Effect of charge reduction on shielding in dusty plasmas

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The shielding of dust particles by each other in a dusty plasma is studied taking into account the effects of dust charging. It is shown that the assumption of a Boltzmann response for dust species is not appropriate under these circumstances. Further, it is shown that in the close-packing limit, dust grains screen each other by charge reduction, and an expression is obtained for this shielding scale length. [DOI: 10.1063/1.1582185]

I. INTRODUCTION

Electrostatic shielding of the dust charge in dusty plasmas in the presence of electrons, ions and other dust charges is an important and interesting problem. In typical laboratory conditions, dust grains acquire a large negative charge due to the differential fluxes of electrons and ions. This large dust charge produces an electric field in the plasma. A question, which then naturally arises, is how this electric field is shielded from the rest of the plasma? In the limit of an isolated dust grain, where there is a single grain in a large volume, the electrons and ions will shield the electric field within a characteristic length scale λ_D where $\lambda_D^2 = T/4\pi e^2(n_e+n_i)^{-1}$, where $n_e(n_i)$ is the electron (ion) density, while T is the temperature of the electrons and ions. This has given rise to the notion of "Yukawa" particles which interact with each other via a shielded potential $\phi_Y = (Q^2/4\pi\epsilon_0 r)e^{-r/\lambda_D}$. This model is widely used in theoretical models and molecular dynamic simulations pertaining to dusty plasmas. However, the notion of a Yukawa particle is strictly valid in the isolated grain limit where $a_d \gg \lambda_D$ ($a_d = (3/4\pi n_d)^{1/3}$ is the Wigner–Seitz radius, n_d is the dust number density).

A case of great experimental interest is where grains are closely packed such that $a_d \leqslant \lambda_D$. In this case, the presence of other grains in the screening process must be taken into account, and the relevant question then is the following: in the limit $a_d \leqslant \lambda_D$, do grains screen each other? To account for screening due to the other grains, some authors[1–4] have extended the notion of Boltzmann's response (where the electric field balances the kinetic pressure) to moderately strongly coupled dust species with a fixed charge q_d and have thus argued that dust grains screen each other via a Debye shielding with a length scale λ_{Dd} where $\lambda_{Dd}^2 = T_d/4\pi n_d q_d^2$ (T_d is the dust kinetic temperature). This has been verified in one-dimensional particle-in-cell simulations of dusty plasma with a fixed dust charge.[3] However, the dust charge q_d is not fixed but is self-consistently determined by the local plasma conditions. In fact, in the regime $a_d \leqslant \lambda_D$ experimental observations show that the dust charge is substantially reduced from its value in the single grain limit.[5] Thus a correct theory of grain charge screening should be based on an appropriate theory of grain charging which takes into account the grain charge reduction in the limit $a_d \leqslant \lambda_D$. In this paper, we focus our attention on the effects of grain charging but do not consider its effects on strong coupling.

The mechanism of grain charging in the plasma is a complicated nonlinear problem involving the solution of Poisson's equations with electrons, ions and N grains (in different locations) and appropriate boundary conditions on grain surfaces and at infinity.[6,7] A two-dimensional numerical solution of this problem by Young *et al.*[8] shows that the grain charge depends on the grain location and the plasma potential which becomes strongly negative within the cloud in the regime $a_d \leqslant \lambda_D$, varies spatially. Fortunately in most situations of practical interest, the average dust charge q_d and the plasma potential ϕ can be obtained as a function of dust density n_d (for fixed background plasma parameters) using a simple, but elegant charging model due to Havnes *et al.*[7] This model considers cases where dust charging is due to plasma thermal fluxes alone. It does not consider effects related to secondary electron or ion or photoelectron emission. For typical laboratory conditions where electron energies are $\leqslant 1$ eV, these effects are not significant.[9,10] They are important, though, in the planetary and astrophysical environment. The prediction of this model regarding charge reduction in the close packing limit have been verified in a number of experiments.[5,11]

In this paper we analyze the problem of dust charge screening using the charging model of Havnes *et al.* and show the following.

(a) The Boltzmann response for the dust species is not appropriate because it relies on balancing the electrical force on the dust species by the dust kinetic pressure gradient. However, these forces cannot balance each other because they can be shown to lie in the same direction.

(b) Dust grains, when packed closely, shield each other via charge reduction. In other words, dust charge reduction is a consequence of the mutual screening of grain charges. The effect is important in the limit $a_d \leqslant \lambda_D$, and the scale length of this screening is λ_c where $\lambda_c^2 = T_{\rm eff}/(4\pi q_d n_d^2)$ where $T_{\rm eff}$ is the temperature associated with "electric pressure" in the dusty plasma.

II. CHARGING MODEL

We begin by considering conditions of dusty plasma assumed in the model of Havnes *et al.*[7] in which a dust cloud is assumed to be imbedded in an infinite plasma background. The plasma potential ϕ is taken to be zero at infinity. Within the cloud ϕ is usually nonzero and negative. Following Havnes *et al.* we assume the Boltzmann response for ions and electrons. In steady state, the dust charge q_d is determined by the condition that the electron thermal flux I_e is equal to the ion thermal flux I_i. Using the theory of orbit limited motion, the expressions for I_e and I_i are given by[6,9,12]

$$I_e+I_i=0, \tag{1}$$

where

$$I_e=-q(\pi a^2)(8T_e/\pi m_e)^{1/2}n_e e^{q\psi/T_e}, \tag{2}$$

$$I_i=q(\pi a^2)(8T_i/\pi m_i)^{1/2}n_i\left[1-\frac{q\psi}{T_i}\right], \tag{3}$$

and

$$n_e=n_0 e^{q\phi/T_e}, \quad n_i=n_0 e^{-q\phi/T_i}. \tag{4}$$

In these equations, a is the radius of the grain and ψ is the dust surface potential relative to the plasma potential experienced by orbiting ions in the vicinity. As stated earlier, this may not be zero within the cloud. It is related to the dust charge q_d via $q_d=\psi a$ and n_0 in Eq. (4) the plasma density at infinity where $\phi=0$ and there is no dust. These expressions are valid when the relative drift speed between the plasma and the dust is small as compared to the thermal velocities of electrons and ions. Further conditions for the validity of these equations are discussed in Refs. 6, 9 and 13. Next, we assume, *a priori*, that the dust charge is appropriately shielded in all regimes and hence, in steady state, one can always assume quasi-neutrality on scales larger than the scale length of the appropriate shielding mechanism. (This will be justified later when we show that indeed the dust charge is always shielded.) Hence,

$$q_d n_d=q(n_i-n_e). \tag{5}$$

As pointed out by Goertz *et al.*[14] for fixed background plasma parameters, i.e., T_e/T_i, m_e/m_i, n_0 and the grain radius a, Eqs. (1) and (5) constitute two equations for the dust surface potential ψ and the plasma potential ϕ as functions of n_d, i.e., $\psi=\psi(n_d)$, $\phi=\phi(n_d)$. The dust density n_d can be parametrized by the dimensionless Havnes parameter ϵZ defined as $\epsilon Z=(q_d n_d/q n_i)$. In the single grain limit, where $\epsilon Z\to 0$, for a hydrogen plasma with $T_e/T_i=1$, one obtains $\psi\simeq -2.51$, and $\phi=0$.[9,14] In the close packing limit where $\epsilon Z\to 1$, one obtains $\psi\to 0$ while $\phi\to -1.9$.[9,14] As mentioned earlier, these effects related to charge reduction as well as the negativity of ϕ in the close packing limit has been verified experimentally[5,11] and seen in simulations.[8] The reason for these effects is that in the limit $a_d \leqslant \lambda_D$, the Debye spheres of various grains overlap giving rise to a nonzero, negative plasma potential. The dust charge is reduced, because in this limit, the dust surface potential with respect to the plasma potential, which determines q_d, is reduced.

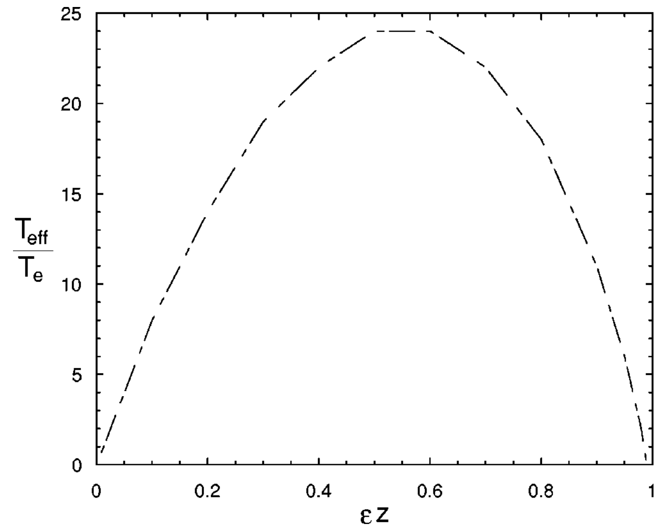

FIG. 1. Plot of $T_{\rm eff}/T_e$ vs ϵZ where $\epsilon Z=(q_d n_d/q n_i)$. Note $T_{\rm eff}/T_e>0$ in the whole range of ϵZ.

The dependence $\phi=\phi(n_d)$ is a peculiar characteristic of a dusty plasma, and has some interesting consequences. Some of these will be explored here. As pointed out by Goertz *et al.*,[14] the dependence $\phi=\phi(n_d)$ implies that the Coulomb force on the grains behaves like a "pressure" force, i.e., $F=q_d E=-q_d\nabla\phi=-T_{\rm eff}/n_d\nabla n_d$, with an effective temperature $T_{\rm eff}$ given by $T_{\rm eff}=q_d n_d\, d\phi/dn_d$ provided it can be shown that $T_{\rm eff}>0$ for all values of ϵZ. The derivative $d\phi/dn_d$ can be evaluated via Eqs. (1)–(5), and yields the expression[15]

$$\frac{T_{\rm eff}}{T_e}=\frac{q_d}{q}\frac{\Psi[(1+\alpha)-\alpha\Psi]f(\Phi)}{[\Psi\{(1+\alpha)-\alpha\Psi\}+f(\Phi)(1+\alpha)(1-\alpha\Psi)]}, \tag{6}$$

where

$$f(\Phi)=(e^{\Phi}-e^{-\alpha\Phi})/(e^{\Phi}+\alpha e^{-\alpha\Phi}), \tag{7}$$

and $\alpha=T_e/T_i$. For typical laboratory conditions, α is either greater than or equal to one. The derivative dq_d/dn_d can also be evaluated from these equations and is given by[15]

$$\frac{n_d}{q_d}\frac{dq_d}{dn_d}=-\frac{(1+\alpha)(1-\alpha\Psi)f(\Phi)}{\Psi[(1+\alpha)-\alpha\Psi]+f(\Phi)(1+\alpha)(1-\alpha\Psi)}. \tag{8}$$

Here ϕ and ψ are normalized so that $\Phi=q\phi/T_e$ and $\Psi=q\psi/T_e$. From Eq. (6), $T_{\rm eff}$ can be determined as a function of ϵZ for a given α and the mass ratio m_e/m_i. In case of grain charging due to thermal fluxes alone, Ψ, Φ and q_d are negative in the entire range of ϵZ. Hence $T_{\rm eff}\geqslant 0$ from Eq. (6), which in turn implies that the Coulomb force and the dust pressure force are in the same direction. As a result, the Boltzmann response, which relies on the balancing of the electrical force by the pressure gradient force, is not valid for the dust species. Clearly, $T_{\rm eff}$ is a measure of average electrostatic energy per dust particle. In Fig. 1, we plot $T_{\rm eff}/T_e$ vs ϵZ for a hydrogen plasma. It is zero for small ϵZ, because $\Phi\to 0$ in the limit $\epsilon Z\to 0$. For large ϵZ, $T_{\rm eff}$ is again zero because q_d is small in this range of ϵZ. $T_{\rm eff}$ attains a maximum for intermediate values of ϵZ where it is greater than the electron temperature by at least an order of magnitude. Thus, in this range of ϵZ, dust grains have maximum electrostatic energy per particle and collective effects are most important in this range. As stated earlier, this electrostatic energy behaves like "pressure," i.e., it expels dust grains from regions of high density.

III. CHARGE REDUCTION AND SHIELDING

In the second part of our paper we show that grains screen each other by charge reduction. In other words, the large negative plasma potential in the regime $a_d \ll \lambda_D$ is screened from the pristine plasma at infinity (where the plasma potential is zero) by charge reduction. To see this, consider a situation where there are a large number of dust grains in the Debye sphere, i.e., $a_d \ll \lambda_D$, so that the usual Debye screening due to electrons and ions is ineffective. Now, imagine a bunch of grains of number density $(n_d + \Delta n_d)$. Because of the density enhancement, two effects occur in the bunch. First, the plasma potential in the bunch becomes more negative with respect to the surrounding regions and a local electric field is created. Second, the dust charge in the bunch is reduced and the reduction in the space charge due to this reduction shields the negative plasma potential and the electric field of the bunch. This shielding can also be seen by considering the space charge balance in the bunch. For small changes in q_d and n_d, we write

$$\Delta \rho_d = \Delta q_d n_d + q_d \Delta n_d = [n_d (dq_d/dn_d) + q_d] \Delta n_d . \quad (9)$$

Since $dq_d/dn_d < 0$, the enhancement in the bunch space charge due to an increase in n_d is balanced or "shielded" by the reduction in bunch space charge due to reduction in q_d.

To obtain a scale length for this screening let us calculate the plasma potential $\Delta\phi$ induced because of the charge reduction. The reduction in space charge due to reduction in q_d is given by $\Delta q_d n_d = (dq_d/dn_d)(\Delta n_d) n_d$. Using the definition of T_{eff}, we may express $\Delta q_d n_d = \Delta\phi / 4\pi\lambda_c^2$, with

$$\lambda_c^2 = \frac{T_{\mathrm{eff}}}{4\pi q_d^2 n_d} \left[\frac{n_d}{q_d} \frac{dq_d}{dn_d} \right]^{-1} . \quad (10)$$

Thus the dust charge reduction induces a potential $\Delta\phi$ which varies on the scale length λ_c. To see the relation of λ_c with λ_D we consider the total space charge in the bunch and write

$$\Delta q_d n_d + q_d \Delta n_d + q \Delta n_i - q \Delta n_e = 0. \quad (11)$$

The condition for the validity of this equation will be discussed shortly. Using the definition of λ_c we obtain

$$q_d \Delta n_d = -(\Delta\phi / 4\pi\lambda_c^2) \times |(n_d/q_d) dq_d/dn_d|^{-1} . \quad (12)$$

Now in the high dust density limit where $a_d \ll \lambda_d$ or $\epsilon Z \to 1$, $\Psi \to 0$ so that from Eq. (8) $(n_d/q_d) dq_d/dn_d = -1$ in this limit. Thus $\Delta q_d n_d + q_d \Delta n_d = 0$ and the screening due to charge reduction is perfect in this limit. If this screening is not perfect, then the residual potential is screened by electrons and ions on a larger scale λ_D. Using Boltzmann's response for Δn_e and Δn_i in Eq. (11), the relationship between λ_c and λ_D can be expressed as

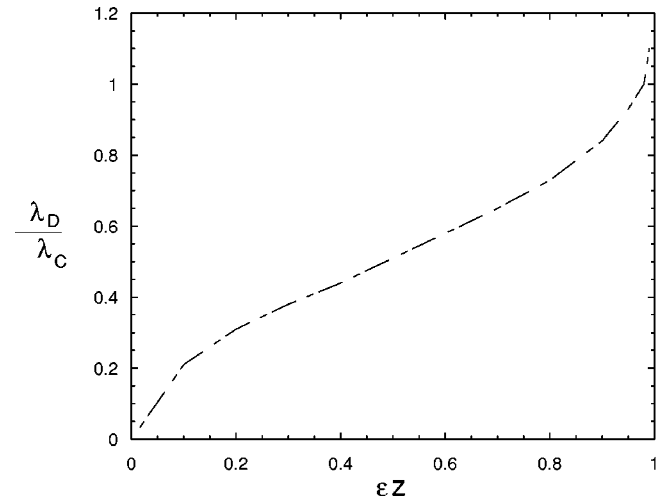

FIG. 2. Plot of λ_D/λ_c vs ϵZ where $\epsilon Z = (q_d n_d / q n_i)$.

$$\left[1 + \frac{n_d}{q_d} \frac{dq_d}{dn_d} \right] = \frac{\lambda_c^2}{\lambda_c^2 + \lambda_D^2} . \quad (13)$$

This equation clearly expresses the relationship between screening and charge reduction. In the single grain limit $dq_d/dn_d \simeq 0$, $\lambda_D \ll \lambda_c$ so that Debye screening due to electrons and ions is more important. In the opposite limit where $a_d \ll \lambda_d$ or $\epsilon Z \to 1$, $(n_d/q_d) dq_d/dn_d \to -1$ hence $\lambda_c \ll \lambda_D$ implying that screening due to charge reduction is more important in this limit. For intermediate values of ϵZ, the screening due to charge reduction strongly competes with screening due to Debye shielding. In (Fig. 2) we plot λ_D/λ_c vs ϵZ using Eqs. (1)–(6) which clearly show this effect. The effective screening length λ_T can thus be expressed as

$$\frac{1}{\lambda_T^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_D^2} . \quad (14)$$

The condition for the validity of the perturbed quasi-neutrality expressed in Eq. (11) is that the scale-length of the bunch $L \gg \lambda_T$. In low dust density limit $\lambda_T \simeq \lambda_D$ while in the opposite limit $\lambda_T \simeq \lambda_c$. Earlier, we had discussed the concept of screening in the context of acoustic modes where the dust charge is fluctuating.[15,16] Now we show that these concepts have much more general and wider applicability *quite independent of acoustic modes or any charge fluctuation effects*. It should be noted that λ_c, which is the scale of the variation of the induced plasma potential (due to charge reduction) must be present in the calculation by Whipple *et al.*[6] This is a complete electrostatic treatment of the problem. However, this calculation is numerical and too involved for a clear delineation of this scale. In the present paper we have obtained this scale by simple physical arguments. Recently, Lampe *et al.*[17] have shown that trapped ions modify the Debye screening in the vicinity of the grain. These conclusions are relevant to our discussion in the limit $a_d \gg \lambda_D$ where there is usual screening of the grain by electrons and ions. However, our main conclusion, that in the close packing limit the screening is mainly due to charge reduction, will not be affected. In this limit the screening, due to electrons and ions is weak.

IV. SUMMARY

To summarize, starting from the charging model due to Havnes *et al.* (which considers charging due to thermal fluxes) we show that the Boltzmann response for the dust is not valid. Hence dust grains do not screen each other by Debye screening when charging physics of the grains is taken into account. Further, we show that the dust grains screen each other by charge reduction. In the limit $a_d \leqslant \lambda_D$, a dust cloud develops a large negative potential which is shielded by charge reduction in the cloud over a scale length λ_c.

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