σ. Another point worth mentioning is that the effective mass governing neutrinoless double beta decay can be written as

$$\begin{aligned} \langle m \rangle &= \left| \langle m \rangle^3 + \langle m \rangle^{\rm st} \right| , \\ \text{where} \quad \langle m \rangle^3 &\equiv \cos^2\theta_\odot\, m_1 + \sin^2\theta_\odot\, m_2\, e^{i\alpha_2} + \sin^2\theta_{\rm CHOOZ}\, m_3\, e^{i\alpha_3} \\ \text{and} \quad \langle m \rangle^{\rm st} &\equiv |U_{e4}|^2\, m_4\, e^{i\alpha_4} + |U_{e5}|^2\, m_5\, e^{i\alpha_5}\ . \end{aligned} \tag{3.4}$$

Obviously, $|\langle m \rangle^3|$ is an effective mass similar to the one analyzed in the usual three-flavor situation [27], cf. table 1. The quantity $|\langle m \rangle^{\rm st}|$ is the contribution from the two sterile states. We will encounter in the following all cases: dominance of the sterile contribution, dominance of the active contribution, and equal-sized contributions, leading potentially to complete cancellation. The same cases are also present for the kinematic neutrino mass m_β, where however no cancellation is possible as it is given by an incoherent sum.

We will discuss now the three mass-related observables for the eight possible mass orderings. We give approximate analytic expressions for these observables in the limit of vanishing smallest mass and $\sin^2\theta_{\rm CHOOZ}$. We use these expressions to give illustrative numerical values in each case for the best-fit values of the oscillation parameters (or for the second illustrative central point from eq. (2.6)). We will also plot the observables as a function of the smallest neutrino mass for the central points as well as by varying the parameters in their corresponding allowed ranges from eqs. (2.3), (2.5), (2.6).

4. Sterile neutrinos heavier than active neutrinos: 2 + 3 scenarios

4.1 Scheme SSN

In this scheme, $m_5 > m_4 > m_3 > m_2 > m_1$. The three lowest states account for the solar and atmospheric neutrino mass-squared differences according to eq. (3.1). We have

$$\Delta m^2_{\rm s1} = m_5^2 - m_1^2 \quad \text{and} \quad \Delta m^2_{\rm s2} = m_4^2 - m_1^2\ . \tag{4.1}$$

Schematically, this scheme is shown in figure 1. We can express the individual masses in terms of the smallest mass m_1 and the independent mass-squared differences:

$$\begin{aligned} m_2 &= \sqrt{\Delta m^2_\odot + m_1^2}\,, \qquad & m_3 &= \sqrt{\Delta m^2_{\rm A} + \Delta m^2_\odot + m_1^2}\,, \\ m_4 &= \sqrt{\Delta m^2_{\rm s2} + m_1^2}\,, \qquad & m_5 &= \sqrt{\Delta m^2_{\rm s1} + m_1^2}\ . \end{aligned} \tag{4.2}$$

[5] In scenarios with three sterile neutrinos one would have 16 possible mass orderings, see appendix A.

The typical masses are therefore $m_2 \simeq \sqrt{\Delta m^2_\odot} \simeq 0.01\,\text{eV}$, $m_3 \simeq \sqrt{\Delta m^2_\text{A}} \simeq 0.05\,\text{eV}$, $m_4 \simeq \sqrt{\Delta m^2_{\text{s}2}} \simeq 0.94\ (0.95)\,\text{eV}$ and $m_5 \simeq \sqrt{\Delta m^2_{\text{s}1}} \simeq 2.55\ (1.38)\,\text{eV}$. The three lightest neutrinos have the same values as in a normal hierarchical 3-flavor framework. The upper left plot in figure 4 shows the individual masses as a function of the smallest mass in this picture. The limit where all five neutrinos are quasi-degenerate comes when the smallest mass is beyond 1 eV.

If this scheme is realized then the sum of neutrino masses which is constrained from cosmology is given as

$$\begin{aligned}\Sigma^{\text{SSN}} &\simeq \sqrt{\Delta m^2_\odot} + \sqrt{\Delta m^2_\text{A}} + \sqrt{\Delta m^2_{\text{s}1}} + \sqrt{\Delta m^2_{\text{s}2}} \\ &\simeq \sqrt{\Delta m^2_{\text{s}1}} + \sqrt{\Delta m^2_{\text{s}2}} \simeq 3.49\ (2.32)\ \text{eV}\ . \end{aligned} \tag{4.3}$$

Neglecting m_1 is a good approximation as long as $m_1 \lesssim 0.1\,\text{eV}$. For such values, Σ lies roughly between 3.3 and 3.8 (or 1.9 and 2.6) eV, but it can reach unrealistically large values of 10 eV for a smallest mass in the eV range.

The kinematic mass is also mainly given by the sterile neutrino contribution:

$$\begin{aligned}m_\beta^{\text{SSN}} &\simeq \sqrt{\sin^2\theta_\odot\,\Delta m^2_\odot + \sin^2\theta_{\text{CHOOZ}}\,\Delta m^2_\text{A} + |U_{e4}|^2\,\Delta m^2_{\text{s}2} + |U_{e5}|^2\,\Delta m^2_{\text{s}1}} \\ &\simeq \sqrt{|U_{e4}|^2\,\Delta m^2_{\text{s}2} + |U_{e5}|^2\,\Delta m^2_{\text{s}1}} \simeq 0.32\ (0.20)\ \text{eV}\ . \end{aligned} \tag{4.4}$$

Both Σ and m_β are shown in figure 5, where the solid lines are these quantities at the best-fit values of eq. (2.5) whereas the bands are obtained by varying the masses and the mixing angles within their allowed ranges. The KATRIN experiment, having a sensitivity of 0.3 eV, will find a positive signal if $m_1 \gtrsim 0.3\,\text{eV}$. For smaller values, however, m_β can lie below 0.3 eV. If m_β larger than 0.5 eV is found then this scenario is ruled out, unless $m_1 \gtrsim 0.2\,\text{eV}$. No qualitatively different features are found for the second point from eq. (2.6).

Finally, neutrinoless double beta decay should be triggered by an effective mass given by

$$\begin{aligned}\langle m\rangle^{\text{SSN}} &\simeq \Big|\sin^2\theta_\odot\sqrt{\Delta m^2_\odot} + \sin^2\theta_{\text{CHOOZ}}\sqrt{\Delta m^2_\text{A}}\,e^{i(\alpha_3-\alpha_2)} \\ &\qquad + |U_{e4}|^2\sqrt{\Delta m^2_{\text{s}2}}\,e^{i(\alpha_4-\alpha_2)} + |U_{e5}|^2\sqrt{\Delta m^2_{\text{s}1}}\,e^{i(\alpha_5-\alpha_2)}\Big| \\ &\simeq \Big||U_{e4}|^2\sqrt{\Delta m^2_{\text{s}2}} + |U_{e5}|^2\sqrt{\Delta m^2_{\text{s}1}}\,e^{i(\alpha_5-\alpha_4)}\Big| \simeq (0.025 \div 0.048)\ \text{eV}\,, \end{aligned} \tag{4.5}$$

where the two sterile neutrinos provide the leading contribution. In case of the second typical point from eq. (2.6) the two additional neutrinos give a leading contribution between 0.008 and 0.031 eV. The upper left panels of figures 6 and 7 show the effective mass as a function of the smallest mass in this scenario. The shaded region inside is drawn for the best-fit values of mass and mixing parameters and varying the Majorana phases between 0 and 2π, while for the outer shaded regions we vary these parameters also in their permissible ranges. The figures show that neglecting the smallest mass is a good approximation as long

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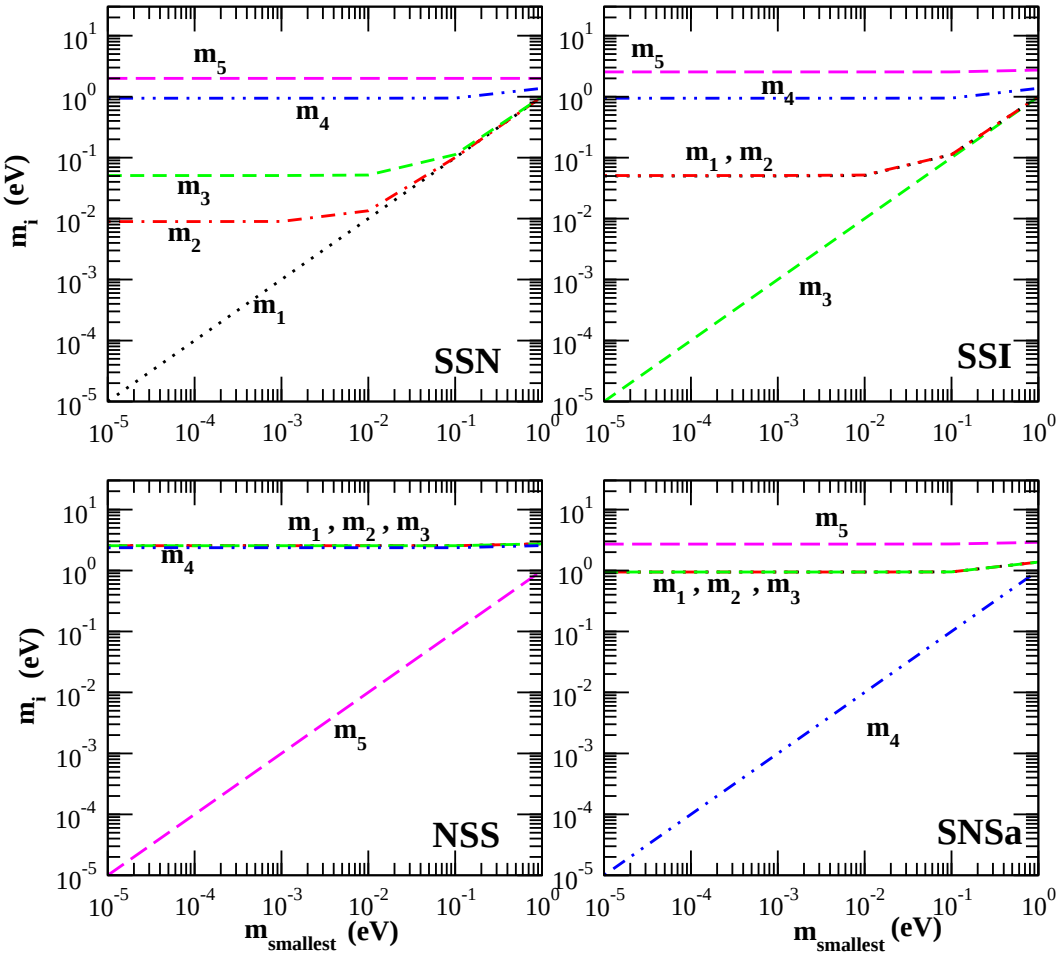

Figure 4: Scenarios with three active and two sterile neutrinos: the individual neutrino masses as a function of the smallest neutrino mass for scenarios SSN, SSI, NSS and SNSa. For the mass-squared differences related to the sterile neutrinos the best-fit point given in eq. (2.5) is used and we assumed that $\Delta m^2_{s1} = \Delta \tilde{m}^2_{s1}$ and $\Delta m^2_{s2} = \Delta \tilde{m}^2_{s2}$. Scenario ISS is indistinguishable from case NSS and SISa from SNSa. The schemes SNSb and SISb are very similar to NSS.

as $m_1 \lesssim 0.01\,\text{eV}$. There is for both central points a cancellation regime (for m_1 between 0.02 and 0.1 eV) in which the effective mass vanishes or becomes unobservably small. If we use the ranges around the two central points of the sterile neutrino parameters, then the two terms can cancel when the conditions $|U_{e4}/U_{e5}|^2 = \sqrt{\Delta m^2_{s1}/\Delta m^2_{s2}}$ and $\alpha_5 - \alpha_4 \simeq \pi$ are fulfilled. For the smallest neutrino mass above 0.1 eV the effective mass cannot vanish due to the non-maximal solar neutrino mixing angle. For smaller values of m_1 the scenario is ruled out if $\langle m \rangle$ is found to be larger than 0.1 eV.

4.2 Scheme SSI

In this scenario, schematically shown in figure 1, it holds $m_5 > m_4 > m_2 > m_1 > m_3$, i.e., the two heavy sterile neutrinos are heavier than the three light neutrinos which enjoy an

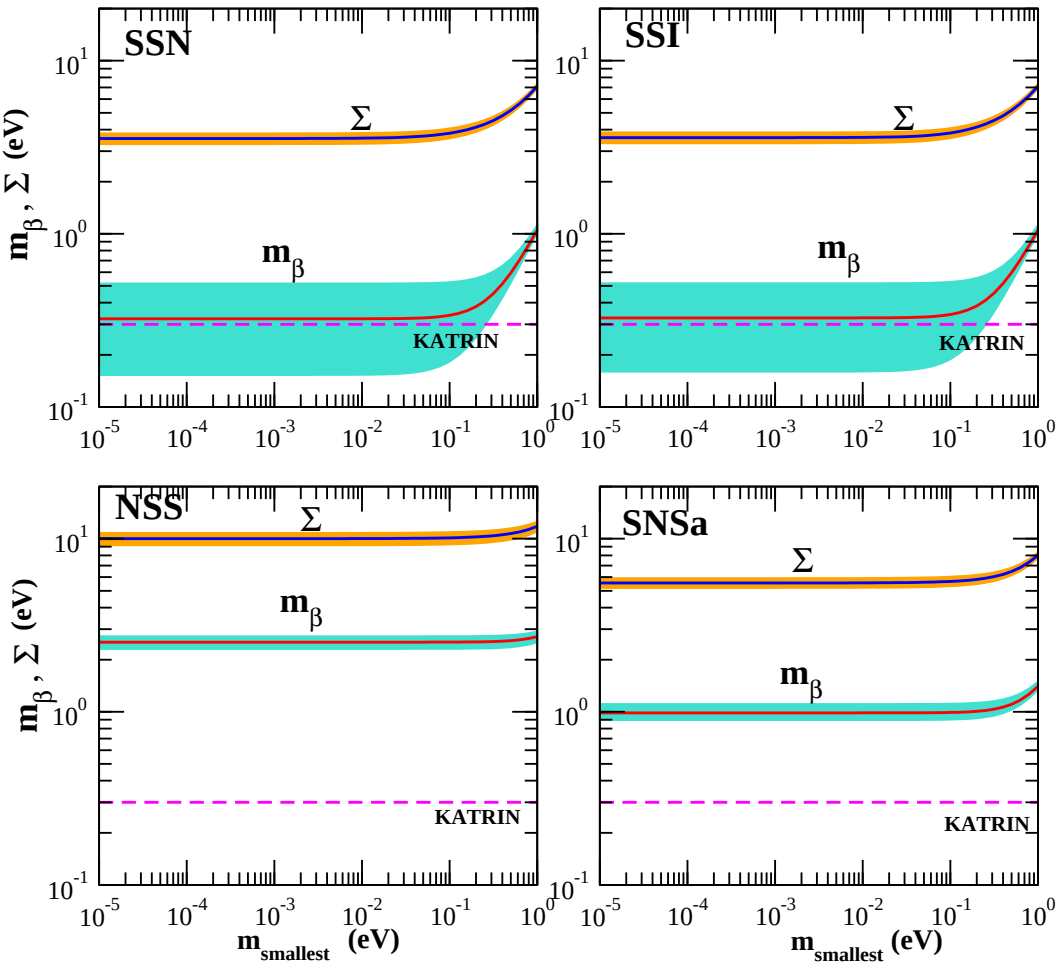

Figure 5: The sum of neutrino masses Σ and the kinematic neutrino mass m_β for scenarios SSN (top left), SSI (top right), NSS (bottom left) and SNSa (bottom right). The solid lines give the values of the respective observable at the best-fit point eq. (2.5)while the shaded regions are obtained by varying the parameters involved in their corresponding ranges from eqs. (2.3) and (2.7). We assumed that $\Delta m^2_{\rm s1} = \Delta\tilde{m}^2_{\rm s1}$ and $\Delta m^2_{\rm s2} = \Delta\tilde{m}^2_{\rm s2}$. Scenario ISS is indistinguishable from case NSS, SISa is indistinguishable from SNSa, and SNSb/SISb are indistinguishable from NSS. For these two observables SSN and SSI give identical results. Also indicated is the KATRIN sensitivity on m_β of 0.3 eV.

inverted hierarchy. Consequently, eq. (3.2) holds. In addition, we have

$$\Delta m^2_{\rm s1} = m_5^2 - m_3^2 \quad \text{and} \quad \Delta m^2_{\rm s2} = m_4^2 - m_3^2\,, \tag{4.6}$$

and the masses in terms of the smallest mass are

$$\begin{aligned} m_1 &= \sqrt{\Delta m^2_{\rm A} + m_3^2}\,, \qquad & m_2 &= \sqrt{\Delta m^2_{\rm A} + \Delta m^2_{\odot} + m_3^2}\,, \\ m_4 &= \sqrt{\Delta m^2_{\rm s2} + m_3^2}\,, \qquad & m_5 &= \sqrt{\Delta m^2_{\rm s1} + m_3^2}\,. \end{aligned} \tag{4.7}$$

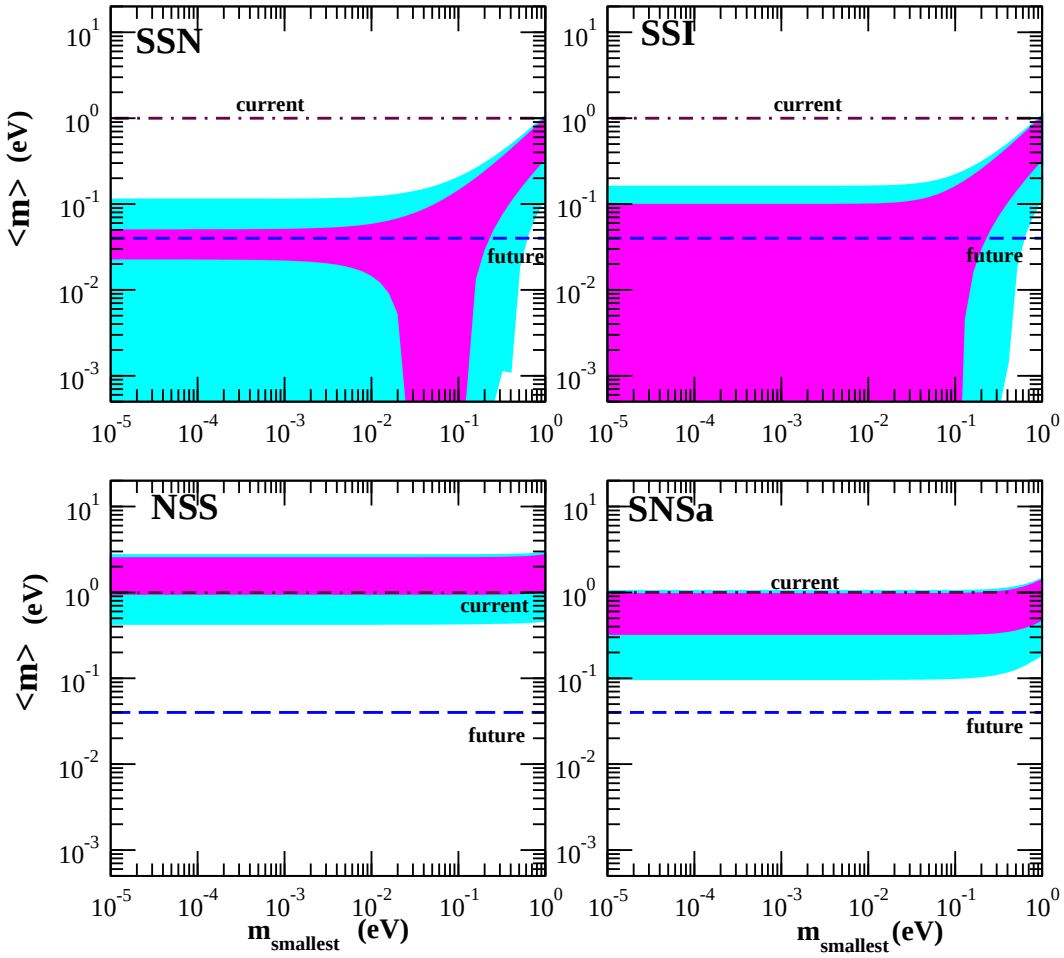

Figure 6: The effective mass $\langle m \rangle$ for scenarios SSN (top left), SSI (top right), NSS (bottom left) and SNSa (bottom right). The magenta shaded (darker) regions correspond to the the best-fit point from eq. (2.5) and allowing the phases to take arbitrary values in the interval $[0 : 2\pi]$. The blue shaded (lighter) regions are obtained by varying the mass and mixing angles as well in the allowed ranges from eqs. (2.3) and (2.7). We assumed that $\Delta m^2_{\rm s1} = \Delta \tilde{m}^2_{\rm s1}$ and $\Delta m^2_{\rm s2} = \Delta \tilde{m}^2_{\rm s2}$. Scenario ISS is indistinguishable from case NSS, SISa is indistinguishable from SNSa, and SNSb/SISb are indistinguishable from NSS. Also indicated is the current limit of 1 eV and a future bound of 0.04 eV.

Neglecting the smallest mass we find typical values of $m_2 \simeq m_1 \simeq \sqrt{\Delta m^2_{\rm A}} \simeq 0.05\,\text{eV}$, $m_4 \simeq \sqrt{\Delta m^2_{\rm s2}} \simeq 0.94\ (0.95)\,\text{eV}$ and $m_5 \simeq \sqrt{\Delta m^2_{\rm s1}} \simeq 2.55\ (1.38)\,\text{eV}$. The active neutrinos behave according to an inverted hierarchy in a 3-generation framework. As a function of the smallest neutrino mass m_3 the other masses are shown in figure 4. One finds that $\Sigma^{\rm SSI}$ is identical to $\Sigma^{\rm SSN}$ in eq. (4.3) ($\Delta m^2_\odot$ has to be replaced with $\Delta m^2_{\rm A}$, which does not make a notable difference). The kinematic mass is also basically identical to the one in scenario SSN, which is given in eq. (4.4). In the effective mass the situation is different,[6] because

[6]There is another non-oscillation probe which can distinguish SSN and SSI, namely the decay of astrophysical high energy neutrinos, treated in appendix B.

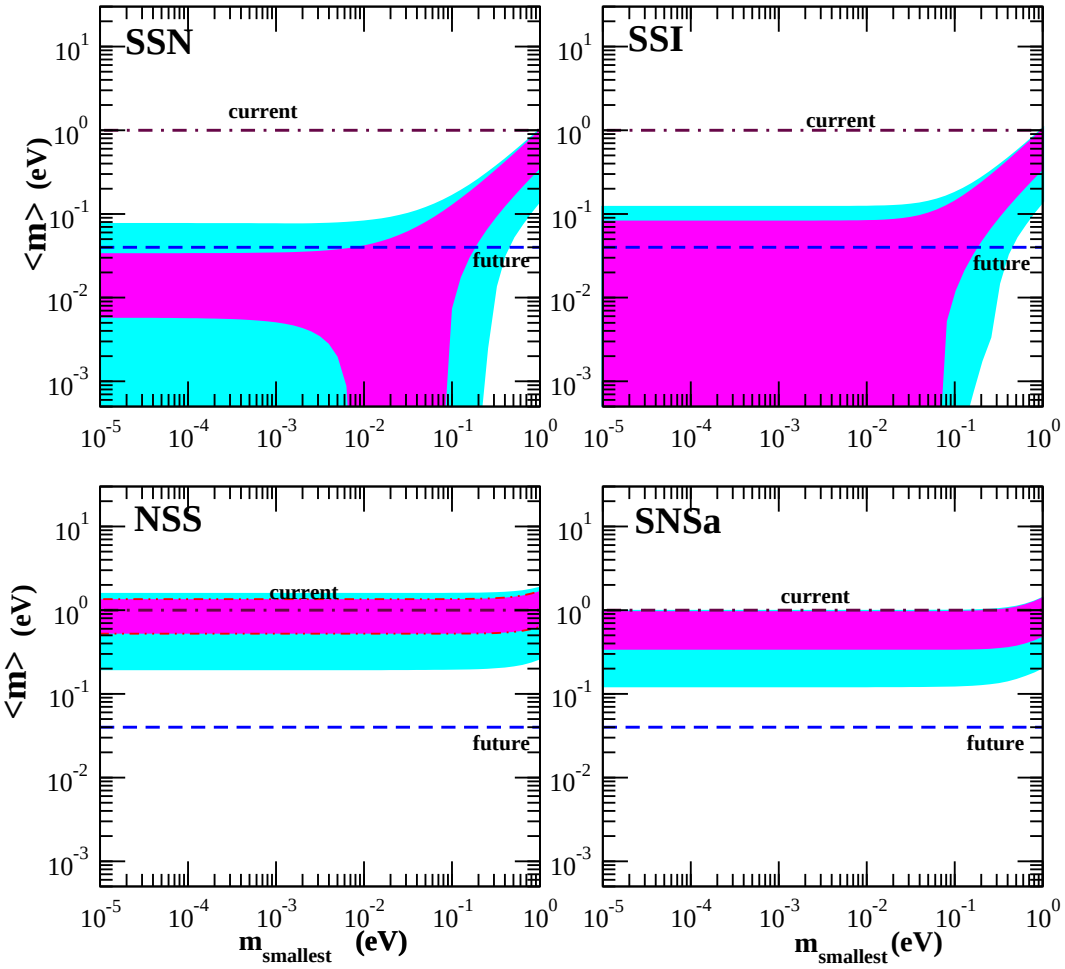

Figure 7: Same as figure 6 for the second typical sterile parameter point and the corresponding range from eq. (2.6).

the light neutrinos obey an inverted hierarchy and therefore a contribution of the same order of magnitude as the sterile ones:

$$\langle m\rangle^{\mathrm{SSI}} \simeq \left|\sin^2\theta_\odot\sqrt{\Delta m^2_{\mathrm{A}}}+\cos^2\theta_\odot\sqrt{\Delta m^2_{\mathrm{A}}}\,e^{i\alpha_2}+|U_{e4}|^2\sqrt{\Delta m^2_{\mathrm{s2}}}\,e^{i\alpha_4}+|U_{e5}|^2\sqrt{\Delta m^2_{\mathrm{s1}}}\,e^{i\alpha_5}\right| . \tag{4.8}$$

The absolute value of the first two terms is (see table 1) $\sqrt{\Delta m^2_{\mathrm{A}}}\,\sqrt{1-\sin^2 2\theta_\odot\,\sin^2\alpha_2/2}$ (between 0.020 and 0.051 eV) while for the best-fit values from eq. (2.5) the absolute value of the last two terms is between 0.025 and 0.048 eV, see eq. (4.5). Hence, the effective mass can vanish completely in this scheme even for the best-fit values and a vanishing smallest neutrino mass. This is borne out by the dark shaded regions in the upper right panels of figures 6 and 7 for the best-fit points of eqs. (2.5) and (2.6). The effective mass can be as large as 0.1 eV (or 0.08 eV for the second typical point from eq. (2.6)) in the small m_1 ($\lesssim 0.1$ eV) regime. Finding a larger $\langle m\rangle$ will rule out scenario SSI.

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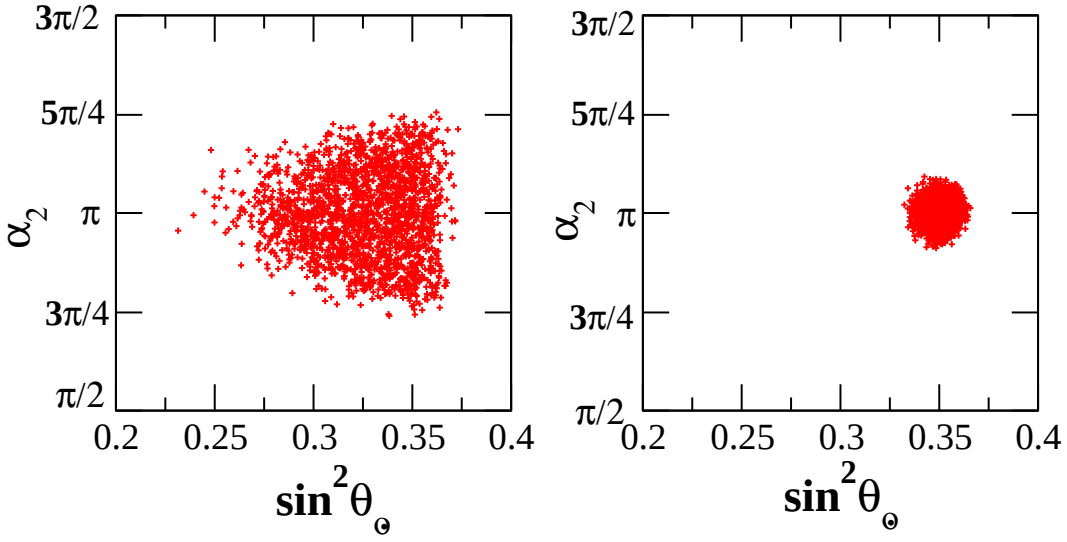

Figure 8: Allowed values of the Majorana phase α_2 and $\sin^2\theta_\odot$ in scenarios NSS and ISS for the current limit on $\langle m\rangle$ of 1 eV (left) and a future limit of 0.5 eV (right). The 3σ ranges from eq. (2.3) and the ranges around the best-fit point from eq. (2.5) are used.

We remark here that only in the schemes SSN and SSI the magnitudes of U_{e4} and U_{e5} are important for the predictions of m_β and $\langle m\rangle$. However, our statements regarding the possible exclusion of scenarios SSN and SSI with future measurements is rather insensitive to the precise values of U_{e4} and U_{e5}.

5. Sterile neutrinos lighter than active neutrinos: 3 + 2 scenarios

5.1 Scheme NSS

In this scheme, $m_3 > m_2 > m_1 > m_4 > m_5$, i.e., the three normally ordered active neutrinos are heavier than the two sterile neutrinos, see figure 2. Apart from eq. (3.1) it holds that

$$\Delta m^2_{\rm s1} = m^2_3 - m^2_5 \quad \text{and} \quad \Delta m^2_{\rm s2} = m^2_3 - m^2_4\,, \tag{5.1}$$

and the masses in terms of the smallest mass and the mass-squared differences are

$$\begin{aligned} m_4 &= \sqrt{\Delta m^2_{\rm s1} - \Delta m^2_{\rm s2} + m^2_5}\,, & m_1 &= \sqrt{\Delta m^2_{\rm s1} - \Delta m^2_{\rm A} - \Delta m^2_{\odot} + m^2_5}\,, \\ m_2 &= \sqrt{\Delta m^2_{\rm s1} - \Delta m^2_{\rm A} + m^2_5}\,, & m_3 &= \sqrt{\Delta m^2_{\rm s1} + m^2_5}\,. \end{aligned} \tag{5.2}$$

We therefore have for a negligible smallest mass three quasi-degenerate neutrinos and another quite massive state. Their values are $m_1 \simeq m_2 \simeq m_3 \simeq \sqrt{\Delta m^2_{\rm s1}} \simeq 2.55$ (1.38) eV and $m_4 \simeq \sqrt{\Delta m^2_{\rm s1} - \Delta m^2_{\rm s2}} \simeq 2.37$ (1.00) eV. As a function of the smallest mass m_5, they are given in figure 4.

Let us neglect the smallest neutrino mass and insert the best-fit values. In this case, the sum of masses as constrained by cosmological observations is given by

$$\Sigma^{\rm NSS} \simeq \sqrt{\Delta m^2_{\rm s1} - \Delta m^2_{\rm s2}} + 3\sqrt{\Delta m^2_{\rm s1}} \simeq 10.0\ (5.14)\ {\rm eV}\ . \tag{5.3}$$

The kinematic mass is given by

$$m_\beta^{\rm NSS} \simeq \sqrt{\Delta m^2_{\rm s1}\,(1 - |U_{e5}|^2)} \simeq 2.55\ (1.38)\ {\rm eV}\ . \tag{5.4}$$

With $|U_{e5}|^2$ being very small there is basically no dependence on the mixing parameters $|U_{ei}|^2$ because of the quasi-degenerateness of the three leading neutrinos. Only for m_5 reaching eV values one observes deviations from the last two equations in figure 5. The cosmological observable Σ is quite large, namely between 9 and 11 (3 and 6) eV. Interestingly, $m_\beta \simeq \sqrt{\Delta m^2_{\rm s1}}$ lies always above 1 eV, and is even above the current bound of 2.3 eV for the best-fit point. Hence, this scenario constraints $\Delta m^2_{\rm s1}$ to lie below $\simeq 5.3\,{\rm eV}^2$. In general, if this scenario is realized, KATRIN will definitely observe a positive signal the absence of which can rule out this scenario.

Finally, for neutrinoless double beta decay we have (in the limit $|U_{e4}| \to 0$)

$$\langle m \rangle^{\rm NSS} \simeq \sqrt{\Delta m^2_{\rm s1}}\,\sqrt{1 - \sin^2 2\theta_\odot\,\sin^2\alpha_2/2} \simeq (1.02 \div 2.55)\ (0.55 \div 1.38)\ {\rm eV}\,, \tag{5.5}$$

which is dominated by the three active quasi-degenerate neutrinos and cannot vanish due to the non-maximality of solar neutrino mixing as is reflected in the lower left panel of figure 6. The effective mass ranges from $\sqrt{\Delta m^2_{\rm s1}}\,\cos 2\theta_\odot$ to $\sqrt{\Delta m^2_{\rm s1}}$. In general, the effective mass is sizable in scenario NSS and already a part of it around the best-fit point is disfavored by the current upper limit of 1 eV. In fact, improving the limit on the effective mass below 0.2 eV rules out this scheme if neutrinos are Majorana particles.

It is possible to set limits on $\Delta m^2_{\rm s1}$ and in particular on $\theta_\odot$ and the Majorana phase α_2 demanding $\langle m \rangle$ to lie within a specific limit. Using the 3σ ranges from eq. (2.3) and the ranges around the best-fit point from eq. (2.5) one can investigate what values are allowed. The result for α_2 and $\sin^2\theta_\odot$ can be seen in figure 8. We took for $\langle m \rangle$ the current limit of 1 eV and a future limit of 0.5 eV. Taking the second central point for the sterile neutrino parameters gives hardly any constraint for a limit of 1 eV, but for $\langle m \rangle \leq 0.5$ eV the plot looks similar to the 1 eV plot of the best-fit point.

5.2 Scheme ISS

In this scheme (see figure 2) it holds $m_2 > m_1 > m_3 > m_4 > m_5$, i.e., the three inversely ordered active neutrinos are heavier than the two sterile neutrinos. Apart from eq. (3.2) we have

$$\Delta m^2_{\rm s1} = m_2^2 - m_5^2 \quad \text{and} \quad \Delta m^2_{\rm s2} = m_2^2 - m_4^2\,, \tag{5.6}$$

and the masses in terms of the smallest mass are

$$\begin{aligned} m_4 &= \sqrt{\Delta m^2_{\rm s1} - \Delta m^2_{\rm s2} + m_5^2}\,, \qquad & m_3 &= \sqrt{\Delta m^2_{\rm s1} - \Delta m^2_{\rm A} - \Delta m^2_\odot + m_5^2}\,, \\ m_1 &= \sqrt{\Delta m^2_{\rm s1} - \Delta m^2_\odot + m_5^2}\,, \qquad & m_2 &= \sqrt{\Delta m^2_{\rm s1} + m_5^2}\ . \end{aligned} \tag{5.7}$$

The results are basically identical to scenario NSS. We therefore do not give the expressions. Distinguishing between scenarios ISS and NSS could for instance be done via matter effects in oscillation experiments or in supernovae [13].

6. One heavy and one light sterile neutrino: $1 + 3 + 1$ scenarios

We will discuss now the mass-related observables when the three active neutrinos are '"sandwiched" between the sterile ones. Recall that the fit from [9] does not apply in this case. We note that apart from the mass-squared differences the mixing matrix elements U_{e4} and U_{e5} might also be different. However, their values do hardly influence the predictions. We will insert for the rest of this section the numerical values from eqs. (2.5), (2.6) for the sterile neutrino parameters, but will indicate that there might be differences by replacing in the expressions $\Delta m^2_{\rm s1} \to \Delta \tilde{m}^2_{\rm s1}$ and $\Delta m^2_{\rm s2} \to \Delta \tilde{m}^2_{\rm s2}$.

6.1 Scheme SNSa

In this scheme, $m_5 > m_3 > m_2 > m_1 > m_4$. Apart from eq. (3.1) we have

$$\Delta \tilde{m}^2_{\rm s1} = m_5^2 - m_1^2 \quad \text{and} \quad \Delta \tilde{m}^2_{\rm s2} = m_1^2 - m_4^2 \,, \tag{6.1}$$

see figure 3. We can express the individual masses as

$$\begin{aligned} m_1 &= \sqrt{\Delta \tilde{m}^2_{\rm s2} + m_4^2} \,, & m_2 &= \sqrt{\Delta \tilde{m}^2_{\rm s2} + \Delta m^2_{\odot} + m_4^2} \,, \\ m_3 &= \sqrt{\Delta \tilde{m}^2_{\rm s2} + \Delta m^2_{\odot} + \Delta m^2_{\rm A} + m_4^2} \,, & m_5 &= \sqrt{\Delta \tilde{m}^2_{\rm s1} + \Delta \tilde{m}^2_{\rm s2} + m_4^2} \,. \end{aligned} \tag{6.2}$$

We therefore have three quasi-degenerate active neutrino masses of order $\sqrt{\Delta \tilde{m}^2_{\rm s2}} \simeq 0.94\ (0.95)\,\text{eV}$ and one very heavy mass around $\sqrt{\Delta \tilde{m}^2_{\rm s1} + \Delta \tilde{m}^2_{\rm s2}} \simeq 2.71\ (1.67)\,\text{eV}$. A plot of the m_i as a function of m_4 is given in figure 4. Neglecting the smallest mass is correct as long as it is below 0.5 eV. We can estimate that

$$\Sigma^{\rm SNSa} \simeq 3\sqrt{\Delta \tilde{m}^2_{\rm s2}} + \sqrt{\Delta \tilde{m}^2_{\rm s1} + \Delta \tilde{m}^2_{\rm s2}} \simeq 5.55\ (4.52)\ \text{eV} \,. \tag{6.3}$$

The magnitude of Σ is quite sizable, and can be between 5 and 6 (4 and 5) eV, if one varies the mass-squared differences in their allowed ranges.

The sterile masses provide the leading contribution not only in this observable, but also in the kinematic mass:

$$m_\beta^{\rm SNSa} \simeq \sqrt{\Delta \tilde{m}^2_{\rm s2} + |U_{e5}|^2 \left(\Delta \tilde{m}^2_{\rm s1} + \Delta \tilde{m}^2_{\rm s2}\right)} \simeq 1.00\ (0.97)\ \text{eV} \,. \tag{6.4}$$

As in scenarios NSS and ISS, $m_\beta \simeq \sqrt{\Delta \tilde{m}^2_{\rm s2}}$ is always above the KATRIN sensitivity of 0.3 eV, therefore a signal in this experiment corresponding to at least 1 eV should be observed if this scheme is realized.

Finally, neutrinoless double beta decay should be triggered by an effective mass given by

$$\begin{aligned} \langle m \rangle^{\rm SNSa} &\simeq \left| \cos^2\theta_\odot \sqrt{\Delta \tilde{m}^2_{\rm s2}} + \sin^2\theta_\odot \sqrt{\Delta \tilde{m}^2_{\rm s2}}\, e^{i\alpha_2} + |U_{e5}|^2 \sqrt{\Delta \tilde{m}^2_{\rm s1} + \Delta \tilde{m}^2_{\rm s2}}\, e^{i\alpha_5} \right| \\ &\simeq \sqrt{\Delta \tilde{m}^2_{\rm s2}} \sqrt{1 - \sin^2 2\theta_\odot \, \sin^2 \alpha_2/2} \,, \end{aligned} \tag{6.5}$$

where the third term can be neglected. Hence, $\langle m \rangle^{\rm SNSa}$ is dominated by the active neutrinos and lies between $\sqrt{\Delta\tilde{m}^2_{\rm s2}}$ and $\sqrt{\Delta\tilde{m}^2_{\rm s2}}\cos 2\theta_\odot$, which is roughly 0.9 and 0.38 eV, respectively. The effective mass for this scenario is plotted in the lower right panel of figure 6 for the best-fit values as well as by varying all parameters within their allowed range. The situation for $\langle m \rangle$ is unfortunately similar to scenarios NSS and ISS, even though here the overall mass scale is $\sqrt{\Delta\tilde{m}^2_{\rm s2}}$, while it was $\sqrt{\Delta m^2_{\rm s1}}$ in the previous cases. The problem is of course the allowed range of the mass-squared differences and the unknown Majorana phase. Only if the condition

$$\sqrt{\Delta m^2_{\rm s1}}\,\cos 2\theta_\odot \geq \zeta\,\sqrt{\Delta\tilde{m}^2_{\rm s2}} \tag{6.6}$$

is fulfilled, then we can distinguish scenarios NSS/ISS and SNSa/SISa via neutrinoless double beta decay. In eq. (6.6) we have included a factor $\zeta \geq 1$, which takes into account the nuclear matrix element uncertainty, a necessity when one tries to distinguish different mass orderings via neutrinoless double beta decay [28]. For the best-fit values and $\zeta = 1$ indeed eq. (6.6) is fulfilled, but already for the second central point one cannot distinguish the schemes anymore as can also be seen from the lower panels of figure 7.

Anyway, if neutrinos are Majorana particles, then we can rule out scenario SNSa if $\langle m \rangle \leq 0.1\,\text{eV}$. One can generate plots as shown in figure 8 in order to obtain constraints on the parameters $\theta_\odot$ and $\sin^2\alpha_2$ from experimental information about $\langle m \rangle$. This requires limits which are stronger by a factor $\sqrt{\Delta m^2_{\rm s1}/\Delta\tilde{m}^2_{\rm s2}}$ than the limits used to generate figure 8.

6.2 Scheme SISa

In this scheme we have $m_5 > m_2 > m_1 > m_3 > m_4$, i.e., the sterile neutrinos are above and below three inversely ordered active neutrinos, see figure 3. One finds

$$\Delta\tilde{m}^2_{\rm s1} = m_5^2 - m_3^2 \quad \text{and} \quad \Delta\tilde{m}^2_{\rm s2} = m_3^2 - m_4^2\,, \tag{6.7}$$

and can express the individual masses as

$$\begin{aligned} m_3 &= \sqrt{\Delta\tilde{m}^2_{\rm s2} + m_4^2}\,, & m_1 &= \sqrt{\Delta\tilde{m}^2_{\rm s2} + \Delta m^2_{\rm A} + m_4^2}\,, \\ m_2 &= \sqrt{\Delta\tilde{m}^2_{\rm s2} + \Delta m^2_{\rm A} + \Delta m^2_\odot + m_4^2}\,, & m_5 &= \sqrt{\Delta\tilde{m}^2_{\rm s1} + \Delta\tilde{m}^2_{\rm s2} + m_4^2}\,. \end{aligned} \tag{6.8}$$

We do not give the expressions for Σ, m_β or $\langle m \rangle$, because the results are indistinguishable from scenario SNSa. Again, matter effects in neutrino oscillation experiments could be used to distinguishing the scenarios.

6.3 Scheme SNSb

In this scheme, $m_4 > m_3 > m_2 > m_1 > m_5$. Apart from eq. (3.1) we have

$$\Delta\tilde{m}^2_{\rm s1} = m_1^2 - m_5^2 \quad \text{and} \quad \Delta\tilde{m}^2_{\rm s2} = m_4^2 - m_1^2\,, \tag{6.9}$$

see figure 3. The individual masses are

$$\begin{aligned} m_1 &= \sqrt{\Delta\tilde{m}^2_{\rm s1} + m_5^2}\,, & m_2 &= \sqrt{\Delta\tilde{m}^2_{\rm s1} + \Delta m^2_\odot + m_5^2}\,, \\ m_3 &= \sqrt{\Delta\tilde{m}^2_{\rm s1} + \Delta m^2_\odot + \Delta m^2_{\rm A} + m_5^2}\,, & m_4 &= \sqrt{\Delta\tilde{m}^2_{\rm s1} + \Delta\tilde{m}^2_{\rm s2} + m_5^2}\,. \end{aligned} \tag{6.10}$$

The mass-related observables are obtained from the formulae for scenario SNSa from section 6.1 with the exchange $\Delta\tilde{m}^2_{\rm s1} \leftrightarrow \Delta\tilde{m}^2_{\rm s2}$. Hence,

$$\begin{aligned}
\Sigma^{\rm SNSb} &\simeq 3\sqrt{\Delta\tilde{m}^2_{\rm s1}} + \sqrt{\Delta\tilde{m}^2_{\rm s1} + \Delta\tilde{m}^2_{\rm s2}} \simeq 10.36\ (5.81)\ {\rm eV}\ , \\
m_\beta^{\rm SNSb} &\simeq \sqrt{\Delta\tilde{m}^2_{\rm s1} + |U_{e5}|^2\,(\Delta\tilde{m}^2_{\rm s1} + \Delta\tilde{m}^2_{\rm s2})} \simeq 2.57\ (1.39)\ {\rm eV}\ , \\
\langle m\rangle^{\rm SNSb} &\simeq \left|\cos^2\theta_\odot\,\sqrt{\Delta\tilde{m}^2_{\rm s1}} + \sin^2\theta_\odot\,\sqrt{\Delta\tilde{m}^2_{\rm s1}}\,e^{i\alpha_2} + |U_{e5}|^2\,\sqrt{\Delta\tilde{m}^2_{\rm s1} + \Delta\tilde{m}^2_{\rm s2}}\,e^{i\alpha_5}\right| \\
&\simeq \sqrt{\Delta\tilde{m}^2_{\rm s1}}\,\sqrt{1 - \sin^2 2\theta_\odot\,\sin^2\alpha_2/2}\ . \qquad (6.11)
\end{aligned}$$

All these expression are almost identical to the ones for scenarios NSS and ISS, because the leading contributions to all observables correspond to a situation with three quasi-degenerate active neutrinos having a mass $\sqrt{\Delta\tilde{m}^2_{\rm s1}}$. Therefore mass-related observables can not distinguish these cases, unless the mass-squared differences $\Delta m^2_{\rm s1}$ and $\Delta\tilde{m}^2_{\rm s1}$ are very much different from each other.

6.4 Scheme SISb

In this scheme we have $m_4 > m_2 > m_1 > m_3 > m_5$, i.e., the sterile neutrinos are above and below three inversely ordered active neutrinos, see figure 3. One finds

$$\Delta\tilde{m}^2_{\rm s1} = m_3^2 - m_5^2 \quad \text{and} \quad \Delta\tilde{m}^2_{\rm s2} = m_4^2 - m_3^2\,, \qquad (6.12)$$

and can express the individual masses as

$$\begin{aligned}
m_3 &= \sqrt{\Delta\tilde{m}^2_{\rm s1} + m_5^2}\,, & m_1 &= \sqrt{\Delta\tilde{m}^2_{\rm s1} + \Delta m^2_{\rm A} + m_5^2}\,, \\
m_2 &= \sqrt{\Delta\tilde{m}^2_{\rm s1} + \Delta m^2_{\rm A} + \Delta m^2_\odot + m_5^2}\,, & m_4 &= \sqrt{\Delta\tilde{m}^2_{\rm s1} + \Delta\tilde{m}^2_{\rm s2} + m_5^2}\ . \qquad (6.13)
\end{aligned}$$

We do not give the expressions for Σ, m_β or $\langle m\rangle$, because the results are indistinguishable from scenario SNSb and therefore also from NSS and ISS. Again, matter effects in neutrino oscillation experiments could be used to distinguish between the scenarios.

7. Discussions and summary

Adding two sterile neutrinos to the three active ones gives rise to eight possible mass orderings, out of which the right one should be identified in order to pin down the flavor structure of the neutrino mass matrix. We have investigated how and if mass-related measurements can do the job. In addition, we studied the general properties of the non-oscillation observables in scenarios with two sterile neutrinos. The possible mass orderings are shown schematically in figures 1), (2 and 3. Apart from the usual 3-generation masses and mixing parameters we have to cope with two additional mixing matrix elements $|U_{e4}|$ and $|U_{e5}|$ as well as with two mass-squared differences $\Delta m^2_{\rm s1}$ and $\Delta m^2_{\rm s2}$. Without loss of generality we can assume $\Delta m^2_{\rm s1} > \Delta m^2_{\rm s2}$ and associate $\Delta m^2_{\rm s1}$ with the state 5 and $\Delta m^2_{\rm s2}$ with state 4, respectively.

We use the following nomenclature for the eight different schemes:

	m_β (eV)	$\langle m \rangle$ (eV)	Σ (eV)
SSN	0.15 ÷ 0.52	0.0 ÷ 0.11	3.33÷ 3.85
SSI	0.16 ÷ 0.52	0.0 ÷ 0.16	3.33 ÷ 3.85
NSS, ISS, SNSb, SISb	2.3 ÷ 2.7	0.43 ÷ 2.72	9.16 ÷ 10.80
SNSa, SISa	0.89 ÷ 1.11	0.09 ÷ 1.03	5.19 ÷ 5.92
Current Bound	2.3 eV (95% C.L.)	$\sim$1 eV (90% C.L.)	$\sim$1 eV

Table 2: The ranges of the predictions for m_β, $\langle m \rangle$ and Σ according to the various mass orderings for $m_{\rm smallest} < 0.1\,{\rm eV}$ and assuming that $\Delta m^2_{\rm s1} = \Delta \tilde{m}^2_{\rm s1}$ and $\Delta m^2_{\rm s2} = \Delta \tilde{m}^2_{\rm s2}$. Also shown are the current bounds on these observables.

(i) SSX, where X = N for a normal and X=I for an inverted ordering of the mostly active neutrinos. In these schemes the two predominantly sterile neutrinos are heavier than the three predominantly active neutrinos (2+3 scenarios);

(ii) XSS (X = N or I as before), where the two predominantly sterile neutrinos are lighter than the three predominantly active neutrinos (3+2 scenarios);

(iii) SXS with X = N or I, where the three active neutrinos are sandwiched between the sterile ones (1+3+1 scenarios). In this class there can be four possible scenarios which we denote as SXSa and SXSb. The scheme SXSa corresponds to the state 5 higher than the three active states and SXSb corresponds to the state 5 lower than the three active states. Those scenarios are strictly speaking not covered by the available analyzes of scenarios with two sterile neutrinos. In absence of any fit of this possibility, we assumed for simplicity that the parameters are the same as for the other scenarios.

The following general comments can be made about the different mass related observables:[7] the sum of neutrino masses depends basically only on the new mass-squared differences, and typical (minimal) values are $\sqrt{\Delta m^2_{\rm s1}} + \sqrt{\Delta m^2_{\rm s2}}$, $3\sqrt{\Delta m^2_{\rm s1}}$ or $3\sqrt{\Delta m^2_{\rm s2}}$, depending on the mass ordering. Given the best-fit values and allowed ranges of masses this is already in conflict with the standard cosmological scenario, as discussed below.

The parameters relevant for neutrinoless double beta decay and direct beta-decay searches can be written as a contribution from the three mostly active states and the two mostly sterile neutrinos:

$$\langle m \rangle = \left| \langle m \rangle^3 + \langle m \rangle^{\rm st} \right| \quad \text{and} \quad m_\beta = \sqrt{(m^3_\beta)^2 + (m^{\rm st}_\beta)^2}\ . \tag{7.1}$$

where $|\langle m \rangle^3|$ and m^3_β are the expressions known from 3-flavor analyzes, see table 1. All cases are possible in eq. (7.1): dominance of the sterile contribution, dominance of the active contribution, and equal-sized contributions, leading potentially to complete cancellation only in $\langle m \rangle$.

[7] Predictions for mass-related observables in the presence of yet another sterile neutrino are discussed in appendix A.

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scheme	feature	KATRIN	$0\nu\beta\beta$
SSN	NH plus ν_{s_1}, ν_{s_2}	maybe	maybe
SSI	IH plus ν_{s_1}, ν_{s_2}	maybe	maybe
NSS, ISS	QD with $\sqrt{\Delta m^2_{\rm s1}}$	yes	yes
SNSb, SISb	QD with $\sqrt{\Delta \tilde{m}^2_{\rm s1}}$	yes	yes
SNSa, SISa	QD with $\sqrt{\Delta \tilde{m}^2_{\rm s2}}$	yes	yes

Table 3: The various schemes with two sterile neutrinos and their meaning for KATRIN and future $0\nu\beta\beta$ experiments. We assumed that $\Delta\tilde{m}^2_{\rm s1}$ and $\Delta\tilde{m}^2_{\rm s2}$ are larger than $0.1\,{\rm eV}^2$.

In general, the mass-related observables can not distinguish between a normal or inverted ordering of the three active neutrinos, with the exception of schemes SSN and SSI, which have different predictions for $\langle m \rangle$. It will however be difficult to test this difference in practise, as precise knowledge of the oscillations parameters is required. Only in the schemes SSN and SSI the magnitude of $|U_{e4}|$ and $|U_{e5}|$ is crucial for the predictions of $\langle m \rangle$ and m_β. For all other mass orderings the dependence on $|U_{e4}|$ and $|U_{e5}|$ is suppressed (Σ does not depend on $|U_{e4}|$ and $|U_{e5}|$). Scenarios SNSb and SISb are indistinguishable from scenarios NSS and ISS if the mass-squared differences are equal or very similar. It turns out that in order to summarize all phenomenology of the mass-related observables it suffices to plot them for four schemes: SSN, SSI, SNSa (covering also SISa) and NSS (covering also ISS, SNSb, SISb). Interestingly, these four cases have also the same phenomenology in what regards decays of high energy astrophysical neutrinos, see appendix B. In table 2 we present for the parameter ranges given in eqs. (2.3) and (2.5) the predictions of the three quantities m_β, $\langle m \rangle$, Σ for the four types of mass orderings in the realistic case when the smallest neutrino mass is smaller than 0.1 eV. Also shown are the current bounds on these observables. table 3 summarizes what the various schemes mean for KATRIN and for future experiments searching for neutrinoless double beta decay. The following conclusions can be drawn:

- scenarios SSN and SSI predict for all observables the smallest values. Hence, they are the easiest to rule out. The other scenarios correspond at leading order to quasi-degenerate 3-neutrino scenarios with the common mass scale given by $\sqrt{\Delta m^2_{\rm s1}}$ (NSS, ISS), $\sqrt{\Delta \tilde{m}^2_{\rm s2}}$ (SNSa, SISa) or $\sqrt{\Delta \tilde{m}^2_{\rm s1}}$ (SNSb, SISb);

- model independent constraints on neutrino masses stem from direct searches in the spectra of beta-decays. The Mainz data give the constraint $m_\beta < 2.2\,{\rm eV}$ at 95% C.L. ($\Delta\chi^2 = 4$) [31]. Following the procedure in [31] the 68% C.L. ($\Delta\chi^2 = 1$) bound is 0.7 eV and the 99.73% C.L. ($\Delta\chi^2 = 9$) bound is 4.0 eV. Scenarios SSN and SSI with both sterile neutrinos being heavier than the active ones can have unobservably small m_β and are consistent with the current bound even at 1σ as can be seen from table 2. The table also shows that scenarios SNSa and SISa are allowed at 2σ, while the scenarios NSS/ISS/SNSb/SISb are consistent with the Mainz result at 3σ. The other six mass orderings will definitely result in a signal in KATRIN. If the

two sterile neutrinos are lighter than the active ones (NSS and ISS), then there is a direct correspondence $m_\beta \simeq \sqrt{\Delta m^2_{\rm s1}}$, which can be used to rule out part of the allowed range of $\Delta m^2_{\rm s1}$ already at the current stage. Scenarios SNSa and SISa predict $m_\beta \simeq \sqrt{\Delta \tilde{m}^2_{\rm s2}}$, which will rule out part of its allowed range in the near future;

- the effective mass governing neutrinoless double beta decay can only vanish when the sterile neutrinos are heavier than the active ones (SSN and SSI, for which the largest value is 0.1 eV). In the other orderings the non-maximality of the solar neutrino mixing angle renders $\langle m \rangle$ non-zero. If the sterile neutrinos are lighter than the active ones (NSS and ISS), then $\langle m \rangle$ is larger than 0.2 eV and has a maximal value above the current limit[8] of ~ 1 eV at 90% C.L. Consequently, this scenario can be ruled out by a stronger limit on $\langle m \rangle$ and one can also constrain parameters with the current limit. This concerns in particular $\sin^2\theta_\odot$ and the Majorana phase α_2. Unfortunately, the sterile neutrino parameters are such that this scenario is hardly distinguishable from the scenarios in which one sterile neutrino is heavier and the other one lighter than the active ones. Whereas telling apart NSS/ISS from SNSb/SISb is basically impossible if $\Delta m^2_{\rm s1}$ is similar to $\Delta \tilde{m}^2_{\rm s1}$, distinguishing SNSa/SISa from NSS/ISS requires a condition of the form $\sqrt{\Delta m^2_{\rm s1}}\cos 2\theta_\odot \geq \zeta \sqrt{\Delta \tilde{m}^2_{\rm s2}}$, where ζ denotes the nuclear matrix element uncertainty.

In general, the sum of neutrino masses in the scenarios under study is always larger than about 2 eV. This minimal value is obtained in the two schemes SSN and SSI in which the sterile neutrinos are heavier than the active ones. In addition, the two mass-squared differences related to the MiniBooNE/LSND experiments should be rather small, because $\Sigma = \sqrt{\Delta m^2_{\rm s1}} + \sqrt{\Delta m^2_{\rm s2}}$. The other scenarios have sizable values of Σ, approaching up to 10 eV if the sterile neutrinos are lighter than the active ones. However, cosmological limits can always be evaded or relaxed. Nevertheless, we add some discussion on typical limits obtained in the literature (for an overview, see [17]), which typically constrain both the sum of neutrino masses and the effective number of neutrino species $N_{\rm eff}$ contributing to the radiation density. Those limits depend also on $N_{\rm m}$, which is the number of *equally massive* species. As one example, we focus on ref. [18], in which likelihood contours are provided in the $N_{\rm eff}$–Σ plane for three cases: (i) $N_{\rm m} = N_{\rm eff}$, (ii) $N_{\rm m} = 3$ and (iii) $N_{\rm m} = 1$. The value $N_{\rm eff} = 5$ is allowed only at about 99% C.L. in all the above cases and the bounds on Σ are 0.62, 0.57 and 0.41 eV, respectively (all at 95% C.L.). The type Ia supernova data from SNLS, large scale structure data from 2DF and SDSS, baryon acoustic oscillation data from SDSS, CMB anisotropy data from WMAP and the smaller scale measurement by the BOOMERANG experiment were included in that analysis. Adding the Lyman-α forest data gives even stronger bounds [19], leaving out the baryon acoustic oscillation relaxes the limits [17]. For the unrealistic case when the lightest neutrino is heavier than 1 eV we have $N_{\rm m} = N_{\rm eff} = 5$ and case (i) applies. The resulting Σ in our scenarios is of course much larger than allowed. Another example is when in scenarios NSS/ISS the

[8]Note that in our analysis we are not using the data coming from the positive evidence claimed by a part of the Heidelberg-Moscow collaboration [32] and thus we have only an upper bound.

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smallest mass can be neglected. Then we have three quasi-degenerate neutrinos, but also one other massive neutrino with mass $\sqrt{\Delta m^2_{s1} - \Delta m^2_{s2}}$. This possibility, as well as the other cases we study, is not covered by the analysis in [18] or in any other paper we are aware of. Nevertheless, we can safely assume[9] that limits on the sum of masses for $N_{\rm eff} = 5$ do not exceed 1 eV. Consequently, and not surprisingly, all scenarios with two sterile neutrinos have serious problems with cosmology and require non-standard physics (primordial lepton asymmetries, low reheating temperature, additional neutrino interactions,...) as described, e.g., in [29, 30].

To conclude, scenarios with two sterile neutrinos offer rich and interesting phenomenology. In six of the eight allowed cases KATRIN and future $0\nu\beta\beta$ experiments will find a signal. Mass-related observables alone, however, can not identify the correct mass ordering completely, which leaves room for further studies in order to disentangle the possibilities by means of oscillation experiments.

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A. On scenarios with three sterile neutrinos

For the sake of completeness we summarize briefly the formulae and results for the mass-related observables in case when three sterile neutrinos are added. No qualitatively new aspects are found in these scenarios. The authors of ref. [9] also performed an analysis of this possibility and it was found that no significant improvement of the fit can be achieved in this way. The best-fit values for the mass-squared differences are

$$\Delta m^2_{s1} = 1.84 \text{ eV} \,, \;\; \Delta m^2_{s2} = 0.83 \text{ eV} \,, \;\; \Delta m^2_{s3} = 0.46 \text{ eV} \,. \tag{A.1}$$

Note that there can be scenarios in which the fit of ref. [9] does not apply. There is no information given on the mixing matrix elements, let us therefore take for simplicity the values

$$|U_{ei}| = 0.1 \quad \text{for } i = 4, 5, 6 \,. \tag{A.2}$$

We will estimate now the values of the mass-related observables for all possible mass orderings. Again, we fix $m_{1,2,3}$ to be responsible for the oscillations of solar and atmospheric

[9] A more quantitative and accurate estimate on the joint constraint on the number of neutrino species and the sum of neutrino masses from cosmology in the various scenarios and estimating the $\Delta\chi^2$ would require a more detailed analysis of the cosmological data sets which is clearly not in the purview of the present analysis.

neutrino oscillations. With the details given in the main text, it is quite easy to obtain the following formulae, which are valid when the smallest neutrino mass is neglected. As for two sterile neutrinos, the non-oscillation probes can not distinguish whether the active neutrinos are normally or inversely ordered. There are in total 16 possible mass orderings, which we group in 4 classes:

(i) the first type of mass spectrum holds when the three sterile neutrinos are all heavier than the active ones (SSSN and SSSI). In this case,

$$\begin{aligned} \Sigma &\simeq \sqrt{\Delta m^2_{\rm s1}} + \sqrt{\Delta m^2_{\rm s2}} + \sqrt{\Delta m^2_{\rm s3}} \simeq 2.9 \text{ eV} \,, \\ m_\beta &\simeq \sqrt{|U_{e4}|^2\, \Delta m^2_{\rm s3} + |U_{e5}|^2\, \Delta m^2_{\rm s2} + |U_{e6}|^2\, \Delta m^2_{\rm s1}} \simeq 0.18 \text{ eV} \,, \\ \langle m \rangle &\simeq \left| |U_{e4}|^2 \sqrt{\Delta m^2_{\rm s3}} + |U_{e5}|^2 \sqrt{\Delta m^2_{\rm s2}}\, e^{i\alpha_{54}} + |U_{e6}|^2 \sqrt{\Delta m^2_{\rm s1}}\, e^{i\alpha_{64}} \right| \lesssim 0.29 \text{ eV} \,, \end{aligned} \tag{A.3}$$

where $\alpha_{54} = \alpha_5 - \alpha_4$ and $\alpha_{64} = \alpha_6 - \alpha_4$ are combinations of Majorana phases. The situation is somewhat similar to scenarios SSN and SSI;

(ii) a second class of spectra is found when the three sterile neutrinos are lighter than the three active ones (scenarios NSSS and ISSS). One has

$$\begin{aligned} \Sigma &\simeq 3\sqrt{\Delta m^2_{\rm s1}} \simeq 4.1 \text{ eV} \,, \\ m_\beta &\simeq \sqrt{\Delta m^2_{\rm s1}} \simeq 1.36 \text{ eV} \,, \\ \langle m \rangle &\simeq \sqrt{\Delta m^2_{\rm s1}} \sqrt{1 - \sin^2 2\theta_\odot \, \sin^2 \alpha_2/2} \simeq (0.54 \div 1.36) \text{ eV} \,. \end{aligned} \tag{A.4}$$

This is similar to the possibilities NSS and ISS;

(iii) two neutrinos can be heavier than the active ones which in turn are heavier than the third sterile state. There are three possibilities for this. If the two heavy sterile neutrinos correspond to $\Delta m^2_{\rm s1}$ and $\Delta m^2_{\rm s2}$, then the resulting schemes SSNSa and SSISa give

$$\begin{aligned} \Sigma &\simeq 3\sqrt{\Delta m^2_{\rm s3}} + \sqrt{\Delta m^2_{\rm s3} + \Delta m^2_{\rm s2}} + \sqrt{\Delta m^2_{\rm s3} + \Delta m^2_{\rm s1}} \simeq 4.7 \text{ eV} \,, \\ m_\beta &\simeq \sqrt{\Delta m^2_{\rm s3} + |U_{e5}|^2\, (\Delta m^2_{\rm s3} + \Delta m^2_{\rm s2}) + |U_{e6}|^2\, (\Delta m^2_{\rm s3} + \Delta m^2_{\rm s1})} \simeq 0.70 \text{ eV} \,, \\ \langle m \rangle &\simeq \sqrt{\Delta m^2_{\rm s3}} \sqrt{1 - \sin^2 2\theta_\odot \, \sin^2 \alpha_2/2} \simeq (0.27 \div 0.68) \text{ eV} \,. \end{aligned} \tag{A.5}$$

Also possible is that the two heavy sterile neutrinos correspond to $\Delta m^2_{\rm s1}$ and $\Delta m^2_{\rm s3}$ (schemes SSNSb and SSISb), in which case

$$\begin{aligned} \Sigma &\simeq 3\sqrt{\Delta m^2_{\rm s2}} + \sqrt{\Delta m^2_{\rm s2} + \Delta m^2_{\rm s3}} + \sqrt{\Delta m^2_{\rm s2} + \Delta m^2_{\rm s1}} \simeq 5.5 \text{ eV} \,, \\ m_\beta &\simeq \sqrt{\Delta m^2_{\rm s2} + |U_{e5}|^2\, (\Delta m^2_{\rm s2} + \Delta m^2_{\rm s3}) + |U_{e6}|^2\, (\Delta m^2_{\rm s2} + \Delta m^2_{\rm s1})} \simeq 0.93 \text{ eV} \,, \\ \langle m \rangle &\simeq \sqrt{\Delta m^2_{\rm s2}} \sqrt{1 - \sin^2 2\theta_\odot \, \sin^2 \alpha_2/2} \simeq (0.36 \div 0.91) \text{ eV} \,. \end{aligned} \tag{A.6}$$

Finally, the two heavy sterile neutrinos can correspond to $\Delta m^2_{\rm s2}$ and $\Delta m^2_{\rm s3}$ (schemes SSNSc and SSISc):

$$\begin{aligned}
\Sigma &\simeq 3\sqrt{\Delta m^2_{\rm s1}} + \sqrt{\Delta m^2_{\rm s1} + \Delta m^2_{\rm s3}} + \sqrt{\Delta m^2_{\rm s1} + \Delta m^2_{\rm s2}} \simeq 7.2 \text{ eV}\,, \\
m_\beta &\simeq \sqrt{\Delta m^2_{\rm s1} + |U_{e5}|^2\,(\Delta m^2_{\rm s1} + \Delta m^2_{\rm s3}) + |U_{e6}|^2\,(\Delta m^2_{\rm s1} + \Delta m^2_{\rm s2})} \simeq 1.37 \text{ eV}\,, \\
\langle m \rangle &\simeq \sqrt{\Delta m^2_{\rm s1}}\,\sqrt{1 - \sin^2 2\theta_\odot\,\sin^2\alpha_2/2} \simeq (0.54 \div 1.36) \text{ eV}\,. \qquad \text{(A.7)}
\end{aligned}$$

(iv) the fourth class of mass orderings are scenarios in which the two sterile neutrinos are lighter than the active ones which in turn are lighter than the last sterile one. As for the previous class of scenarios, three possibilities are present. In scenarios SNSSa and SISSa the two light sterile neutrinos correspond to $\Delta m^2_{\rm s2}$ and $\Delta m^2_{\rm s3}$:

$$\begin{aligned}
\Sigma &\simeq 3\sqrt{\Delta m^2_{\rm s2}} + \sqrt{\Delta m^2_{\rm s2} - \Delta m^2_{\rm s3}} + \sqrt{\Delta m^2_{\rm s1} + \Delta m^2_{\rm s2}} \simeq 5.0 \text{ eV}\,, \\
m_\beta &\simeq \sqrt{\Delta m^2_{\rm s2} + |U_{e5}|^2\,(\Delta m^2_{\rm s2} - \Delta m^2_{\rm s3}) + |U_{e6}|^2\,(\Delta m^2_{\rm s1} + \Delta m^2_{\rm s2})} \simeq 0.93 \text{ eV}\,, \\
\langle m \rangle &\simeq \sqrt{\Delta m^2_{\rm s2}}\,\sqrt{1 - \sin^2 2\theta_\odot\,\sin^2\alpha_2/2} \simeq (0.36 \div 0.91) \text{ eV}\,. \qquad \text{(A.8)}
\end{aligned}$$

If the neutrinos associated with $\Delta m^2_{\rm s1}$ and $\Delta m^2_{\rm s3}$ are lighter (scenarios SNSSb and SISSb), then

$$\begin{aligned}
\Sigma &\simeq 3\sqrt{\Delta m^2_{\rm s1}} + \sqrt{\Delta m^2_{\rm s1} - \Delta m^2_{\rm s3}} + \sqrt{\Delta m^2_{\rm s1} + \Delta m^2_{\rm s2}} \simeq 6.9 \text{ eV}\,, \\
m_\beta &\simeq \sqrt{\Delta m^2_{\rm s1} + |U_{e5}|^2\,(\Delta m^2_{\rm s1} - \Delta m^2_{\rm s3}) + |U_{e6}|^2\,(\Delta m^2_{\rm s1} + \Delta m^2_{\rm s2})} \simeq 1.37 \text{ eV}\,, \\
\langle m \rangle &\simeq \sqrt{\Delta m^2_{\rm s1}}\,\sqrt{1 - \sin^2 2\theta_\odot\,\sin^2\alpha_2/2} \simeq (0.54 \div 1.36) \text{ eV}\,. \qquad \text{(A.9)}
\end{aligned}$$

Finally, in scenarios SNSSc and SISSc the heaviest neutrino corresponds to $\Delta m^2_{\rm s3}$:

$$\begin{aligned}
\Sigma &\simeq 3\sqrt{\Delta m^2_{\rm s1}} + \sqrt{\Delta m^2_{\rm s1} - \Delta m^2_{\rm s2}} + \sqrt{\Delta m^2_{\rm s1} + \Delta m^2_{\rm s3}} \simeq 6.6 \text{ eV}\,, \\
m_\beta &\simeq \sqrt{\Delta m^2_{\rm s1} + |U_{e5}|^2\,(\Delta m^2_{\rm s1} - \Delta m^2_{\rm s2}) + |U_{e6}|^2\,(\Delta m^2_{\rm s1} + \Delta m^2_{\rm s3})} \simeq 1.37 \text{ eV}\,, \\
\langle m \rangle &\simeq \sqrt{\Delta m^2_{\rm s1}}\,\sqrt{1 - \sin^2 2\theta_\odot\,\sin^2\alpha_2/2} \simeq (0.54 \div 1.36) \text{ eV}\,. \qquad \text{(A.10)}
\end{aligned}$$

Hence, except for scenarios SSSN and SSSI one can expect a signal in KATRIN and in neutrinoless double beta decay searches. The latter has in this case only three main predictions, given by a quasi-degenerate scenario with a common mass scale $\sqrt{\Delta m^2_{\rm s1}}$, $\sqrt{\Delta m^2_{\rm s2}}$ or $\sqrt{\Delta m^2_{\rm s3}}$. In order to distinguish these cases, conditions in analogy to eq. (6.6) should be fulfilled. Note that strictly speaking the analysis of ref. [9] does not apply to the scenarios discussed in items (iii), (iv) and (v). The reason is the same as the one discussed in section 3 for schemes SNSa/SISa/SNSb/SISb.

B. An alternative non-oscillation probe of neutrino spectra: decay of astrophysical neutrinos

Another interesting non-oscillation probe to distinguish different mass orderings is the decay of astrophysical neutrinos [33]. Albeit such an analysis depends crucially on the non-trivial assumption that neutrinos decay, it is an interesting exercise to investigate the implications of such a situation.[10] Different astrophysical sources can generate a neutrino flux with a certain initial composition in ν_e, ν_μ and ν_τ. Assuming that all neutrino states except for the lightest ν_i decay, leads to

$$\Phi_e : \Phi_\mu : \Phi_\tau = |U_{ei}|^2 : |U_{\mu i}|^2 : |U_{\tau i}|^2 \,, \tag{B.1}$$

where Φ_α with $\alpha = e, \mu, \tau$ is the flux of neutrinos and anti-neutrinos of flavor α which reaches Earth. Note that in the decay scenario this final flux is independent on the initial flavor composition. The crucial observation is that the surviving lightest neutrino state being ν_i, the flux Φ_α, which is proportional to $|U_{\alpha i}|^2$, may differ in the eight neutrino mass orderings under study. For scenario SSN we have $i = 1$, while for SSI it holds $i = 3$. This corresponds to the situation in the three-flavor scenarios studied in refs. [33]. In contrast, for scenarios NSS, ISS, SNSb and SISb we have $i = 5$, while in the orderings SNSa and SISa it holds that $i = 4$. There are therefore four different possibilities, and the mass orderings sharing the same phenomenology are in fact the same as the ones sharing the same phenomenology of the mass-related observables.

What is eventually measured in neutrino telescopes like IceCube [35] are ratios of fluxes, and here for illustrative purposes we will focus on the ratio

$$R_{e\mu} \equiv \frac{\Phi_e}{\Phi_\mu} \,, \tag{B.2}$$

which can be obtained by comparing the rate of shower and muon events [36]. Taking ratios including ν_τ into account will complicate the situation considerably, as the mixing elements $|U_{\tau 4}|$ and $|U_{\tau 5}|$ enter the game, which are basically unconstrained (one could however use these ratios to obtain information on these elements). We assume maximal atmospheric neutrino mixing, take the best-fit values from eqs. (2.3), (2.5) and use from ref. [9] that $|U_{\mu 5}| = 0.12$ as well as $|U_{\mu 4}| = 0.16$. It follows for the ratios that $R_{e\mu}^{\rm SSN} = 2/\tan^2\theta_\odot = 4.7$, $R_{e\mu}^{\rm SSI} = 2\sin^2\theta_{\rm CHOOZ} = 0$, $R_{e\mu}^{\rm NSS,\,ISS,\,SNSb,\,SISb} = |U_{e5}|^2/|U_{\mu 5}|^2 = 1$ and $R_{e\mu}^{\rm SNSa,\,SISa} = |U_{e4}|^2/|U_{\mu 4}|^2 = 0.47$. The ratios are easily distinguishable from each other, in particular SSN and SSI, which we have shown in section 4 to be very similar in the mass-related observables. Note however that for standard astrophysical sources (an initial composition of $1:2:0$) and no decay the ratio $R_{e\mu}$ is equal to 1 for maximal atmospheric mixing and $U_{e3} = 0$, i.e., identical to $R_{e\mu}^{\rm NSS,\,ISS,\,SNSb,\,SISb}$. In addition, taking the uncertainty of the mixing matrix elements into account complicates the situation further. Taking the 3σ ranges from ref. [16] (in particular $\sin^2\theta_{\rm A} = (0.32 \div 0.64)$ for atmospheric neutrino mixing) and assuming again a 50% uncertainty on the sterile neutrino parameters gives that

[10] For other uses of sterile neutrinos in neutrino telescopes, see [34].

$R_{e\mu}^{\rm SSN} = (2.2 \div 10.8)$, $R_{e\mu}^{\rm SSI} = (0 \div 0.13)$, $R_{e\mu}^{\rm NSS,\,ISS,\,SNSb,\,SISb} = (0.11 \div 9.0)$ and $R_{e\mu}^{\rm SNSa,\,SISa} = (0.06 \div 4.0)$. The standard scenario predicts $R_{e\mu}$ between 0.73 and 1.19, a range which is covered by all decay scenarios except for SSN and SSI. The ratios are now overlapping and except for the cases of measuring very small ($R_{e\mu} \leq 0.06$) or large ratios ($R_{e\mu} \geq 9$) no possibility can be unambiguously identified. However, certain cases can be ruled out, for instance scenarios SSI, SNSa and SISa for an observation of $R_{e\mu} \geq 4$. To fully disentangle the different cases the mixing parameters the mixing matrix elements should be known much more precisely.

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