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Gravitational equilibrium and the mass limit for dust clouds

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Abstract. We show the existence of a new class of astrophysical objects where the self-gravity of the dust is balanced by the force arising from shielded electric fields on the charged dust. The problem of equilibrium dust clouds is formulated in terms of an equation of hydrostatic force balance together with an equation of state. Because of the dust charge reduction at high dust density, the adiabatic index reduces from two to zero. This gives rise to a mass limit M_{AS} for the maximum dust mass that can be supported against gravitational collapse by these fields. If the total mass M_D of the dust in the interstellar cloud exceeds M_{AS} , the dust collapses, while in the case $M_D < M_{AS}$, equilibrium may be achieved. The physics of the mass limit is similar to the Chandrasekhar's mass limit for compact objects, such as white dwarfs and neutron stars.

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1. Introduction

The gravitational collapse of dust in interstellar clouds plays an important role in the formation of stars and other planetary bodies [1, 2]. In the cloud the interstellar radiation fields and other mechanisms ionize the gas and hence the dust grains acquire a net non-negligible electric charge [3, 4]. The gravitational collapse of the charged dust in the astrophysical context has been investigated by a number of authors [5]–[11]. One of the main conclusions from these studies is that the threshold for the Jeans instability is set up by the dust acoustic waves [12] rather than by the neutral sound waves. Recently, the importance of gravitational-like instabilities in dusty plasmas has been recognized in processes which are responsible for star and planet formation [13].

As is well known, one of the major problems encountered in the study of the gravitational collapse of matter is the formation of a legitimate equilibrium. Since the dust kinetic pressure is too weak, it cannot balance the self-gravity of the dust on astrophysically relevant scales. One way to resolve this problem is to ignore the equilibrium altogether and consider the local stability of perturbations much smaller than the global scale. This study is of limited scope, as it does not address the stability of global eigenmodes which are expected to play a major role in the collapse. The other alternative, frequently used hitherto, is to balance the self-gravity of the dust by charge separation electric fields in the background plasma. Since the dimensions of the cloud are typically much larger than the plasma Debye radius λ_d , the plasma–dust system will be quasi-neutral and hence the charge separation electric fields are expected to be weak.

In this paper, we discuss the possibility of a new way of balancing the self-gravity of the dust. Specifically, we show that the self-gravity of the dust can be balanced by the pressure P_E arising from shielded electric fields in the background plasma. This pressure arises essentially due to an inhomogeneous distribution of the dust in the background plasma. Such inhomogeneities are bound to occur during the collapse of the dust in a cloud. The importance of this effect lies in the fact that the dust equilibrium constructed by balancing the self-gravity with P_E are of astrophysically relevant length and mass scales.

We present an equation of state for P_E , which relates it to the dust number density n_d , i.e. $P_E = P_E(n_d)$. Equilibrium configurations are obtained by balancing P_E with the self-gravity of the dust. Specifically, we construct one-parameter family of spherically symmetric and static dust configurations. Furthermore, we show that as the dust number density increases during the collapse, P_E also increases. However, due to the dust charge reduction, which occurs at high dust density [14]–[17], the increase in the pressure is not sufficient to balance the self-gravity. This results in an upper limit M_{AS} on the total mass M_D of the dust in the cloud for the equilibrium. For $M_D > M_{AS}$, the dust-cloud collapses, while for $M_D < M_{AS}$ an equilibrium may be obtained. The physics of this mass limit is very similar to the Chandrasekhar’s mass limit for compact objects, such as white dwarfs and neutron stars [18]–[20]. In the dust equilibria near the mass limit, the dust is ‘compactly’ packed at high densities. These dust clouds, which are a new class of astrophysical objects where gravity is supported by shielded electric fields, may exist inside small (<0.1 pc) interstellar clouds.

2. The model

The simple model we study consists of a dust cloud of finite extent imbedded in background electron–ion plasma of much larger extent. Our model is governed by

$$0 = -qn_i \nabla \varphi - \nabla p_i, \quad (1)$$

$$0 = qn_e \nabla \varphi - \nabla p_e, \quad (2)$$

$$\rho_d \frac{dv_d}{dt} = -q_d n_d \nabla \varphi - \rho_d \nabla \Phi, \quad (3)$$

$$\nabla^2 \Phi = 4\pi G \rho_d \quad (4)$$

and

$$qn_i + q_d n_d - qn_e = 0, \quad (5)$$

where $n_e(n_i)$ are the electron (ion) densities, $p_e(p_i)$ are the electron (ion) pressure, $n_d(\rho_d)$ is the dust number (mass) density and $\varphi(\Phi)$ are the electro-static (gravitational) potential. On the slow timescale of the collapse, we neglect the inertia of electrons and ions in equations (1) and (2). We have also neglected the dust kinetic pressure in equation (3) and assumed quasi-neutrality in equation (5). This is appropriate for the dust cloud of dimensions much larger than the plasma Debye radius. The first term due to the electric field in the right-hand side of equation (3) is finite and arises due to an inhomogeneous distribution of charged dust grains. We will show shortly that this force can be expressed as the gradient of P_E and can balance the self-gravity given by the second term in the right-hand side of equation (3). The dust charge q_d in equation (3) is not constant but is a function of n_d [14, 15]. It decreases with the increase in n_d . It is important to take into account this effect as the dust charge reduction at high dust density encountered during the collapse may be substantial. The equilibrium dust charge q_d is given by

$$I_e + I_i + I_{PH} = 0, \quad (6)$$

where $I_e(I_i)$ are the electron (ion) thermal flux impinging on the dust grain surface, and I_{PH} is the flux of photoelectrons [11]. These charging currents are given by

$$I_e = -q\pi a^2 \left(\frac{8T}{\pi m_e} \right)^{1/2} n_e e^{\frac{q\psi}{T}}, \quad (7)$$

$$I_i = q\pi a^2 \left(\frac{8T}{\pi m_i} \right)^{1/2} n_i \left(1 - \frac{q\psi}{T} \right), \quad (8)$$

$$I_{PH} = q\pi a^2 \Gamma_{PH}, \quad (9)$$

where a is the grain radius, T is the background plasma temperature, $m_e(m_i)$ are the electron (ion) mass, ψ is the dust surface potential relative to the local plasma potential φ , and Γ_{PH} is the flux of photoelectrons per unit area from the dust surface. If the ions and electrons are thermalized within the interstellar cloud, which is a reasonable assumption [15], then n_i and n_e are given by the Boltzmann relation: $n_i = n_0 \exp(-q\varphi/T)$ and $n_e = n_0 \exp(q\varphi/T)$. The local plasma potential φ is taken to be zero at infinity, where $n_d = 0$ and $n_i = n_e = n_0$. However, within the cloud, φ may not be zero and an electric field arises due to an inhomogeneous dust distribution. As shown by Havnes *et al* [15], equations (7)–(9) together with (1), (2) and (5) can be combined to obtain two equations which can be solved to relate φ and ψ as functions of n_d (for given n_0 , a and T) i.e. $\varphi = \varphi(n_d)$ and $\psi = \psi(n_d)$. Thus, the Coulomb force on the dust behaves like pressure [16, 17]. The dust number density n_d or the dust mass density ρ_d is parametrized by a dimensionless number $p = aTn_d/q^2n_0$. Due to lighter mass of electrons, the dust charge q_d , ψ and the plasma potential φ are negative in the presence of thermal fluxes. By solving the aforementioned equations, Havnes *et al* [21] have obtained simple polynomial approximations to φ and ψ as a function of p , given by

$$\bar{\psi} = -\frac{q\psi}{T} = \frac{a_0 + a_1p}{1 + b_1p + b_2p^2}, \quad \bar{\varphi} = -\frac{q\varphi}{T} = \frac{c_1p + c_2p^2}{1 + d_1p + d_2p^2}, \quad (10)$$

where constants a_j , b_j , c_j and d_j depend on the species of the interstellar gas and I_{PH} . These constants are tabulated in [21]. In the isolated dust grain limit $p \rightarrow 0$, $\bar{\varphi} \rightarrow 0$ and $\bar{\psi} \rightarrow 2.5$ for H ions, and $I_{PH} = 0$. As p increases, $\bar{\psi}$ and q_d approach zero, while $\bar{\varphi}$ increases monotonically and saturates, i.e. $\bar{\varphi} \rightarrow c_2/d_2 = 1.87$. The dust charge reduction at high dust density is due to the mutual screening of the grains [22], and has been verified in laboratory experiments and numerical simulations [23, 24].

3. Equilibrium configurations

We begin our calculations by defining the pressure P_E due to the shielded electric fields. Since $q_d n_d$ is a function of φ , we may rewrite (3) as

$$\rho_d \frac{dv_d}{dt} = -\nabla P_E - \rho_d \nabla \Phi, \quad (11)$$

where P_E is an effective pressure defined by $P_E = \int q_d n_d d\varphi$. The latter, which expels charged dust particles from regions of high density, may balance the self-gravity of the dust in the cloud

to provide equilibrium, i.e.

$$\nabla P_E = -\rho_d \nabla \Phi. \quad (12)$$

This is the equation of force balance for the dust. In this paper, we construct equilibria where dust is static i.e. $v_d = 0$. Eliminating $q_d n_d$ through the quasi-neutrality condition (5) and imposing the condition $P_E = 0$ at $\bar{\varphi} = 0$ (the pressure is purely due to the electrostatic potential), we obtain

$$P_E = 2n_0 T (\cosh \bar{\varphi} - 1). \quad (13)$$

Here, P_E is a function of n_d via $\bar{\varphi}$ in equation (10) (for given n_0 , a and T). Thus, equation (13) together with (10) constitutes an equation of state for our problem. The other equation between P_E and n_d is obtained by taking the divergence of equation (12) (after dividing by ρ_d) and using equation (4) to eliminate Φ . For a particular species of an interstellar gas, these two equations can be solved with appropriate boundary conditions (mentioned later) to obtain the spatial profiles of n_d and P_E in the equilibrium configuration.

4. The mass limit

In this section, we show that there is an upper limit on the total mass of the dust in the cloud for the equilibrium and point out that the physics of this limit is similar to Chandrasekhar's mass limit of compact objects [18]–[20]. In order to see the mass limit we note from equations (10) and (13) that P_E increases with n_d . Now, in the low dust density limit $p \ll 1$, we have $\bar{\varphi} \rightarrow c_1 p \ll 1$, $P_E \approx n_0 T \bar{\varphi}^2$, and thus $P_E \propto p^2$, i.e. P_E increases with the increase in n_d , with an adiabatic index $\Gamma = 2$. On the other hand, in the high-density limit $p \rightarrow \infty$, $\bar{\varphi} \rightarrow c_2/d_2$, P_E saturates and the adiabatic index Γ goes to zero. This reduction in Γ is responsible for the limit on the total dust mass M_D in the cloud for the equilibrium. Basically, if M_D is large then the gravity compresses grains to high densities and de-charges (substantial charge reduction) them before the pressure P_E has had a chance to stop the gravity. Subsequent collapse then takes place as if the dust is uncharged. On the other hand, if M_D is small, the charge reduction is not substantial and P_E may balance the gravity. Because of the reduction of the adiabatic index for the pressure from two to zero due to the charge reduction, the pressure is not able to resist the self-gravity as the density increases, resulting in the mass limit. This scenario is very similar to the physics of Chandrasekhar's mass limit [18]–[20]. In the latter, the adiabatic index for the hydrostatic pressure reduces from 5/3 to 4/3 due to relativistic effects, resulting in the mass limit for white dwarfs. In the next section, we construct equilibrium solutions and calculate the mass limit.

5. Solutions

The problem of the equilibrium of dust clouds may be formulated in terms of a fluid with the force balance $\nabla P_E = -\rho_d \nabla \Phi$, together with an equation of state. We look for spherically symmetric and static ($v_d = 0$) solutions. Taking the divergence of equation (12) (after dividing by ρ_d) and using Poisson's equation (4), the force balance equation in spherical coordinates can be

written as

$$\frac{1}{r^2} \frac{d}{dr} r^2 \frac{dP_E}{p dr} = -2c_1^2 p, \quad (14)$$

where we have normalized pressure P_E by $n_0 T$, the dust density by p , and the distance r by the scale length L , given by

$$L = \sqrt{2c_1^2 a^2 T^3 / 4\pi G m_d^2 q^4 n_0}. \quad (15)$$

This parameter gives a typical dimension of the dust cloud in terms of other parameters. As stated earlier, equation (14) is supplemented with the equation of state given by

$$P_E = 2(\cosh \bar{\varphi} - 1), \quad \bar{\varphi} = \frac{c_1 p + c_2 p^2}{1 + d_1 p + d_2 p^2}. \quad (16)$$

Equations (14) and (16) may be solved to obtain $P(r)$ and $p(r)$ for a given species of interstellar gas and a value of I_{PH} . The boundary conditions at $r = 0$ are

$$p = p_c, \quad \frac{dp}{dr} = 0, \quad \text{at } r = 0, \quad (17)$$

where p_c is the normalized central dust density in the cloud. Using these boundary conditions, we construct one-parameter family of spherically symmetric and static solutions characterized by p_c . The normalized radius R of the configuration is given by the condition $p(R) = 0$ and the total mass M_D of the dust cloud is given by

$$M_D = (L^3 m_d / \lambda_d^2 a) I, \quad \text{where } I = \int_0^R p r^2 dr \quad (18)$$

and $\lambda_d = (T/4\pi n_0 q^2)^{1/2}$ is the plasma Debye radius. The integral I , which is a dimensionless number of order unity, should be evaluated by solving (14) and (16). Thus, as in the theory of stellar configurations, there is a definite mass–radius relationship for these configurations [18]. We next discuss the low-density limit.

5.1. Low dust density limit

In this limit $p \ll 1$, $\varphi < 1$ and $P = (c_1 p)^2$ with $\Gamma = 2$. Thus, equation (14) is linear in p and is given by

$$\frac{1}{r^2} \frac{d}{dr} r^2 \frac{dp}{dr} = -p. \quad (19)$$

The solution of (19), with the boundary conditions given in equation (17), is

$$p = p_c \sin r/r, \quad I = p_c \pi \quad \text{and} \quad R = \pi. \quad (20)$$

These tenuous dust equilibria are characterized by a radius, which is independent of p_c while $M_D \propto p_c$ (ρ_c is the central dust mass density). For arbitrary p_c , equations (14) and (16) may be solved numerically to obtain the value of the integral I and the equilibrium solution.

5.2. The mass limit

Next, we discuss the mass limit M_{AS} . To evaluate this limit, we consider the case $I_{PH} = 0$, where P_E is strongest. In this case, all charges are of the same sign (which is negative) and maximum for given values of n_0 , a and T . From the tables given in [21], we choose the values of the constants and d_j corresponding to $I_{PH} = 0$ and H ions, i.e. $c_1 = -1.26$, $c_2 = -0.21$, $d_1 = 1.04$ and $d_2 = 0.112$. The equations are solved by choosing an initial value of $p_c = 10^{-11}$ (corresponding to a typical value in the interstellar cloud), which is increased up to hundred by increasing n_d , but keeping n_0 , a and T constant. For small values of $p_c (< 0.01)$, I increases linearly with p_c as in this limit $M_D \propto \rho_c$. For large values of $p_c (> 0.01)$, we find that I saturates with a broad maximum $I_{\max} = 0.85$ at $p_c = 2$. The value of R is constant and is equal to π for $p_c < 0.01$. In the range $0.01 < p_c < 2$ it decreases from π to 2.27. After this it decreases very slowly for equilibria on the other side of the maximum. In dust clouds with $p_c \leq 2$, dust is compactly packed at high density, the dust charge is small and electrons are depleted; they are absorbed by the dust.

From the values of I and p_c obtained above, we may plot a universal equilibrium curve in terms of the normalized variables as well as a specific curve in physical units of M_D and ρ_c . Substituting the maximum value of I , and expressing the constants L and λ_d in terms G , n_H , T and a , we obtain the mass limit M_{AS} and L as

$$M_{AS} = 0.85 \frac{c_1^3 T^{7/2}}{G^{3/2} q^4 n_H^{1/2}} a^{-4}, \quad L = 0.033 \frac{c_1 T^{3/2}}{G^{1/2} q^2 n_H^{1/2}} a^{-2}, \quad (21)$$

which are the main results of our paper. The important scalings of M_{AS} and L against different parameters are evident in equation (21). Specifically, we note that the strong dependence on a is important, as any error in a will severely affect the values of M_{AS} and L .

To plot the equilibrium curve in terms of M_D and ρ_c , we choose the parameters which characterize a standard interstellar cloud in the HII region [4]: namely $n_0 = n_H = 10^7 \text{ m}^{-3}$ and $T = 5 \times 10^3 \text{ K}$. For the dust size we take $a = 3 \times 10^{-7} \text{ m}$. The equilibrium curve is plotted in figure 1. For these parameters, $L = 5 \times 10^9 \text{ m}$ and in figure 1 there is broad maximum at $M_D \approx 8 \times 10^{18} \text{ kg}$. Thus, the value of the mass limit for the parameters of HII cloud is $M_{AS} \approx 8 \times 10^{18} \text{ kg}$. In the dust equilibria near the mass limit, which are roughly of the mass $M_D \leq M_{AS} \approx 8 \times 10^{18} \text{ kg}$ and sizes $\leq L = 5 \times 10^9 \text{ m}$, the dust is compactly packed with a central density $n_{dc} \sim 2 \times 10^5 \text{ m}^{-3}$. This is about eleven orders of magnitude higher than the average interstellar grain density $\sim 10^{-6} \text{ m}^{-3}$. At the centre of these clouds, $p \approx 1$ and $\bar{\varphi} \approx 0.7$. Thus, in the vicinity of the centre of such clouds the ion density is roughly two times and the electron density is half of its value outside the cloud which is $n_0 = 10^7 \text{ m}^{-3}$. These compact dust objects are roughly of the size of the Sun and contain dust mass equal to that of a small satellite. However, as it is seen from equation (21), these parameters are sensitive functions of the dust size a and the plasma temperature T . Any small error in these parameters will severely affect the values of M_D , M_{AS} and L . For instance, if we increase the dust size three times, i.e. $a = 10^{-6} \text{ m}$, then the mass of the equilibrium increases by four orders of magnitude and $M_{AS} \approx 10^{22} \text{ kg}$. This is roughly equal to the mass of a small planet like Pluto.

The effect of Coulomb correlations defined by the parameter $\Gamma_c = q_d^2 / d T_d$ (d is the mean inter-particle distance and T_d is the kinetic temperature associated with the random motion of the dust grain) may become significant in the compact dust configurations. As a function of p , Γ_c has a maximum at $p = 0.2$. Depending on whether we take T_d to be equal to the gas temperature

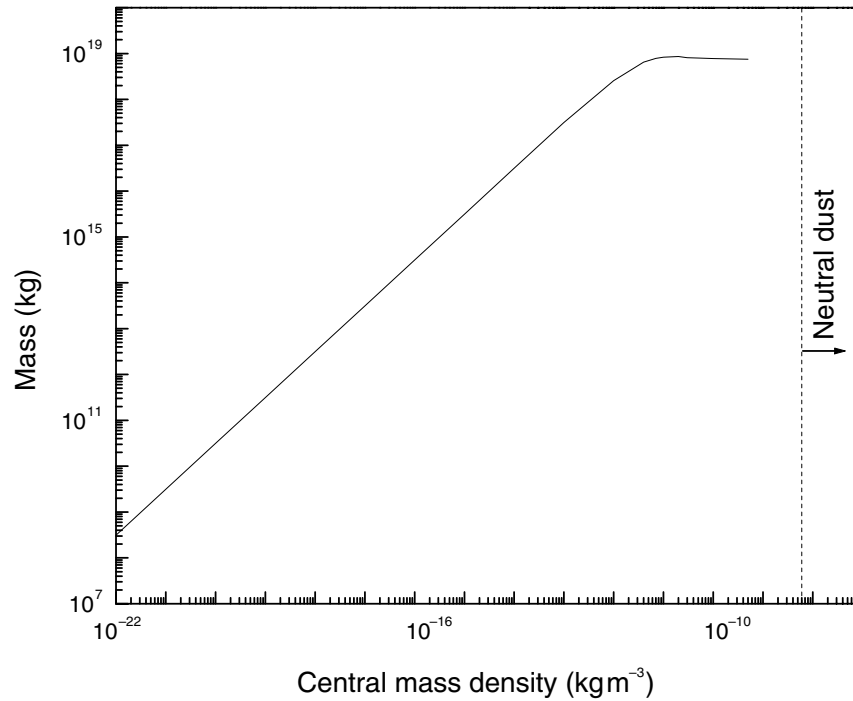


Figure 1. The equilibrium curve M_d versus ρ_c , displaying the mass limit M_{AS} for a constant dust grain size. For small ρ_c , $M_D \propto \rho_c$. The value of M_d decreases very slowly on the other side of the maximum; hence the latter is barely visible. For $\rho_c > 10^{-9} \text{ kg m}^{-3}$, dust becomes neutral as the dust charge is reduced to a value less than one electronic charge.

or lower, $\Gamma_c^{\text{max}} \sim 5$ or higher. Thus, a significant portion of compact dust equilibrium (with p_c in the range 0.1–2) may be in a strongly correlated Coulomb liquid or even a crystalline state.

The overall dimensions of the dust cloud ranges from 0.01 to 50 pc [4]. The dust in these clouds is typically silicate or polycyclic aromatic hydrocarbon (PAH) distributed inhomogeneously. Now, if as a conservative estimate, we take the average interstellar dust mass density $\sim 3 \times 10^{-28} \text{ kg m}^{-3}$ [4] then in a small-sized (~ 0.1 pc) dust cloud the total dust mass is $M_D \sim 3 \times 10^{19} \text{ kg}$, which is roughly equal to the mass limit $M_{AS} \approx 8 \times 10^{18} \text{ kg}$. Hence, in small interstellar clouds with dimensions < 0.1 pc the electric fields are sufficiently strong to hold the dust against gravity in a stable equilibrium, while in bigger clouds with dimensions > 0.1 pc the dust will collapse. Of course these numbers and conclusions depend sensitively on the values of the dust radius and T , and the fact that the whole of the dust cloud is characterized by the HII region parameters given above. These numbers may also be affected by a finite I_{PH} , which is a function of the interstellar UV radiation field and the work function of the grain material.

What are the astrophysical implications of these considerations? This requires further investigation. However, one possibility is that since the compact dust equilibria are fluffy aggregates of a ‘Coulombic matter’ that is held together by Coulomb correlations, these could be a precursor to a proto-planetary or proto-stellar core formation. For example, if the electric fields become weak over a period of time then these aggregates will slowly contract and become denser to give rise to van der Waals correlations and the formation of a more solid body.

It should be noted that the steady-state dust cloud with a constant dust charge requires constant recycling of the background plasma at the grain surface. This in turn requires a balance between the absorption of plasma and re-ionization of subsequently produced neutral H by the UV radiation in the cloud. In the equilibria near the mass limit, the dense core is usually much smaller than the tenuous envelope. Hence, the optical depth in the core $\tau = n\sigma z$ (where σ is the cross-section and z is roughly the radius of the core) is expected to be small so that UV radiation intensity is enough to ionize the neutral hydrogen. However, this point requires further careful consideration which will be taken in a subsequent publication.

6. Conclusions

We have obtained the equilibrium configurations of dust clouds imbedded within plasma background by balancing the pressure P_E due to shielded electric fields with the self-gravity of the dust. The pressure P_E arises due to an inhomogeneous distribution of charged dust grains within the cloud, and tries to expel dust from the regions of high dust density. Furthermore, we have derived an equation of state which relates P_E and the dust number density n_d . The problem of equilibrium can be posed in terms of an equation for the force balance together with an equation of state. Due to the charge reduction at high dust number density, the adiabatic index is shown to reduce from two to zero. This results in an upper limit for the total mass of the dust in the dust cloud equilibrium. The physics of this mass limit is similar to Chandrasekhar's mass limit. This mass limit is numerically evaluated for the HII region parameters. It is found to depend critically on the dust radius and the gas temperature. Based on this limit it is argued that in clouds smaller than 0.1 pc, dust may be held in equilibrium by shielded electric fields, while in bigger clouds it will collapse. An implication of this scenario to planet formation is briefly discussed. In our model, we have ignored a number of effects, e.g. dust rotations, ambient magnetic fields, dust size distribution, etc. These will be covered in a subsequent paper.

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