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Two-dimensional gravity, matrix models and nonperturbative string theory

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Abstract. We review some of the recent developments in nonperturbative string theory and discuss their connections with black hole physics and low dimensional fermi systems.

1. Introduction

In this section I shall briefly motivate the need to consider nonperturbative string theory. Perturbative string theory, as defined by a sum over two-dimensional surfaces of different genera (number of handles) [1] provides the first ever (and so far the only) example of a theory of interacting gravitons that is equivalent to Einstein's theory at low energies [2] and is finite in perturbation theory under appropriate circumstances. While this is an important step towards finding a quantum theory of gravity, there are several questions that remain unanswered in the perturbative approach. To pick a few:
(a) given the fact that perturbative string theory is defined necessarily in a fixed background geometry, is there a nonperturbative formulation (lagrangian or otherwise) that can determine the background geometry itself?
(b) if the background geometry involves strong curvature then one expects the coupling constant to grow large which makes perturbation theory in such backgrounds unreliable; how does one define the theory nonperturbatively in such cases?
(c) a related question is: in the regions of large curvature since string theory becomes strongly coupled, the notion of an expectation value of the metric itself becomes unreliable thanks to strong quantum fluctuations of the metric and its strong mixing with the higher string modes; clearly the picture of "spacetime" needs a revision — what is the new picture?

Besides these there are questions related to non-gravity aspects of string theory which also require a nonperturbative definition. For instance, one needs to have a nonperturbative mechanism of supersymmetry breaking because breaking SUSY at any finite order necessarily spoils the finiteness of string theory (at least till date). There is also this interesting observation [3] that string perturbation theory, even when finite, is non-Borel summable in a certain way, one interpretation of which is the existence of instanton contributions that are stronger than in conventional field theories and which go as $\exp(-1/g_{string})$. A nonperturbative formulation is clearly tailored to probe such effects.

The only workable nonperturbative string theory at the moment is defined in terms of the so-called "matrix models". In the rest of the talk I will define and describe matrix models and discuss how they address some of the questions listed above.

2. Matrix models and dynamically triangulated random surfaces

Random matrices have been used in nuclear physics for a long time [4] largely because of the connection between nuclear spectral density and eigenvalue density of random matrices. The connection between functional integral over matrix-valued fields and two-dimensional surfaces of different topologies was first made in the path-breaking papers [5] by 't Hooft on dual string models and large-N QCD. To understand this connection let us first discuss the discrete (lattice) approach to quantum gravity.

The subject of quantum gravity in the continuum is fraught with several basic questions of principle. Quite besides the issue of nonrenormalizability which is the major problem is four dimensions, there is the issue of the metric-dependence of the short-distance cut-off (needed to ensure general coordinate invariance in the quantum theory) which makes even the *definition* of the theory problematic in all dimensions. One of the early remedies considered was to think of lattice gravity or "dynamically triangulated random surfaces" (DTRS). As in the case of lattice gauge theories which are automatically gauge invariant, the DTRS formulation is also *a priori* general coordinate invariant. In two dimensions, the original ideas of Regge [6] were adopted in the following form. The lattice regularization of Polyakov path integral [1]

$$Z = \sum_h \int \mathcal{D}g_{ab}(\xi)\mathcal{D}X^\mu(\xi)\exp[-\int d^2\xi\sqrt{g}\{\beta + \frac{\beta'}{4\pi}R + \frac{1}{2}g^{ab}\partial_a X^\mu \partial_b X^\mu\}] \quad (1)$$

is taken to be [7]

$$\begin{aligned} Z &= \sum_G \Gamma(G) e^{-\beta p(G) - \beta' h(G)}, \\ \Gamma(G) &= \int \prod_i \frac{d^D \mathbf{x}_i}{\pi^{D/2}} \exp[-\frac{1}{2}\sum_{\langle ij \rangle}(\mathbf{x}_i - \mathbf{x}_j)^2]. \end{aligned} \quad (2)$$

In the first equation there is an explicit summation over genus h; since $(4\pi)^{-1}\int \sqrt{g}R = (2-2h)$ one could pull out the β' term outside to display the sum over genus as $\sum_h \exp[-\beta'(2-2h)]$. In the second equation the sum is over all possible triangulations G. We denote the number of links (edges) of G as $l(G)$, the number of sites (vertices) as $s(G)$ and the number of plaquettes (faces or triangles) as $p(G)$. The notation $h(G)$ stands for the genus of the simplex G, defined by the Euler formula

$$h(G) = s(G) - l(G) + p(G). \quad (3)$$

The connection with matrix models comes about as follows. Consider the following partition function

$$Z(g) = \int dM \exp[-\mathrm{tr}V(M)], \quad V(M) = (\frac{1}{2}M^2 + gM^3). \quad (4)$$