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Dual giant gravitons in AdS$_m$ × Y^n (Sasaki-Einstein)

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Dual giant gravitons in $\mathrm{AdS}_m \times \mathrm{Y}^n$ (Sasaki-Einstein)

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ABSTRACT: We consider BPS motion of dual giant gravitons on $AdS_5 \times Y^5$ where Y^5 represents a five-dimensional Sasaki-Einstein manifold. We find that the phase space for the BPS dual giant gravitons is symplectically isomorphic to the Calabi-Yau cone over Y^5, with the Kähler form identified with the symplectic form. The quantization of the dual giants therefore coincides with the Kähler quantization of the cone which leads to an explicit correspondence between holomorphic wavefunctions of dual giants and gauge-invariant operators of the boundary theory. We extend the discussion to dual giants in $AdS_4 \times Y^7$ where Y^7 is a seven-dimensional Sasaki-Einstein manifold; for special motions the phase space of the dual giants is symplectically isomorphic to the eight-dimensional Calabi-Yau cone.

KEYWORDS: D-branes, AdS-CFT Correspondence.

Contents

1. Introduction

Supersymmetric giant gravitons [1–3] in $\text{AdS}_m \times S^n$ have been studied extensively in the literature. Some papers of relevance to the present work are [4–13]. Recently significant progress has been made in quantizing these objects [10–13]. One of the main motivations for these works is to understand the quantum states of the boundary theory in terms of constructions in the bulk, which may also help in the crucial question of understanding microstates of a black hole (e.g. the 1/16-BPS ones in [14] or the BTZ black hole) from the bulk point of view. It has been found that for supersymmetries equivalent to 1/8-BPS and higher, the quantum states of the boundary theory can be completely accounted for in terms of giant gravitons or dual giant gravitons [11, 12, 15, 16]. Indeed, it appears that the descriptions in terms of giants and dual giants are dual to each other. In case of 1/2-BPS states, there is a precise isomorphism between the Fock spaces of giant gravitons, dual giant gravitons and the boundary excitations (fermions) [13].

It would be fruitful to extend the above correspondence to less supersymmetric backgrounds in which case the boundary theories are more exotic. It may turn out that the giant graviton analysis is more tractable than the gauge theory and can teach us something about the latter, especially about finite N effects.

In this paper we will consider dual giant gravitons in $\text{AdS}_m \times Y^n$ where Y^n is a Sasaki-Einstein manifold. In section 2 we consider BPS dual giants in $\text{AdS}_5 \times Y^5$ where Y^5 is a five-dimensional Sasaki-Einstein manifold. As in the case of $AdS_5 \times S^5$, we find that the coordinate space $R_+ \times Y^5$ reduces to a phase space with symplectic structure identical to that of the Calabi-Yau cone over Y^5. In section 3 we use this result to perform Kähler quantization of the BPS dual giant gravitons and find an explicit map between their wavefunctions and gauge invariant operators in the corresponding boundary theory. We do this analysis explicitly for $T^{1,1}$ and indicate how this generalizes to higher $Y^{p,q}$ spaces. In section 4 we consider special motion of dual giant gravitons (arguably supersymmetric) in $\text{AdS}_4 \times Y^7$ and find again that the symplectic structure of the dual giant configuration space $R_+ \times Y^7$ becomes identical to the Kähler structure of the eight-dimensional Calabi-Yau cone over Y^7.

While this paper was being finished, two papers [17, 18] appeared which have material related to the present paper. In particular [18] has substantial overlap with parts of the present paper dealing with $\text{AdS}_5 \times Y^5$.

2. Dual giant gravitons on $\text{AdS}_5 \times \text{Y}^5$

In this section we consider dual giant gravitons on $\text{AdS}_5 \times Y^5$ where Y^5 is a 5-dimensional Sasaki-Einstein manifold. We will consider for concreteness $Y^5 = Y^{p,q}$, although the method of calculation of section 4 for the seven-dimensional Sasaki-Einstein manifold Y^7 suggests that the results of this section should be generally valid (see section 4.2).

We will take the AdS_5 metric to be

$$ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2(d\chi^2 + \cos^2\chi d\phi_1^2 + \sin^2\chi d\phi_2^2)$$

$$V(r) = 1 + \frac{r^2}{l^2} \tag{2.1}$$

The $Y^{p,q}$ has a metric

$$\frac{1}{l^2}ds^2 = \frac{1-y}{6}\left(d\theta^2 + \sin^2\theta d\phi^2\right) + \frac{1}{w(y)q(y)}dy^2 + \frac{q(y)}{9}\left(d\psi^2 - \cos\theta d\phi\right)^2$$

$$+ w(y)\left(d\alpha + f(y)(d\psi - \cos\theta d\phi)\right)^2 \tag{2.2}$$

where

$$w(y) = \frac{2(a-y^2)}{1-cy}, \; q(y) = \frac{a - 3y^2 + 2cy^3}{a - y^2}, \; f(y) = \frac{ac - 2y + y^2 c}{6(a - y^2)} \tag{2.3}$$

Non-zero values of c will be set to 1 by a rescaling of coordinates. The case $c = 0$ corresponds to $Y^{1,0} = T^{1,1}$ which we will deal with separately. The parameter a can take a countably infinite number of values between 0 and 1, specified by two integers p and q (see, e.g. [20] for details). The coordinate y varies between the two smallest roots of $q(y) = 0$.

The five-form RR field strength is given by

$$F^{(5)} = -\frac{4}{l} \left[e^0 \wedge e^1 \wedge e^2 \wedge e^3 \wedge e^4 + \Omega(Y^5) \right]. \tag{2.4}$$

The corresponding 4-form potential can be written as

$$C^{(4)} = r^4 \cos\chi \sin\chi \, dt \wedge d\chi \wedge d\phi_1 \wedge d\phi_2 + l^4 C_{Y^5}^{(4)} \tag{2.5}$$

The $e^i, i = 0, \ldots, 4$ represent vielbeins of AdS$_5$ and will be taken to be

$$e^0 = V^{1/2}(r) \, dt, \ e^1 = V^{-1/2}(r) \, dr, \ e^2 = r d\chi, e^3 = r \cos\chi d\phi_1, e^4 = r \sin\chi d\phi_2 \tag{2.6}$$

$\Omega(Y^5)$ represents the volume form of Y^5 and is written locally as $dC_{Y^5}^{(4)}$; its specific form will not be important for us.

A dual giant graviton [2, 3, 12] is a D3-brane wrapped on the $S^3 \in AdS_5$. Its embedding is given by

$$t = \sigma_0 = \tau, \ r = r(\tau), \ \chi = \sigma_1, \ \phi_1 = \sigma_2, \ \phi_2 = \sigma_3,$$
$$y = y(\tau), \theta = \theta(\tau), \phi = \phi(\tau), \alpha = \alpha(\tau), \psi = \psi(\tau) \tag{2.7}$$

The pull-back of $C^{(4)}$ onto the world-volume is $C_{\sigma_0\sigma_1\sigma_2\sigma_3}^{(4)} = l^{-1}r^4 \cos\sigma_1 \sin\sigma_1$. The DBI plus WZ Lagrangian density is

$$\mathcal{L} = -\frac{N}{2\,\pi^2\,l^4} r^3 \cos\sigma_1 \sin\sigma_1 \left[\Delta^{1/2} - \frac{r}{l} \right] \tag{2.8}$$

where

$$\Delta = V(r) - \frac{\dot{r}^2}{V(r)} - l^2 \left[\frac{1-y}{6} \left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2 \right) \right.$$
$$\left. + \frac{1}{w(y)q(y)}\dot{y}^2 + \frac{q(y)}{9} \left(\dot{\psi} - \cos\theta\dot{\phi} \right)^2 + w(y) \left(\dot{\alpha} + f(y)(\dot{\psi} - \cos\theta\dot{\phi}) \right)^2 \right] \tag{2.9}$$

We can integrate over the angles σ_i to get Action $= \int dt \, L$ where the effective point-particle Lagrangian L is

$$L = \int d\sigma_1 \, d\sigma_2 \, d\sigma_3 \ \mathcal{L} = -\frac{N}{l^4} r^3 \left[\Delta^{1/2} - \frac{r}{l} \right] \tag{2.10}$$

The conjugate variables of the classical mechanics model are

$$P_r = \frac{N \, r^3 \, \dot{r}}{l^4 \, V(r) \, \Delta^{1/2}}, \ \ P_y = \frac{N \, r^3 \, g_{yy}\dot{y}}{l^4 \, \Delta^{1/2}}, \ \ P_\theta = \frac{N \, r^3 \, g_{\theta\theta} \, \dot{\theta}}{l^4 \, \Delta^{1/2}},$$

$$P_\phi = \frac{Nr^3}{l^2\Delta^{1/2}} \left[\frac{1-y}{6} \sin^2\theta\dot{\phi} - \frac{1}{9} \cos\theta q(y) \left(\dot{\psi} - \cos\theta\dot{\phi} \right) - w(y)f(y)\cos\theta \left(\dot{\alpha} + f(y)(\dot{\psi} - \cos\theta\dot{\phi}) \right) \right]$$

$$P_\alpha = \frac{N \, r^3}{l^2\Delta^{1/2}} w(y) \left(\dot{\alpha} + f(y)(\dot{\psi} - \cos\theta\dot{\phi}) \right)$$

$$P_\psi = \frac{N \, r^3}{l^2\Delta^{1/2}} \left[\frac{q(y)}{9} \left(\dot{\psi} - \cos\theta\dot{\phi} \right) + w(y) \left(\dot{\alpha} + f(y)(\dot{\psi} - \cos\theta\dot{\phi}) \right) \right] \tag{2.11}$$

Henceforth we will put $l = 1$ for simplicity. In the following we will look at solutions of the equation of motion which preserve supersymmetry. The simplest way to do this is to consider motion along the so-called Reeb Killing vector field (we will explain the connection to supersymmetry in section 2.2)

$$V_R = 3\frac{\partial}{\partial\psi} - \frac{1}{2}\frac{\partial}{\partial\alpha} \tag{2.12}$$

See, e.g. [20, 21] for various properties of this vector field. The corresponding dynamical trajectory is

$$\dot{r} = \dot{y} = \dot{\theta} = 0$$
$$\dot{\phi} = 0, \dot{\psi} = 3, \dot{\alpha} = -\frac{1}{2} \tag{2.13}$$

Note that on this trajectory

$$\dot{f}(r, y, \theta, \phi, \psi, \alpha) = 3\frac{\partial}{\partial\psi}f - \frac{1}{2}\frac{\partial}{\partial\alpha}f = V_R(f) \tag{2.14}$$

for any function f. It is a lengthy but straightforward calculation to check that (2.13) actually solves the equations of motion coming from the Lagrangian in (2.10).

The reverse trajectory

$$\dot{r} = \dot{y} = \dot{\theta} = 0, \dot{\phi} = 0, \dot{\psi} = -3, \dot{\alpha} = \frac{1}{2} \tag{2.15}$$

is also a solution, with the opposite momenta. This solution also turns out to be super-symmetric.

2.1 Reduced phase space

The equations (2.13) translate to the following constraints on the 12-dimensional phase space:

$$P_r = P_y = P_\theta = 0$$
$$P_\phi = -\frac{Nr^2}{3}(1-y)\cos\theta, \ P_\alpha = -2Nr^2y, \ P_\psi = \frac{Nr^2}{3}(1-y) \tag{2.16}$$

Obtaining the second line involves significant amount of algebra. A useful identity is

$$q(y) + \frac{w(y)}{4}(6f(y) - 1)^2 = 1 \tag{2.17}$$

The constraints (2.16) reduce the original 12 dimensional phase space to a six dimensional phase space which can be parametrized by the six coordinates $\zeta^a = (r, y, \theta, \phi, \alpha, \psi)$.

2.1.1 Symplectic structure

We would like to derive the symplectic form on the reduced phase space. This can be done by treating (2.16) as Dirac constraints ($C^i, i = 1, \ldots, 6$). Calculation of Dirac brackets among the six independent coordinates proceeds as in the $AdS_5 \times S^5$ case [12, 10, 9]. Thus

$$\{\zeta^a, \zeta^b\}_{DB} = \{\zeta^a, \zeta^b\}_{PB} - \{\zeta^a, C^i\}_{PB} \, M_{ij}^{-1} \, \{C^j, \zeta^b\}_{PB}$$
$$M^{ij} \equiv \{C^i, C^j\}_{PB} \tag{2.18}$$

Note that ζ^a are coordinates and hence their PB's are zero. However, the Dirac brackets are non-zero: the supersymmetry constraints transform the coordinate space into a phase space, as in the case of $AdS_5 \times S^5$.

The symplectic form is defined as

$$\omega = \omega_{ab} d\zeta^a \wedge d\zeta^b,$$
$$(\omega^{-1})^{ab} \equiv \{\zeta^a, \zeta^b\}_{DB} \tag{2.19}$$

The result of this exercise is that

$$\omega = d\phi \wedge d\left[-\frac{Nr^2}{3l^2}(1-y)\cos\theta\right] + d\psi \wedge d\left[\frac{Nr^2}{3l^2}(1-y)\right] + d\alpha \wedge d\left[-N2\frac{r^2}{l^2}y\right] \tag{2.20}$$

Let us compare this with the Kähler form of the Calabi Yau cone. The cone is defined by a Kähler metric

$$ds_6^2 = d\rho^2 + \rho^2 ds_{Y^5}^2 \tag{2.21}$$

with the Kähler form[1]

$$J = d\phi \wedge d\left[\frac{\rho^2}{6l^2}(1-y)\cos\theta\right] + d\psi \wedge d\left[-\frac{\rho^2}{6l^2}(1-y)\right] + d\alpha \wedge d\left[\frac{\rho^2}{l^2}y\right] \tag{2.22}$$

Thus, we obtain one of our main results:

$$\omega = NJ, \text{ provided } \quad \rho^2 \equiv 2r^2 \tag{2.23}$$

The factor of N is similar to that in the case of $AdS_5 \times S^5$ [12, 9, 10], and corresponds to $1/\hbar$ (note that the Dirac bracket ω^{-1}); e.g., note the factor of N in Eqns (3.22) and (3.23) of [10].

2.2 Hamiltonian, R-charge and supersymmetry

The hamiltonian on the reduced subspace is

$$H = \dot{\zeta}^a P_{\zeta^a} - L = \frac{1}{l}\left(3P_\psi - \frac{1}{2}P_\alpha\right) \tag{2.24}$$

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[1]Our convention differs in sign from [20]. We choose this sign to ensure that $H = 3P_{\psi'}$ is positive (see (2.27)).

This follows simply, because $L = 0$. It is useful to make the change of variable $(\alpha, \psi) \rightarrow (\alpha', \psi') = (\alpha + \frac{1}{6}\psi, \psi)$; in these coordinates

$$\dot{\alpha}' = 0, \dot{\psi}' = \{\psi', H\} = \{\psi', 3P_\psi/l\} = \frac{3}{l} \qquad (2.25)$$

The Reeb vector now is simply

$$V_R = 3\frac{\partial}{\partial\psi'} \qquad (2.26)$$

It is easy to see that

$$H = \frac{3}{l}P_{\psi'} \qquad (2.27)$$

Since the momentum along the Reeb Killing vector is dual to the R-charge [20, 21] in the boundary field theory, eq. (2.27) is a BPS relation and therefore (2.13) must represent BPS dual giant gravitons. An independent verification of this fact could be done by checking the kappa-symmetry projection; we will not do this here.[2]

Besides the Reeb vector $3\partial/\partial\psi'$, there are two other Killing vectors $\partial/\partial\alpha', \partial/\partial\phi$. These correspond to (linear combinations of) the "toric" T^3 action on these manifolds and are dual to the three $\mathcal{U}(1)$ charges in the dual field theory.

2.3 The case of $T^{1,1}$

In this case the metric is [23]

$$ds^2|_{T^{1,1}} = \frac{1}{6}(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2 + d\theta_2^2 + \sin^2\theta_2 d\phi_2^2) + \frac{1}{9}(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2 \quad (2.28)$$

The Kähler cone with the above base is given by the metric

$$ds_6^2 = d\rho^2 + \rho^2 ds^2|_{T^{1,1}} \qquad (2.29)$$

and a Kähler form [24]

$$J = \frac{\rho^2}{6}(\sin\theta_1 d\theta_1 \wedge d\phi_1 + \sin\theta_2 d\theta_2 \wedge d\phi_2) - \frac{1}{3}\rho d\rho \wedge (d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2) \quad (2.30)$$

Once again the symplectic form ω obtained from quantizing BPS dual giant gravitons satisfies (2.23). The hamiltonian is given by (2.27). Since the angle ψ (equivalently, ψ') has a range $(0, 4\pi)$, it is customary to define an angle $\nu = \psi/2$ [20]. In terms of this, the classical BPS relation (2.27) becomes

$$H = \frac{3}{2l}P_\nu \qquad (2.31)$$

2.4 Summary of this section

We find that the phase space for the BPS dual giant gravitons moving in $\text{AdS}_5 \times Y^5$ is symplectically isomorphic to the Calabi-Yau cone (2.21), with the identification (2.23).

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[2]It has been shown in [18] that the kappa-symmetry projection equation is indeed satisfied for the motion (2.25).

3. Quantization: counting BPS dual giants

3.1 The case of $T^{1,1}$

To get oriented, we will discuss the case of $T^{1,1}$ first. The cone (2.29) is given by the following equation in C^4

$$w_i w_i = 0, \ \vec{w} \in C^4 \tag{3.1}$$

The Kähler form is obtainable from the following Kähler potential [24]

$$f = \frac{3}{2}(w_i \bar{w}_i)^{2/3} \equiv \rho^2 \tag{3.2}$$

The classical hamiltonian is given by (see (2.16))

$$H = \frac{3}{l}P_{\psi'} = \frac{N}{l^3}r^2 = \frac{N}{2l^3}\rho^2 \tag{3.3}$$

Since the symplectic form for BPS dual giants is the same as the Kähler form for the cone (2.29) or (3.1), we can quantize this space according to the standard procedure of geometric quantization of Kähler manifolds [25, 26]. According to this method, we define the quantum Hilbert space in terms of holomorphic functions $\psi(w)$ with a norm defined by the restriction of that on C^4 to the surface (3.1). The operator corresponding to (3.3) is given by [26]

$$\hat{H} = -i\hbar X_H + \langle \theta, X_H \rangle + H \tag{3.4}$$

where θ is the symplectic potential which we can define as $\theta = i\partial f$. X_H denotes the symplectic (Hamiltonian) flow of H. It turns out that the last two terms cancel each other, as in the case of a simple harmonic oscillator (except that the algebra is more complicated) and we are left with just the first term which corresponds to the classical symplectic flow for H. This gives (identifying $\hbar$ of (3.4) with $1/N$, see remarks below (2.23)

$$\hat{H} = \frac{3}{2l}\hat{R}, \ \hat{R} \equiv w_i \frac{\partial}{\partial w_i} \tag{3.5}$$

where $\hat{R}$ is the operator form for the R-charge. The operator form is determined by the fact that the R-charge each w_i is 1 [24, 22]. Comparing (2.31) with (3.5), we get

$$\hat{P}_\nu = \hat{R} \tag{3.6}$$

which expectedly identifies the momentum along the Reeb vector with the R-charge at the boundary, as already mentioned in the previous section.

3.1.1 Wavefunctions

It is easy to solve the eigenvalue problem of $\hat{R}$ (which is equivalent to solving the eigenvalue problem of $\hat{H}$, because of (3.5)). The solutions can be expressed in a basis of monomials

$$\hat{R} \ \psi_{\vec{j}}(\vec{w}) = j\psi_{\vec{j}}(\vec{w}), \quad \psi(w) = w_1^{j_1} \dots w_4^{j_4}, j_1 + \dots + j_4 = j \tag{3.7}$$

For multiple dual giants, we will have

$$\hat{R} \, \psi(w^{(a)}) = \sum_a w_i^{(a)} \frac{\partial \psi}{\partial w_i^{(a)}} = \sum_a j^{(a)} \psi(w^{(a)}) \tag{3.8}$$

where the wavefunctions are obtained by taking symmetrized product of monomials as in (3.7), e.g.

$$\psi_{\vec{j}^1, \vec{j}^2}(\vec{w}^1, \vec{w}^2) = \frac{1}{2} \left(\psi_{\vec{j}^1}(\vec{w}^1) \psi_{\vec{j}^2}(\vec{w}^2) + \psi_{\vec{j}^1}(\vec{w}^2) \psi_{\vec{j}^2}(\vec{w}^1) \right) \tag{3.9}$$

3.1.2 Explicit counting

Clearly the spectrum of (3.7) is integral and the eigenfunctions are monomials in w_i (modulo the relation (3.1)). Some examples are:

- Wavefunctions of degree 1 (corresponding to R-charge =1 operators in the boundary theory):

  There are 4 eigenfunctions given by

  $$\psi(w) = w_1, w_2, w_3, w_4 \tag{3.10}$$

  or alternatively, by using a linear change, the wavefunctions are

  $$\omega_1 = w_3 + iw_4, \omega_2 = w_1 - iw_2, \omega_3 = w_1 + iw_2, \omega_4 = -w_3 + iw_4 \tag{3.11}$$

  eq. (3.1) now becomes

  $$\omega_1 \omega_4 = \omega_2 \omega_3 \tag{3.12}$$

  In the boundary theory also, there are precisely 4 gauge invariant operators of R-charge 1, namely

  $$Tr(W_1), Tr(W_2), Tr(W_3), Tr(W_4) \tag{3.13}$$

  where [23]

  $$W_1 = A_1 B_1, W_2 = A_1 B_2, W_3 = A_2 B_1, W_4 = A_2 B_2 \tag{3.14}$$

- Wavefunctions of degree 2 (corresponding to R-charge 2 in the boundary theory):

  There are 19 wavefunctions, out of which 10 are two-particle wavefunctions

  $$\psi(\vec{\omega}^{(1)}, \vec{\omega}^{(2)}) = \omega_i^{(1)} \omega_j^{(2)} + \omega_j^{(1)} \omega_i^{(2)}, \, i \le j, \, i,j = 1,2,3,4 \tag{3.15}$$

  Recall that *the dual giant gravitons are bosonic*. The remaining 9 are one-particle wavefunctions:

  $$\psi(\vec{\omega}) = \omega_i \omega_j, \, i \le j, \, i,j = 1,2,3,4 \tag{3.16}$$

  where we have used (3.12).

  Again the counting matches with the boundary theory for R-charge 2, where we have 10 double trace operators

  $$Tr(W_i)Tr(W_j), \, i \le j, \, i,j = 1,2,3,4 \tag{3.17}$$

in precise correspondence with (3.15) and 9 single-trace operators

$$Tr(W_i W_j), \ i \le j, \ i,j = 1,2,3,4 \tag{3.18}$$

in precise correspondence with (3.16). The relation (3.12) corresponds to (see [23])

$$W_1 W_4 = A_1 B_1 A_2 B_2 = A_1 B_2 A_2 B_1 = W_2 W_3 \tag{3.19}$$

The above discussion suggests the correspondence

$$\omega_i \leftrightarrow W_i \tag{3.20}$$

- Wavefunctions of degree 3 (corresponding to R-charge 3 in the boundary theory):

  Counting the same way as before, we find that there are 16 1-particle, 36 2-particle and 4 3-particle wavefunctions. Similarly in the gauge theory there are 16 single trace, 36 double trace and 4 triple trace operators.

The above pattern continues for higher R-charges as well. The main point here[3] is that the operators W_i all commute (as a consequence of F-term equations of motion) and hence gauge invariant operators written in terms of these commuting operators are in one-to-one correspondence with wavefunctions of the variables ω_i. Hence, *dual giant states are in one-to-one correspondence with gauge-invariant operators in the boundary theory.* In other words,

$$\prod_i \omega_i^{n_i} \leftrightarrow Tr\left(\prod_i W_i^{n_i}\right) \tag{3.21}$$

where the l.h.s. corresponds to all wavefunctions of the dual giant gravitons and r.h.s. corresponds to all gauge-invariant operators of the boundary theory at large N.

3.1.3 Finite N

The above correspondence was argued for at large N, where in the boundary theory we could treat traces of arbitrary high powers as independent and, correspondingly in the bulk, we could consider arbitrary number of particles. As we argued in case of $AdS_5 \times S^5$, let us assume that the maximum number of giant gravitons is given by N, the rank of the gauge group. Like in case of 1/8-BPS dual giants of $AdS_5 \times S^5$ [12], the correspondence mentioned above appears to work for finite N here as well. e.g. if we choose $N = 2$ in the degree 3 case above, we lose 4 states in the bulk and 4 operators in the boundary theory. It would be interesting to construct a general proof of the finite N correspondence for arbitrary multiparticle states and for general $Y^{p,q}$.

3.2 Higher $Y^{p,q}$'s

We will only sketch the argument here, leaving details for later. The Calabi Yau cone will again be described by some number r of relations $R_a(\omega) = 0$, $a = 1, \ldots, r$, among n complex coordinates ω_i, $i = 1, 2, \ldots, n$. The Kähler polarization is described by analytic functions

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[3]This has been emphasized to us by Shiraz Minwalla.

in the ω_i. Since the symplectic form in the phase space of the dual giant gravitons agrees with the Kähler form, by the rules of geometric quantization wavefunctions will again be given by monomials in these ω_i modulo the relations. It has been shown in [19] that in the gauge theory there are exactly n commuting adjoint operators $W_i, i = 1, 2, \ldots n$ satisfying the same r relations and the monomials $\prod_i \omega_i^{n_i}$ correspond, in a one-to-one fashion, to $Tr(\prod_i W_i^{n_i})$ (these gauge invariant operators exclude operators which can be written using the invariant epsilon tensor of SU(N)). Hence the argument used for $T^{1,1}$ will again apply, establishing a one-to-one correspondence between the dual giant states and gauge invariant operators.

4. AdS$_4 \times$ Y^7

In this section we will consider dual giant gravitons in a second setting, namely $AdS_4 \times Y^7$ backgrounds of M-theory, where we will take Y^7 to be a Sasaki-Einstein manifold. The basic steps are similar; so we will be brief.

In [27, 28] a recursive procedure is presented for constructing $2n + 3$ dimensional Sasaki-Einstein manifolds Y^{2n+3}, given a $2n + 1$ dimensional Sasaki-Einstein manifold Y^{2n+1}. Thus, the metric of Y^7 is given by (our R is the ρ of [27])

$$ds_7^2 = ds_6^2 + \left(d\psi' + \sigma\right)^2$$
$$ds_6^2 = \frac{dR^2}{u(R)} + R^2 u(R)(d\tau - A(x))^2 + R^2 ds_4^2 \tag{4.1}$$

The five-dimensional Sasaki-Einstein manifold, not explicitly written, is a circle fibration (the circle is parametrized by τ) over a 4-dimensional Kähler manifold

$$ds_4^2 = g_{ij}(x)dx^i dx^j \tag{4.2}$$

The Kähler forms J_6, J_4 for ds_6^2, ds_4^2 are given by

$$J_6 = \frac{1}{2}d\sigma, \ J_4 = \frac{1}{2}dA \tag{4.3}$$

A Ricci-flat Kähler metric can be constructed on the Calabi-Yau cone over Y^7, given by the metric

$$ds_8^2 = d\rho^2 + \rho^2 ds_7^2 \tag{4.4}$$

and a Kähler form

$$J_8 = \rho d\rho \wedge (d\psi' + \sigma) + \rho^2 J_6 \tag{4.5}$$

The AdS$_4$ metric will be given by

$$ds^2|_{AdS_4} = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2 d\Omega_2^2, \ V(r) = 1 + \frac{r^2}{\tilde{l}^2} \tag{4.6}$$

The AdS$_4$ radius of curvature $\tilde{l}$ can be determined from the special case of the AdS$_4 \times S^7$ (see eq. (3) of [2]): $\tilde{l} = l/2 = 1/2$ (we are using $l = 1$ units).

The dual giant graviton this time is an M2-brane wrapped on the 2-sphere of AdS$_4$ whose center of mass is free to move along the radial direction r and along Y^7. The Lagrangian is given by (cf. [2])

$$L = -4\tilde{N}[r^2\sqrt{\Delta} - 2r^3] \tag{4.7}$$

where Δ has an expression analogous to (2.9):

$$\Delta = V(r) - \dot{r}^2/V(r) - g_{ab}\dot{\varphi}_a\dot{\varphi}_b \tag{4.8}$$

Here $\varphi_a, a = 1, \ldots, 7$ denote the coordinates of Y^7. $\tilde{N}$ denotes the flux through the two-sphere of AdS$_4$.

As in the case of AdS$_5 \times Y^5$ we will focus on motions along a special Killing vector (see [27], eq. (4.1) and the comments immediately preceding it)

$$V = \frac{\partial}{\partial\psi'} \tag{4.9}$$

which will play the role of (2.26) in the present case.

Motion of a dual giant along this vector field implies the following condition on the eight velocities:

$$\dot{\psi}' = 1, \dot{x}^i = \dot{\tau} = \dot{R} = \dot{\rho} = 0 \tag{4.10}$$

These constraints boil down to eight conditions on the sixteen-dimensional phase space:

$$P_\rho = P_R = 0$$
$$P_{\psi'} = 2r, P_\tau = 2r(3/4 - R^2)$$
$$P_{x^i} = 2rR^2 A_i(x) \tag{4.11}$$

The reduced phase space of dimension eight can be entirely parametrized by the eight coordinates. One can find the symplectic structure of the reduced phase space again by computing the Dirac brackets. We simply quote the final result (in $l = 1$ units):

$$\omega = \tilde{N}J_8, \text{ provided } \quad \rho^2 \equiv 4r \tag{4.12}$$

The reduced hamiltonian is given by

$$H = P_{\psi'} = 2r \tag{4.13}$$

Thus we find that the conclusions of section 2.4 generalize to the present case, namely, the symplectic form for the dual giants agrees with the Kähler form for the eight-dimensional Calabi-Yau cone under the above identification (4.12) of AdS$_4$ radial coordinate with the radial coordinate ρ of the cone. Because of this, the dual giant gravitons in this special subspace of motions can be quantized again using Kähler quantization.

4.1 Supersymmetry

The relation $H = P_{\psi'}$ strongly suggests supersymmetry. However, since we are not aware of any explicit connection between $P_{\psi'}$ and an R-charge, the only way to make sure of the supersymmetry is to check the kappa-symmetry projection equation. We hope to come back to this elsewhere.

4.2 Back to more general Y^5

Note that the methods in this section do not make any specific assumption about the Sasaki-Einstein manifold in arriving at the main result (4.12). Applying a similar calculation to section 2 it should be possible to generalize the results of that of that section to a general Sasaki-Einstein space in five dimensions.

5. Conclusions

In this paper we considered BPS dual giant gravitons in the type IIB string theory on $AdS_5 \times Y^5$ where Y^5 is a five-dimensional Calabi-Yau manifold. We considered the case of $Y^{p,q}$ spaces explicitly in section 2 and indicated the more general calculation in section 4. We computed the symplectic structure of the dual giant phase space $R_+ \times Y^5$ and showed that the space becomes symplectically isomorphic to the Calabi-Yau cone (with its Kähler form identified as the symplectic form). The radial coordinate of AdS_5 gets identified with the radial coordinate of the cone.

Using the above result, the problem of quantizing BPS dual giant gravitons became that of quantizing the cone as a Kähler manifold. We worked out the case of $T^{1,1}$ explicitly and showed that the wavefunctions correspond to monomials of the embedding complex coordinates w_i which corresponded in a one-to-one fashion to gauge invariant operators. We showed, using results of [19], how to generalize this construction to $Y^{p,q}$. We discussed examples of this correspondence at finite N.

We also discussed dual giant gravitons in AdS_4 times a seven-dimensional Sasaki-Einstein manifold Y^7. Once again we found that the for a special subspace of motions, presumably supersymmetric, the symplectic structure of the dual giant configuration space $R_+ \times Y^7$ coincided with the Kähler structure of the eight-dimensional Calabi-Yau cone over Y^7.

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