Nonlinear Interaction of Compressional Waves in a 2D Dusty Plasma Crystal

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Nonlinear mixing and harmonic generation of compressional waves were studied in a 2D Yukawa (screened Coulomb) triangular lattice. The lattice was a monolayer of highly charged polymer microspheres levitated in a plasma sheath. Two sinusoidal waves with different frequencies were excited in the lattice by pushing the particles with modulated Ar^+ laser beams. Waves at the sum and difference frequencies and harmonics were observed propagating in the lattice. Phonons interacted nonlinearly only above an excitation-power threshold due to frictional damping, as predicted by theory.

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Two-dimensional ordered lattices are found in a variety of physical systems, including Langmuir monolayers [1], electrons on the surface of liquid helium [2], rare gas atoms absorbed on graphite [3], colloidal suspensions [4], and dusty plasmas [5–8]. In the latter two examples, particles interact through a screened Coulomb repulsion or Yukawa potential.

Sound waves, or phonons, are well studied in the linear or low-amplitude limit, both theoretically and experimentally, for a 2D Yukawa lattice. There are two kinds of waves, compressional and transverse, and at high frequency they exhibit dispersion, i.e., the frequency ω is not proportional to wave number k. However, for low frequencies $\omega < 1.3\omega_0$, the propagation of linear compressional sound waves has little dispersion [4,9,10], and a speed $C_L = \omega/k$ [9]. Here, $\omega_0^2 = Q^2/4\pi\epsilon_0 m a^3$ is a 2D analog of the plasma frequency, where Q and m are the particle charge and mass, respectively, and a is the interparticle spacing.

A dusty plasma is a convenient model system to study waves in 2D lattices. Micron-size particles become highly charged when suspended in a plasma. Because of mutual repulsion and the plasma's weaker radial electric fields, they arrange themselves in a structure, called a plasma crystal, with crystalline or liquidlike order. In the presence of gravity, particles can settle in a 2D monolayer. Usually the plasma includes neutral gas, which applies a frictional drag to moving particles. The particles can be imaged directly, and their positions and velocities calculated, which allows studying the lattice microscopically.

The properties of nonlinear waves have not been studied as completely as for linear waves, in dusty plasmas. It has been shown theoretically that nonlinear pulses can take the form of solitons in weakly [11] and strongly coupled [12–14] dusty plasmas, although frictional gas damping can suppress soliton formation [13,14]. In experiments with large amplitudes, nonlinear pulses [12,15] and harmonic generation [16] have been observed in 2D lattices.

Here, we report an experimental observation of nonlinear three-wave mixing in a 2D dusty plasma crystal, and compare our results to a corresponding 1D theory. The experimental conditions for our particles were similar to those in Ref. [17], where the interparticle potential was experimentally shown to be nearly Yukawa, $U(r) = Q(4\pi\epsilon_0 r)^{-1}\exp(-r/\lambda_D)$, where λ_D is the screening length. A lattice is characterized by the screening parameter $\kappa = a/\lambda_D$. Our plasma crystal had a triangular structure with hexagonal symmetry.

The three-wave mixing problem we investigate in a 2D triangular lattice has two externally excited compressional pump waves, propagating along a common axis. They have frequencies $f_1 < f_2$, and wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively. We analyze three-wave nonlinear mixing, resulting in the generation of the waves with combination frequencies $f_{\mathrm{sum,diff}} = f_2 \pm f_1$, and second harmonic generation $f_{\mathrm{sum}} = 2f_1$ or $2f_2$.

In general, three-wave mixing requires satisfying the phase-matching conditions $\mathbf{k}_{\mathrm{sum,diff}} = \mathbf{k}_2 \pm \mathbf{k}_1$, which are equivalent to momentum conservation for phonons. Moreover, all the waves involved should satisfy the dispersion relation. For compressional waves propagating along a common axis in an infinite 2D Yukawa lattice, these conditions require that all the waves travel in the same direction and have wavelengths long enough to satisfy the wave's dispersion relation in its acoustic limit, where $\omega \propto k$.

Using the same vacuum chamber as in Ref. [16], we levitated a monolayer suspension of particles in the plasma (Fig. 1). The particles had a diameter of 8.09 ± 0.18 μm [18] and a mass density 1.514 g/cm^3. The particle suspension had a diameter of about 60 mm. The suspension was not a closed system, because the particles lost energy by frictional drag on the neutral gas. To achieve a low damping rate, we used Ar at a pressure of 5 mTorr, so that the gas drag, which is accurately modeled [18] by the Epstein expression, was only $\nu_d = 0.87$ s^{-1}. The plasma was sustained by a 13.56 MHz rf voltage with a peak-to-peak amplitude of 168 V and a self-bias of -115 V.

The particles in the suspension arranged themselves in a triangular lattice. The interparticle spacing was $a = 675 \pm 14$ μm. The lattice was in an ordered state; the pair

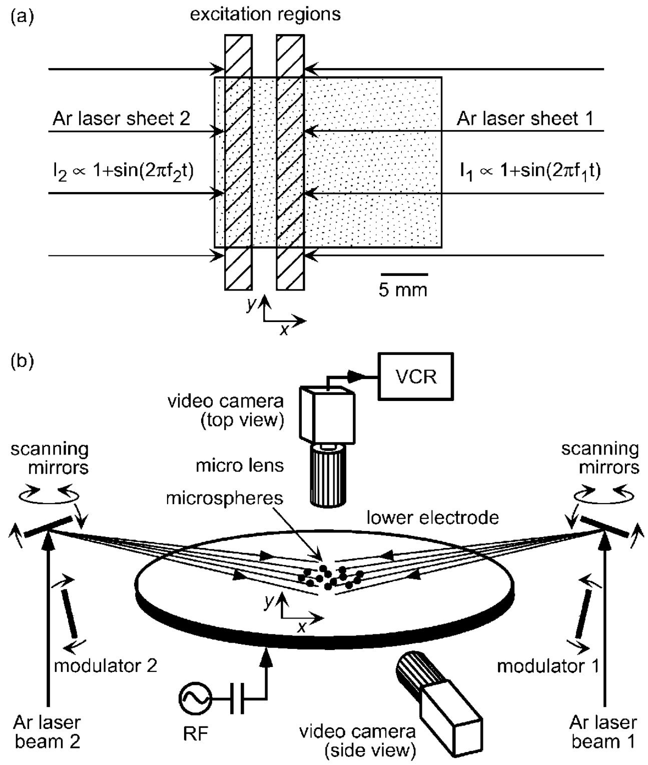

FIG. 1. Experimental apparatus. (a) Particles arranged in a triangular 2D lattice. Atop this image is a sketch showing where the radiation pressure force from two modulated Ar^+ laser sheets pushes particles, exciting sinusoidal compressional pump waves. (b) The particles are polymer microspheres, levitated as a monolayer above the lower electrode in a capacitively coupled rf plasma.

correlation function $g(r)$ had many peaks, and it had translational order length of $16a$ in an undisturbed lattice, although this diminished to $4a$ when we excited large-amplitude waves. We used a pulse technique [19] to measure $C_L = 21.6 \pm 1.5$ mm/s, $Q = -9400 \pm 900e$, and $\lambda_D = 0.73 \pm 0.10$ mm at the particles' height.

The particles were imaged through the top window by a video camera, and they were illuminated by a horizontal He-Ne laser sheet. We digitized movies of 68.3–136.7 s duration using a digital VCR at 29.97 frames per second. The 24×18 mm field of view included 1000–1100 particles [Fig. 1(a)]. Particle coordinates and velocities were then calculated in each frame using the moment method [20].

We used a laser-manipulation method [21] to excite two sinusoidal compressional pump waves with parallel wave fronts in the plasma crystal (Fig. 1). Particles were pushed by the radiation pressure force, which is proportional to an incident laser intensity [18]. An Ar^+ laser beam was split in two, and the intensity of each beam was sinusoidally modulated with a separate scanning mirror that partially blocked the laser beam. The total power of the two laser beams incident on the lattice was varied up to 3.69 W. At their foci at the particle location, the laser beams had a diameter of 0.5 mm measured at the level of $1/e^2$ of their maximum intensity, at a power of 0.61 W. Two more scanning mirrors, oscillated rapidly at 200 Hz, were used to raster each of the two laser beams into sheets, which struck the lattice at an angle of 10° with respect to the horizontal lattice. The sheets extended 4.5 mm beyond each edge of the camera's field of view, and their intensity was uniform $\pm 10\%$ in the y direction within the field of view.

The excitation regions for the two pump waves were the stripes where the two laser sheets struck the lattice [Fig. 1(a)]. The two stripes were separated by 5.5 mm in the x direction. We verified, by using a side-view video camera, that there was no out-of-plane buckling of our 2D lattice, thereby confirming that all the waves we observed were in-plane.

To analyze the wave propagation, we spatially averaged the x component of the particle velocity within 40 rectangular bins elongated along the y axis. We then computed the power spectrum $|v_x(f)|^2$ for particle motion, for each of the 40 bins in the x direction.

One of our chief experimental results is an observation of sum and difference frequencies [Fig. 2(a)]. These combination frequencies, $f_2 + f_1$ and $f_2 - f_1$, are a signature of three-wave mixing between the pump waves. Higher combination frequencies $2f_2 - f_1$, $2f_1 + f_2$, $2f_2 + f_1$, etc. are also present, and these are the result of either three-wave or four-wave mixing.

The spatial profile of the sum-frequency wave $f_2 + f_1$ in Fig. 3(a) reveals how energy is transferred from the pump waves to the wave at the sum frequency. Between the excitation regions, where the two pump waves propagate oppositely, the sum frequency is weak. Elsewhere, the two pump waves propagate in the same direction, and the sum-frequency amplitude grows with distance, as energy is extracted from the pump waves, reaches a maximum value, and then declines, due to pump depletion and damping.

A threshold is seen in Fig. 4 for $f_1 + f_2$ and $2f_1 + f_2$. The combination frequencies were present only when the excitation laser power P_{laser} was sufficient for nonlinear effect to overcome damping. While the amplitude at the pump is almost proportional to P_{laser}, the amplitude at the combination frequencies grew faster than linearly, above the threshold.

As in Ref. [16], our modulation method does not provide a single pump frequency. Second and third harmonics were present in the laser modulation, with $<1\%$ of the intensity, and this complicates the identification of mixing due to harmonic generation in the lattice. Observation of combination frequencies such as $f_2 \pm f_1$, on the other hand, is a convincing indication of mixing, because they are not present in the laser modulation. Higher combination frequencies such as $2f_1 + f_2$ are also a convincing indication of mixing, although we cannot identify whether they are the result of four-wave mixing

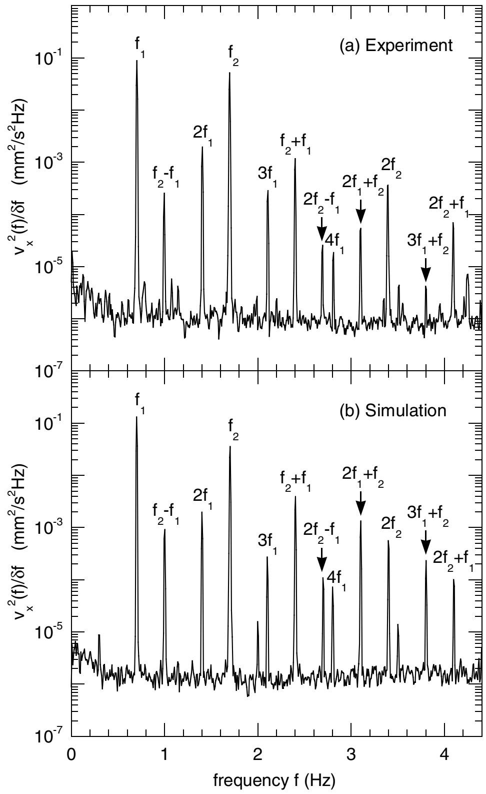

FIG. 2. Power spectrum of the particle speed $|v_x(f)|^2$ averaged over the camera's field of view, for the highest excitation laser power of 3.69 W. (a) In the experiment, sinusoidal compressional pump waves were excited in the plasma crystal with frequencies $f_1 = 0.7$ Hz and $f_2 = 1.7$ Hz. Combination frequencies and harmonics were generated at high excitation laser power, due to nonlinear mixing of the pump waves. (b) 2D MD simulation, described near the end of this Letter.

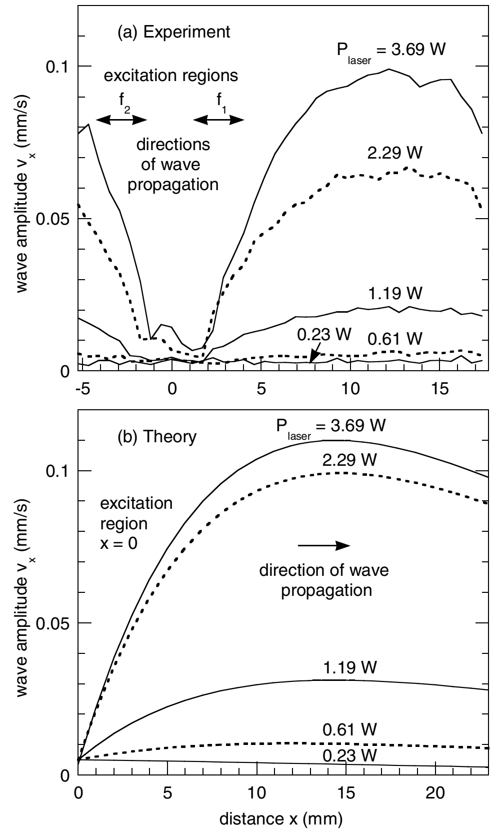

FIG. 3. Spatial profiles of the amplitude at the sum frequency within a bandwidth of 0.06 Hz. (a) In the experiment, sinusoidal compressional pump waves with frequencies $f_1 = 0.7$ Hz and $f_2 = 1.7$ Hz were excited in the plasma crystal at the locations shown by arrows, which were spatially separated. (b) Theory, described near the end of this Letter, with both pump waves excited at $x = 0$. In both the experiment and theory, amplitudes at combination frequencies grew where the pump waves propagated in the same direction.

of pump waves at f_1 and f_2 or three-wave mixing of f_2 with a harmonic $2f_1$ that was present in the laser modulation. The spatial profiles of the harmonics $2f_1$ and $2f_2$ in our experiment, not shown in Fig. 3, are similar to Fig. 4 of Ref. [16].

In a test of the phase-matching conditions, by varying the higher pump frequency f_2, we found that the amplitudes of all combination frequencies increased with f_2 up to $f_2 \approx 1.3\omega_0/2\pi$, and even beyond for some combination frequencies. Nonlinear mixing was observed for f_2 as high as $3\omega_0/2\pi$, even though the compressional waves have significant dispersion for much of this frequency range. For pure plane waves with discrete wave numbers, one would not expect mixing when there is dispersion and $\omega \propto k$ is not satisfied. However, in our experiment, the wave vectors $\mathbf{k}$ had a bandwidth in both direction and magnitude, due to the finite width of excitation regions, and the finite size of the lattice and damping, respectively. We estimate that damping by itself provides a bandwidth as large as $\Delta k = 2\nu_d/C_L$. Within these bandwidths, there are wave numbers that satisfy the phase-matching conditions.

We compare our experimental results to a theory based on the 1D chain model developed in Ref. [14]. To apply this theory to waves propagating along a common axis in a 2D lattice, we substituted the 1D chain's sound speed with its value for a 2D lattice [14].

Our model begins with a nonlinear equation of motion for an infinite chain, Eq. 15 of Ref. [14]. Accounting for the nature of our experimental system, which is not closed, this equation includes gas damping. A uniform particle spacing is assumed, with a binary interaction for four nearest neighbors, and the linear and quadratic nonlinear terms are retained after expanding the interparticle force for small displacement. Here, we omit terms that would result in dispersion. Assuming sinusoidal waves yields $\partial v_1/\partial x + \nu_d v_1/C_L = -iA\omega_1[v_2 v_{\mathrm{diff}} + v_{\mathrm{sum}} v_2^*]/2C_L^2$, describing the spatial evolution of the amplitude v_1 of the first pump wave, measured as the particle speed. Here, ν_d is the gas friction; v_2, v_{sum}, and v_{diff} are the amplitudes at ω_2, $\omega_1 + \omega_2$, and $\omega_2 - \omega_1$,

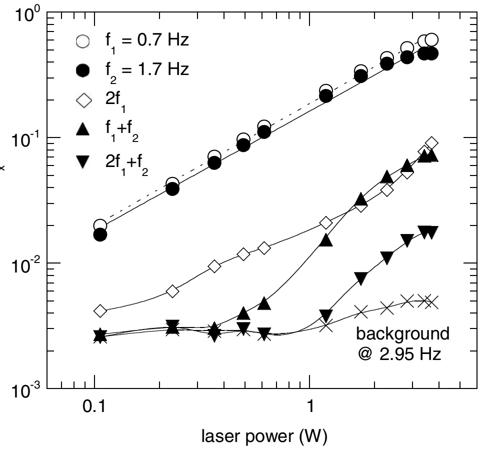

FIG. 4. Wave amplitudes, averaged over the camera's field of view, within a bandwidth of 0.06 Hz. The amplitude of the pump waves at $f_1 = 0.7$ Hz and $f_2 = 1.7$ Hz was almost proportional to $P_{\rm laser}$. In agreement with theory, sum frequencies were generated only above an excitation-power threshold depending on the sum frequency. Pump wave harmonics were present at any $P_{\rm laser}$, perhaps due to imperfect modulation of the laser's intensity. Also shown is the background amplitude at a frequency where there is no laser excitation or combination frequency.

respectively, and $A = (\kappa^3 + 3\kappa^2 + 6\kappa + 6)/(\kappa^2 + 2\kappa + 2)$ is a coefficient for the nonlinear term, for a Yukawa potential. The model also yields four similar equations for the amplitudes of the waves at $2\omega_1$, ω_2, $\omega_1 + \omega_2$, and $\omega_2 - \omega_1$. To calculate the theoretical curves in Fig. 3(b), we solved the system of five equations, yielding $v_{\rm sum}(x)$, $v_1(x)$, $v_2(x)$, $v_{\rm diff}(x)$, and $v_{2\omega_1}(x)$. We assumed both pump waves were excited at $x = 0$, unlike the experiment where the two excitation regions were displaced. We used experimental values for C_L, ν_d, A, $v_1(0)$, $v_2(0)$, $v_{\rm sum}(0)$, $v_{\rm diff}(0)$, and $v_{2\omega_1}(0)$, where the three latter values were assumed to be the thermal noise levels.

We found that this theory yields the following predictions, which are in agreement with our experiment. First, three-wave mixing can occur, but not if pump waves propagate in opposite directions. Second, the amplitudes of the combination-frequency waves grow and then decline with distance (Fig. 3). Third, due to frictional damping, the generation of combination frequencies requires that the pump amplitude must exceed a certain threshold. Fourth, amplitudes at the combination frequencies increase with the pump amplitude and frequency, as long as the dispersion is weak.

We performed a 2D MD simulation using parameters similar to those of the experiment. Unlike the theory, which is a solution of a wave equation including a nonlinear term, this simulation is a solution of the equation of motion for 5000 particles, including a binary interparticle Yukawa repulsion, which is the only source of nonlinearity. Particles are confined by an external harmonic potential. The simulation retains features of the experiment including nonuniform particle spacing, finite-size effects, the laser intensity profile, and lattice defects. The laser force was chosen 9% smaller than calculated in Ref. [18], to match the amplitudes of the pump waves in the experiment.

Simulation results for the power spectrum $|v_x(f)|^2$ in Fig. 2(b) are similar to the experimental results in Fig. 2(a), suggesting that our simulation incorporated the essential physics of our experiment. However, the amplitudes of the peaks in Figs. 2(a) and 2(b) do not all agree. While the amplitudes at f_1, f_2, $2f_1$, $2f_2$, $3f_1$, and $3f_2$ are similar, the amplitudes at the combination frequencies are smaller in the experiment than in the simulation. We cannot fully account for this difference, but one possibility is that the actual interparticle potential is not exactly Yukawa.

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