Large 1-3 leptonic mixing and Renormalization Group Effects

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Large 1-3 leptonic mixing and Renormalization Group Effects

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Abstract. Starting from tri-bimaximal mixing at high scale, we investigate if it is possible to generate a sizable value of $|U_{e3}|$, while at the same time keeping solar neutrino mixing near its measured value, which is close to $\sin^2\theta_{12} = \frac{1}{3}$ through renormalization group corrections. Generation of indicated values of $|U_{e3}| \approx 0.1$ via this effect requires the neutrinos to be quasi-degenerate in mass. If we consider Standard Model (SM) as the low energy effective theory then the required value of the mass scale is beyond 2.3 eV which is the current constraint from tritium β-decay. On the other hand considering the Minimal Supersymmetric Standard Model (MSSM) as the effecitve theory at low energy it is possible to generate the non-zero U_{e3} values hinted by the current global analysis for lower m_0 values. The consistency with the allowed range of $\sin^2\theta_{12}$ together with large running of $|U_{e3}|$ forces one of the Majorana phases to be close to π. This implies large cancellations in the effective Majorana mass governing neutrino-less double beta ($(\beta\beta)_{0\nu}$-)decay, constraining it to lie near its minimum allowed value of $m_0\cos 2\theta_{12}$, where $m_0 \gtrsim 0.1$ eV.

1. Introduction

Assuming three neutrino flavours there are 9 parameters that define the low energy neutrino mass matrix - three masses, three mixing angles and three phases. Experimental data from neutrino oscillation experiments have determined the two mass squared differences as $\Delta m^2_\odot = 7.67^{+0.16,\,0.52}_{-0.19,\,0.53} \times 10^{-5}\text{eV}^2, |\Delta m^2_\text{A}| = 2.39^{+0.11,\,0.42}_{-0.08,\,0.33} \times 10^{-3}\text{eV}^2$. Note that the sign of $|\Delta m^2_\text{A}| \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}|$, i.e., the ordering of neutrino masses is still not known and there can be three options: normal hierarchy (NH) with $m_1 \ll m_2 \ll m_3$, inverted hierarchy (IH) with $m_3 \ll m_1 \simeq m_2$, or quasi-degenerate neutrinos (QD) with $m_0^2 = m_1^2 \simeq m_2^2 \simeq m_3^2 \gg \Delta m^2_\odot, |\Delta m^2_\text{A}|$. The latter requires that $m_{1,2,3} \gtrsim 0.10$ eV. For the QD case also one can still ask the question whether m_1 or m_3 is the lowest mass, i.e. whether $\Delta m^2_\text{A} > 0$ or $\Delta m^2_\text{A} < 0$.

The current values of the mixing angles are close to tri-bimaximal mixing (TBM) [1] with $\theta_{12} = \sin^{-1}(1/\sqrt{3})$, $\theta_{23} = \pi/4$ and $\theta_{13} = 0$.

Recently the global fit of the solar and KamLAND experiment data showed a preference for a non-zero θ_{13}. The origin of this can be understood by observing that for $\theta_{13} = 0$ solar and KamLAND prefer different values of θ_{12} with no overlap at 1σ level. For $\theta_{13} > 0$ solar prefer higher θ_{12}, KamLAND prefer lower θ_{12} and the disagreement is reduced. From a combined analysis the following best-fit value and 1σ range was obtained in [2]:

$$\sin^2\theta_{13} = 0.016 \pm 0.010\,, \tag{1}$$

or $|U_{e3}| = \sin\theta_{13} = 0.126^{+0.035}_{-0.049}$, or $\theta_{13} = \left(7.3^{+2.0}_{-2.8}\right)^\circ$. In [3] the global fit gives. However, the hint in the atmospheric data has been questioned [4], but the recent MINOS data show a 0.7σ excess of electron events [5]. Thus the value of θ_{13} still very much remains an open question to be decided by future experimental data. We explore the effect of renormalization group (RG) corrections to TBM [7, 8, 9] assuming this to have been generated at some high energy scale. Our aim is to check if the RG effects can accommodate the non-zero θ_{13}indicated by the current data at the same time keeping all other parameters within their allowed ranges.

2. Renormalization Group Effects on Tri-bimaximal mixing

In general, the corrections to the mixing angles can be expressed as [6, 7, 8]:

$$\theta^\lambda_{ij} \simeq \theta^\Lambda_{ij} + C\, k_{ij}\, \Delta_\tau + \mathcal{O}(\Delta^2_\tau)\,, \tag{2}$$

where Λ is the high scale at which TBM is implemented and λ is the low energy scale at which measurements take place. We will indicate high scale values by a superscript Λ in the following, and omit, for simplicity, the superscript λ, which would indicate low scale values. Hence we have $\theta^\Lambda_{12} = \sin^{-1}\sqrt{1/3}$, $\theta^\Lambda_{23} = \pi/4$ and $\theta^\Lambda_{13} = 0$. We consider the RG evolution of the neutrino masses and the mixing parameters in the effective theory and take the high scale to be $\Lambda = 10^{12}$ GeV. The low scale is taken to be $\lambda = 10^2$ GeV when the effective theory is the Standard Model (SM), while we take $\lambda = 10^3$ GeV when the effective theory at low energy is the MSSM. The constant C in Eq. (2) is given by $C = -3/2$ for the SM and $C = +1$ for the MSSM. The result in Eq. (2) is obtained in first order in the parameter

$$\Delta_\tau \equiv \begin{cases} \frac{m_\tau^2}{8\pi^2 v^2}\,(1+\tan^2\beta)\,\ln\frac{\Lambda}{\lambda} \simeq 1.4\cdot 10^{-5}\,(1+\tan^2\beta) & (\text{MSSM})\,, \\ \frac{m_\tau^2}{8\pi^2 v^2}\,\ln\frac{\Lambda}{\lambda} \simeq 1.5\cdot 10^{-5} & (\text{SM})\,, \end{cases} \tag{3}$$

with $\Delta_{e,\mu}$ having been neglected since $m_{e,\mu} \ll m_\tau$ and the vev of the Higgs is taken to be $v/\sqrt{2} = 174$ GeV. The k_{ij}'s are given as [10, 7, 8],

$$\begin{aligned} k_{12} &= \frac{\sqrt{2}}{6}\frac{\left|m_1 + m_2\, e^{i\alpha_2}\right|^2}{\Delta m^2_{21}}, \\ k_{23} &= -\left(\frac{1}{3}\frac{\left|m_2 + m_3\, e^{i(\alpha_3-\alpha_2)}\right|^2}{\Delta m^2_{32}} + \frac{1}{6}\frac{\left|m_1 + m_3\, e^{i\alpha_3}\right|^2}{\Delta m^2_{31}}\right)\,, \\ k_{13} &= -\frac{\sqrt{2}}{6}\left(\frac{\left|m_2 + m_3\, e^{i(\delta+\alpha_3-\alpha_2)}\right|^2}{\Delta m^2_{32}} - \frac{\left|m_1 + m_3\, e^{i(\delta+\alpha_3)}\right|^2}{\Delta m^2_{31}} - \frac{4\, m_3^2\, \Delta m^2_{21}}{\Delta m^2_{31}\,\Delta m^2_{32}}\,\sin^2\frac{\delta}{2}\right)\,, \end{aligned} \tag{4}$$

where we have used $\theta_{ij} = \theta^\Lambda_{ij}$, which is correct up to $\mathcal{O}(\Delta_\tau)$, and $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$. The running of the masses has been neglected in the above expressions for the k_{ij}. The masses are decreasing from high to low scale and run as

$$|m_i| = I_K\left(|m^\Lambda_i| + \mu_i\,\Delta_\tau\right)\,, \tag{5}$$

where I_K is a scalar factor that depends on the SU(2) and U(1) gauge coupling constants and the Yukawa matrix in the up quark sector [11, 12, 10] and μ_i are $\mathcal{O}(1)$ numbers. Thus, neglecting the running of masses introduces an error $\mathcal{O}(\Delta_\tau)$ in k_{ij} and hence $\mathcal{O}(\Delta^2_\tau)$ in θ_{ij}. One also observes that for $|\Delta_\tau| \gtrsim ((m^\Lambda_2)^2 - (m^\Lambda_1)^2)/(m^\Lambda_0)^2$, the $\mathcal{O}(\Delta^2_\tau)$ terms dominate over the $\mathcal{O}(\Delta_\tau)$ terms in the evolution of $m_2^2 - m_1^2$ [10]. For such cases Eqs. (4) will no longer be valid. Thus, for the validity of these equations, we require $(m^\Lambda_0)^2\,\Delta_\tau \lesssim (m^\Lambda_2)^2 - (m^\Lambda_1)^2$, which may not be satisfied if $(m_2^\Lambda)^2 - (m_1^\Lambda)^2$ is indeed very small. We will therefore use the full running equations for the mass matrix itself for the plots and numerical values to be presented. Analytical estimates are made with the expressions of the k_{ij} and, as we show, these estimates can explain the numerical results with a sufficient degree of correctness. There is a subtle issue involved when we consider k_{13} in Eq. (4). As is seen, k_{13} depends on the Dirac CP phase δ which is unphysical for the case of $\theta_{13} = 0$ at the high scale Λ. However as discussed in [10, 13], the value of δ at this point depends on the values of the masses and the Majorana phases and RG evolution takes care of that automatically. For analytical estimates, it is convenient to consider the shift of the mixing angles θ_{ij} from their initial values. From the above expressions for the k_{ij}, and in the limit of $|k_{ij}\,\Delta_\tau| \ll 1$, one obtains the following expressions for the observables:

$$|\sin\theta_{13}| \simeq |C\,k_{13}\,\Delta_\tau| \ , \ \ \sin^2\theta_{23} \simeq \frac{1}{2} - C\,k_{23}\,\Delta_\tau \ , \ \ \sin^2\theta_{12} - \frac{1}{3} \simeq \frac{2\sqrt{2}}{3}\,C\,k_{12}\,\Delta_\tau \ . \qquad (6)$$

In the spirit of our analysis we require 1)) that $|C\,k_{13}\,\Delta_\tau| = 0.077 - 0.161$, while $-C\,k_{12}\,\Delta_\tau = 2.8\cdot 10^{-3} - 4.2\cdot 10^{-2}$. Note that for the 1σ range we are taking, $C\,k_{12}\,\Delta_\tau$ (and therefore C) is supposed to be negative. Hence, within the MSSM the required deviation from TBM cannot be realized. Therefore we use the 3σ ranges for $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ for the purposes of illustration [14]. If indeed the trend of $\sin^2\theta_{12} < \frac{1}{3}$ continues then it will not be possible to account for a high scale value of $\sin^2\theta_{12} = \frac{1}{3}$ solely by RG effects within the MSSM From Eqs. (2) and (4) it is evident that whether the angles θ_{ij} will decrease or increase during evolution will depend on the effective theory (SM or MSSM, through the factor C) and also on the sign of $\Delta m^2_{31}(\simeq \Delta m^2_{32})$ for θ_{23}. Since k_{12} is always positive, θ_{12} at the low scale is always larger (smaller) than that at high scale for the MSSM (SM). The size of the RG corrections will depend on the values of the Majorana phases, the neutrino masses and, in case of the MSSM, $\tan\beta$. One can at the outset make some interesting observations from Eqs. (4). It is easy to see that only quasi-degenerate neutrinos will be able to lead to values of $|U_{e3}|$ around 0.1. Note also that in this case the running of solar neutrino mixing is in general enhanced by a factor $|\Delta m^2_{\rm A}|/\Delta m^2_\odot$ with respect to the running of the other mixing angles. We further can expect that the deviation from maximal θ_{23} is of the same order than the deviation from zero $|U_{e3}|$. In the following we will quantify these statements.

3. Resuls

We have performed a detailed analysis, by numerically solving the RG running equations for the effective neutrino mass matrix and then diagonalizing it to extract the masses, mixing angles and phases at low scale. At the high scale Λ, the angles θ_{ij}^Λ are fixed by the requirement of the TBM scenario, while the masses and the CP phases are chosen randomly so that after the RG evolution at low scale the parameters are consistent with the chosen ranges of the current experimental data. We have used the following ranges for the high scale values of the mass-squared differences: $(\Delta m^2_\odot)^\Lambda = 10^{-6} - 10^{-3}$ eV^2 and $|\Delta m^2_{\rm A}|^\Lambda = 1.5\times 10^{-3} - 10^{-2}$ eV^2, while the phases are varied over the full range of $0 - 2\pi$.

Starting with the SM, Fig. 1 shows the allowed region in the $m_0 - \sin^2\theta_{13}$ plane at the low scale λ, after performing the RG evolution, for both NH (left panel) and IH (right panel). Recall that $m_0^2 \gg |\Delta m^2_{\rm A}|$ is the common neutrino mass scale for quasi-degenerate neutrinos. As can be seen, to generate values of $|U_{e3}|$ within the range of interest, neutrino masses should exceed the direct limit of 2.3 eV from tritium decay [15], and hence also the more stringent but model-dependent limits from cosmology. We conclude that a high scale value of $\theta_{13} = 0$ is incompatible with the indicated range of $|U_{e3}|$. The dependence of this statement on the initial values of θ_{12}^Λ and θ_{23}^Λ is moderate and hence this statement is valid in general.

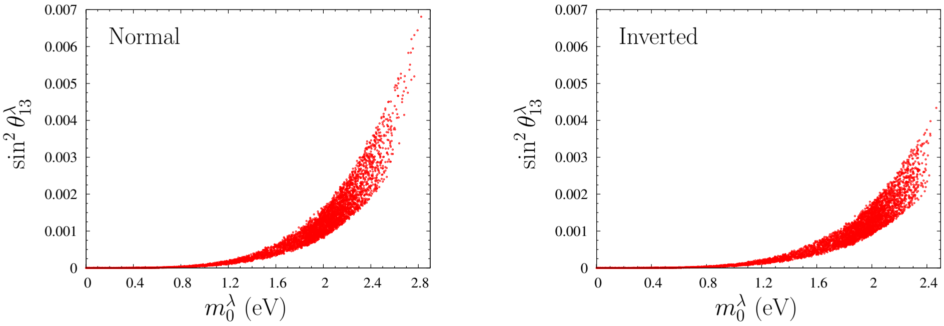

Figure 1. The running of $|U_{e3}|^2 = \sin^2\theta_{13}$ in SM for both NH (left panel) and IH (right panel). The high scale values of mixing angles are kept fixed at TBM values while the masses and phases are varied randomly such that after RG evolution the parameter values are within current experimental ranges.

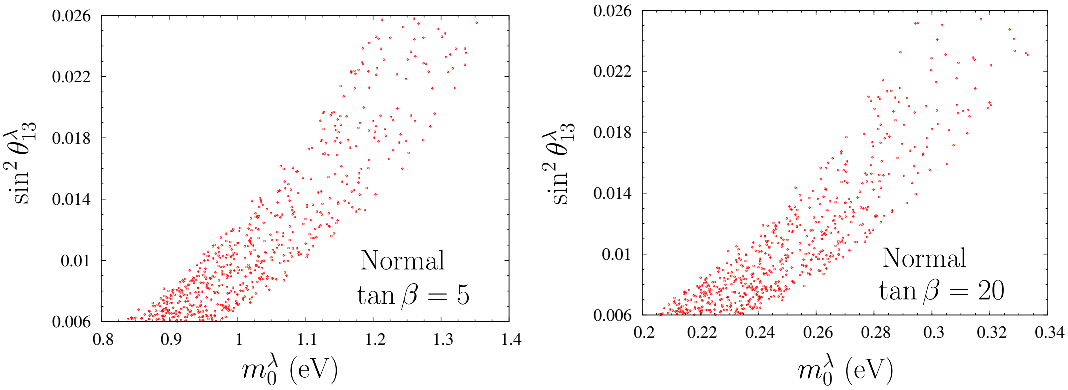

Figure 2. Scatter plots showing the running of $\sin^2\theta_{13}$ with m_0, for MSSM with NH and $\tan\beta = 5, 20$. For a given $\tan\beta$, the allowed regions are the same as above for IH.

Fig. 2 shows the allowed region in the $m_0 - \sin^2\theta_{13}$ plane, when the effective theory is the MSSM, for $\tan\beta = 5, 20$ and NH. The left panel shows that $\sin^2\theta_{13}$ lies in the required range when 0.8 eV $\lesssim m_0 \lesssim$ 1.4 eV for $\tan\beta = 5$, while the allowed mass range becomes 0.2 eV $\lesssim m_0 \lesssim$ 0.34 eV for $\tan\beta = 20$, as can be seen from the right panel. Thus, the relevant range of $m_0 \tan\beta$ is given by (see below for analytical estimates) $4.1 \lesssim (m_0/\text{eV})\tan\beta \lesssim 6.9$. Hence the allowed mass ranges depend strongly on $\tan\beta$ and for higher values of $\tan\beta$, lower values of m_0 are sufficient to produce the required running of θ_{13}. It has been checked that for a fixed $\tan\beta$ value, there is no significant dependence on the mass ordering, other than the direction of the correction to θ_{23}. From the allowed mass ranges obtained in Fig. 2 it is seen that to have $\sin^2\theta_{13}$ in the 1σ range under consideration, we need the neutrinos to be quasi-degenerate even for the MSSM with $\tan\beta = 20$. Fig. 3 shows scatter plots of the allowed region of the neutrino mass scale m_0 and the Majorana phase α_2, which is particularly important for the running of θ_{12} [10] (see also [16]). We compare the allowed regions at high and low scale for NH and $\tan\beta = 5, 20$. The scattered plots obtained for IH show the same characteristics. We see that $|\alpha_2|$ is restricted in a narrow region around $|\alpha_2| = \pi$ for all cases.

Fig. 4 shows the correlation between the low scale values of $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$. For NH $\sin^2\theta_{23} > \frac{1}{2}$, whereas for IH $\sin^2\theta_{23} < \frac{1}{2}$. For NH θ_{13} and θ_{23} are correlated, i.e., a higher value of θ_{13} requires a higher value of θ_{23}. For IHthe predicted values of the two angles are

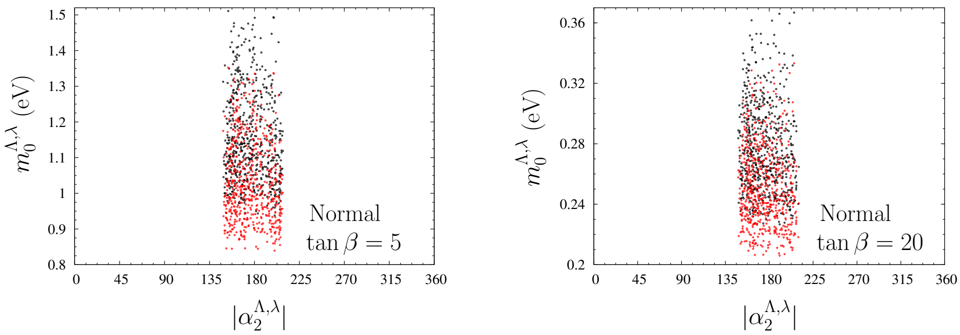

Figure 3. Scatter plots in the $m_0 - |\alpha_2|$ plane for MSSM ($\tan\beta = 5, 20$) and NH, both for high (black circles) and low (red squares) energy scales. Same variation is obtained for IH.

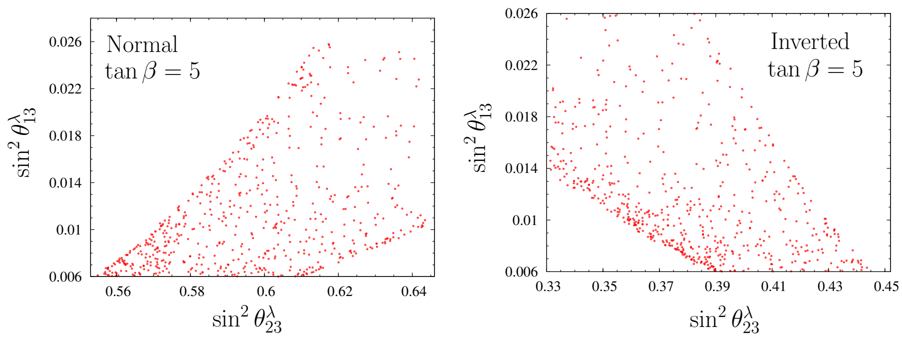

Figure 4. Scatter plots showing the correlation between $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ for MSSM with $\tan\beta = 5, 20$. The left panel shows the case with NH while the right panel is for IH.

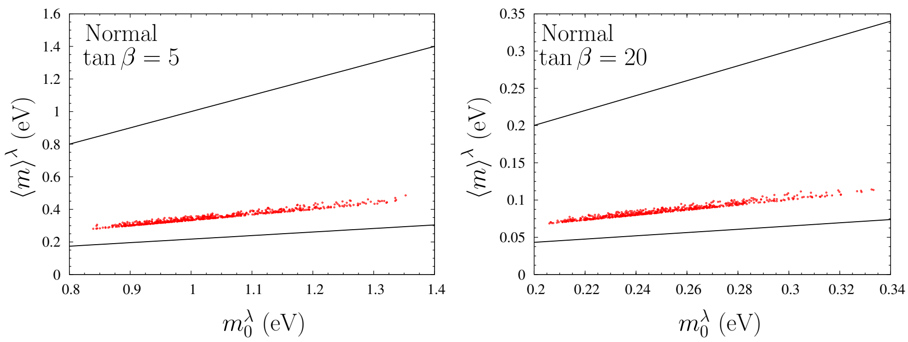

Figure 5. Scatter plot for the effective neutrino mass $\langle m \rangle$ that contributes to neutrino-less double beta decay as a function of m_0, in MSSM with $\tan\beta = 5, 20$ and NH. The solid (black) lines indicate the maximum and minimum possible values of $\langle m \rangle$ for given m_0, obtained by varying the oscillation parameters in their current 3σ range and the phases between 0 to 2π. The cases with IH show same characteristics.

anti-correlated. The plots obtained with $\tan\beta = 20$ are identical to those shown in Fig. 4 for $\tan\beta = 5$, when the mass ordering is the same. For a different $\tan\beta$ the value of m_0 adjusts itself to comply with the low energy cuts on the parameters and the allowed points in the $\sin^2\theta_{23} - \sin^2\theta_{13}$ plane remain same. We note here that maximal atmospheric neutrino mixing is not possible.

In Fig. 5 we plot the effective Majorana mass which governs the rate of $(\beta\beta)_{0\nu}$-decay at low energy. The scatter points show the values of $\langle m\rangle$ allowed by the low energy neutrino oscillation data after RG analysis. The solid (black) lines indicate the maximum and minimum possible values of $\langle m\rangle$ at low scale for a given m_0, obtained by varying the oscillation parameters in their current 3σ range and the phases between 0 to 2π. The plots show that the effective mass obtained after RG analysis lies close to its minimum allowed range. As can also be seen from Fig. 5, for $\tan\beta = 5$, $\langle m\rangle$ takes values between 0.26 and 0.50 eV, to be compared with the general upper and lower limits of 0.2 eV and 1.4 eV. If $\tan\beta = 20$, then 0.07 eV $\lesssim \langle m\rangle \lesssim$ 0.11 eV, while in general the effective Majorana mass could be in between 0.05 eV and 0.34 eV.

4. Conclusions

Global oscillation data imply two large (θ_{12} and θ_{23}) one small (θ_{13}) mixing angle Seesaw mechanism which postulates a new particle with a heavy mass scale can explain the smallness of neutrino mass. However since the measurements are at low energy one needs to include RG effects which even if small are important in view of the onset of precision era in neutrino physics. In this work, we study the RG effects starting with TBM at a high scale and specifically examine whether the current non-zero hint of θ_{13} can be accommodated. RG effects are found to be small in SM In MSSM there is an enhancement by $\tan^2\beta$. Non-zero θ_{13} from current global fit cannot be accommodated in TBM + RG effects if the effective theory is SM but it is possible in MSSM for large m_0 and high $\tan\beta$ Compatibility with current 3σ range of oscillation parameters implies the Majorana phase $\alpha_2 \approx \pi$. Precision measurements of m_0, θ_{12},θ_{13} are expected to play an important role according to our analysis.

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