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Dynamics of self-gravitating dust clouds and the formation of planetesimals

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Abstract

Due to the gravitational force, clouds of dust and gas in the interstellar medium can contract and form stars and planet systems. Here we show that if the dust grains are electrically charged then the self-gravitation can be balanced by the "electrostatic pressure" and the collapse can be halted. In this case, the dust cloud may form soft dust planets, having the weight of a small moon or satellite, but a radius larger than of our Sun. There exist a critical mass beyond which the dust cloud collapses and forms a solid planet. We here present a simple model for the dynamics and equilibrium of self-gravitating dust clouds and apply the model to typical parameters for dust in molecular clouds and in the interstellar medium.

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It has become more and more clear that dust plays a central role in the formation of stars and planets [1–5] and that dust is abundant both in planetary rings in our Solar system [6–8] and in the Earth's atmosphere [9], as well as in laboratory and processing plasmas [10,11]. The standard model for the formation of a solar system is that a cloud of gas and dust collapses to form a central star. Due to the angular momentum in the material, some of the material forms a proto-planetary disk of dust and gas which spins around the star. Dust and ice particles grow by agglomeration [12] or by colliding and sticking to each other. Eventually, larger objects are formed which starts attracting material by their gravitational forces. Observations of the young (20 Myr) star β Pictoris reveal that it is surrounded by a dust disk which shows features of ~ 10 μm-sized crystalline silica and olivine grains near the star, as well as of sub-μm dust grains in bands at a distance of 6, 16 and 30 AU from the star [1]. It is believed that most of this material has been generated from comets and planetesimals after that the original proto-planetary disk was formed [13]. This might be an early stage of a Solar system.

We here present a scenario which could lead to the formation of planets directly from interstellar dust without the initial formation of a central star. This could happen if the dust cloud has a mass of a satellite or of a small planet, i.e., much less than the mass of a star. If the dust particles are immersed in an ionized gas, they will be charged electrically—typically the electric charge is negative due to the attachment of electrons onto the grain surface [14]. Various turbulent processes in the dilute interstellar medium [15] can produce density fluctuations in the dust. The self-gravity in the dust then leads to an instability [16–18] where the dust contracts into separate dust clouds in space. When the negatively charged dust grains become densely enough packed in the dust cloud, a large part of the electrons are absorbed by the dust grains and there will be an overweight of free positively charged ions compared to free electrons. A negative potential is then set up in the cloud that balances the ion pressure and prevents the ions from escaping the dust cloud. The Coulomb force on the dust due to this potential behaves like an effective pressure force, i.e., it expels particles from the regions of high density. Hence it balances the gravitational force to halt the collapse of the dust cloud. In Fig. 1 we illustrate the geometry of a dust cloud where the gravitational force acting on

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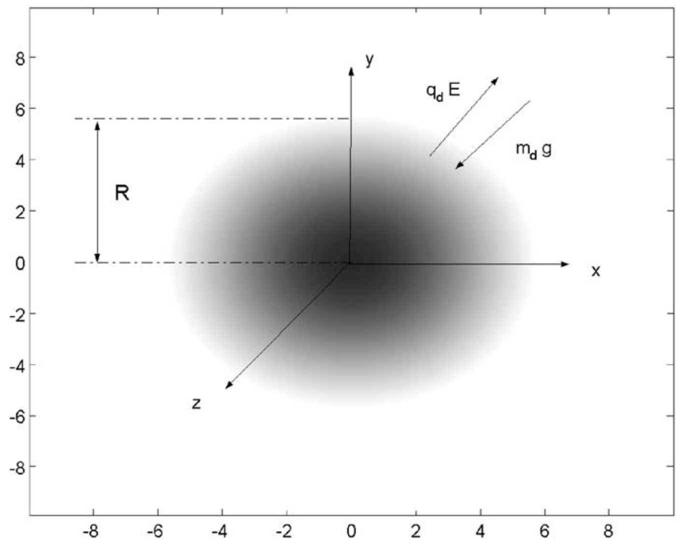

Fig. 1. The geometry of a spherically symmetric dust cloud with radius R. At equilibrium, the attractive gravity force $m_d g$ acting on a dust grain is balanced by the repulsive electric force $q_d E$. Here, m_d and q_d are the mass and electric charge, respectively, of the dust grain, while g and E are the gravity and the electric field, respectively.

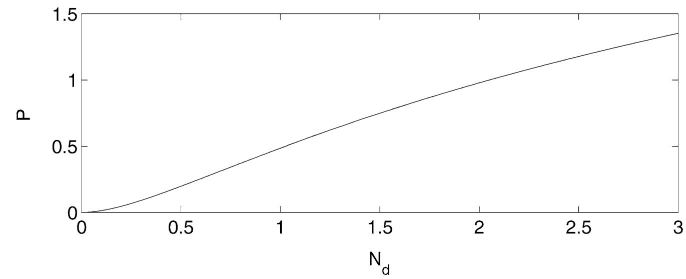

Fig. 2. The electrostatic pressure P as a function of the dust number density N_d. At low densities, the pressure depends on density as $P = c_1^2 N_d^\gamma$ with the effective "heat ratio" $\gamma = 2$, while for large dust densities, γ decreases.

the dust grains is balanced by the electric force. Based on this physics, dust cloud equilibria have recently been constructed [19]. Here we concentrate on the time dependent dynamics and the stability of the dust cloud. It should be noted that these equilibria are strictly quasi-neutral and hence are different from the earlier work where gravitation was balanced by the electric field arising due to charge separation [16–18].

In a fluid description, the dynamics of a spherically symmetric dust cloud can be described by the dimensionless continuity and momentum equations

$$\frac{\partial N_d}{\partial t} + \frac{1}{r^2}\frac{\partial (r^2 N_d v_r)}{\partial r} = 0, \tag{1}$$

and

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} = -\frac{1}{N_d}\frac{\partial P}{\partial r} - \frac{\partial \psi}{\partial r}, \tag{2}$$

respectively. Here N_d is the dust particle density, v_r is the radial velocity of the dust particles, and $r = \sqrt{x^2 + y^2 + z^2}$ is the radial coordinate. The spatial variables x, y, z and r have been normalized by $L = (a^2 T^3 / 4\pi G m_d^2 n_0 e^4)^{1/2}$ where the gravitation constant is $G = 6.67 \times 10^{-8}\ \mathrm{cm^3\,s^{-2}\,g^{-1}}$, the time t by $(aT/4\pi G m_d n_0 e^2)^{1/2}$, the velocity v_r by $\sqrt{aT^2/m_d e^2}$, the dust particle density N_d by $n_0 e^2/aT$, Z by Ta/e^2, the pressure-like term P by $n_0 T/m_d$, the electric potential φ by T/e, the gravitational potential ψ by $aT^2/m_d e^2$, and the mass M by $M' = 4\pi m_d n_0 e^2 L^3/aT$. The gravitational potential ψ is given by Poisson's equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) = N_d, \tag{3}$$

where the right-hand side represents the mass density of the dust cloud which acts as a source for the gravitational potential. The term P in Eq. (2) which we will denote the *electrostatic pressure* arises from the electrostatic potential in the dust cloud which balances the ion pressure. It is derived in the following manner. In the large-scale dust cloud, there will be a quasi-neutral equilibrium where the charge density of the dust will balance almost exactly the ones of the ions an electrons. This can be expressed as $N_i = Z_d N_d + N_e$ where the subscripts "i" and "e" denotes ions and electrons, respectively. The coefficient Z_d represents the number of electrons attached to the surface of the dust grain; it will vary with the density of the dust grains, as discussed below. If the ions and electrons are in thermal equilibrium with each other, their densities can be assumed to obey a Boltzmann distribution, viz. $N_e = \exp(\varphi)$ and $N_i = \exp(-\varphi)$, where φ is the normalized electrostatic potential. We thus have $Z_d N_d = \exp(-\varphi) - \exp(\varphi) = -2\sinh(\varphi)$. The electric force acting on the dust fluid is related to the electrostatic pressure P as

$$F = Z_d N_d \frac{\partial \varphi}{\partial r} = -2\sinh(\varphi)\frac{\partial \varphi}{\partial r} \equiv -\frac{\partial P}{\partial r}, \tag{4}$$

where $P = 2[\cosh(\varphi) - 1]$ enters into the first term of the right-hand side of Eq. (2). The electrostatic potential φ is related directly to the background density N_d through the quasi-neutrality condition mentioned above and the condition that the dust grain should be in charge equilibrium with its surrounding, so that the net electric current to the dust grain vanishes. By balancing the ion and electron currents, $I_i + I_e = 0$, one can obtain a condition which relates the dust charge Z_d to the surface potential of the dust grain and to the potential of the plasma surrounding the dust grain. Approximate formulas have been derived by Havnes [20], which relate the electrostatic potential φ to the dust number density via the rational function $\varphi = (c_1 N_d + c_2 N_d^2)/(1 + d_1 N_d + d_2 N_d^2)$. Numerical values on the constants are tabulated in Table 1 of Ref. [20] for a few cases; here it is assumed that the plasma consists of electrons and protons, and that dust charging by photoelectric effects can be ignored, so that $c_1 = -1.26$, $c_2 = -0.21$, $d_1 = 1.04$ and $d_2 = 0.112$. We note that the relation between P and N_d (via the relation between φ and N_d) constitutes an equation of state in a similar manner as in thermodynamics. In Fig. 2, we have plotted P as a function of N_d. For low dust particle densities, $N_d \ll 1$, the electrostatic pressure depends on the dust density as $P = c_1^2 N_d^\gamma$ with the heat ratio $\gamma = 2$, while for large dust densities, the pressure will increase more slowly with increasing dust densities. In the language of thermodynamics, the heat ratio γ will decrease from 2 to zero, and this will set a limit M_{AS}

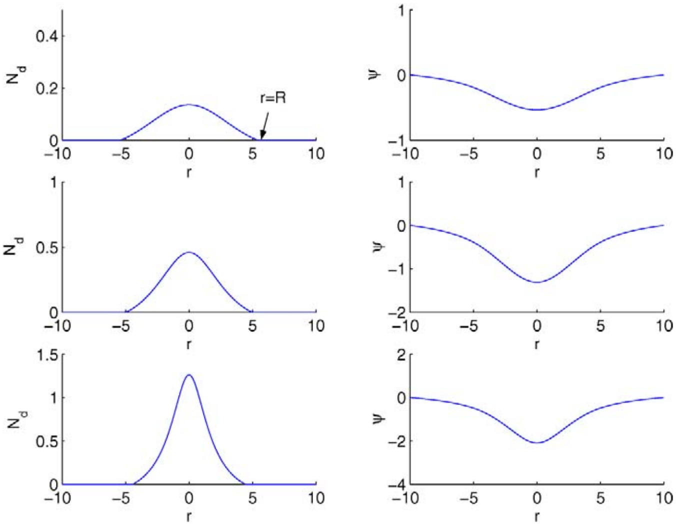

Fig. 3. The dust particle number density (left panels) and gravity potential (right panels) for the total cloud mass $M = 1.95$ (upper panels), $M = 3.9$ (middle panels) and $M = 4.90$ (lower panels).

on the total mass of the dust cloud above which the electrostatic pressure cannot balance the gravitational force [19]. The reason for the instability of the dust clouds with large masses is that the dust grains become less electrically charged when they are densely packed. An estimate for the charge state is given by [20] $Z = (a_0 + a_1 N_d)/(1 + b_1 N_d + b_2 N_d^2)$ with $a_0 = 2.5$, $a_1 = 0.764$, $b_1 = 1.09$ and $b2 = 0.12$, which is a decreasing function of N_d. When the charge of the dust grains decreases, the electric force acting on the dust grains weakens and can no longer balance the gravitational force, with the result that the dust cloud collapses and forms a small planet with a hard surface.

An expression for the equilibrium ($\partial/\partial t = 0$ and $v_r = 0$) inside the dust cloud is obtained by setting the left-hand side of Eq. (2) to zero, and taking the divergence of its right-hand side. The result, after reordering of terms, is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{N_d}\frac{\partial P}{\partial r}\right) = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \psi}{\partial r}\right) = -N_d, \tag{5}$$

where the last equality comes from Eq. (3). In the limit of low density ($N_d \ll 1$), we have the approximate equation of state $P = c_1^2 N_d^2$ and one can show that the exact solution of Eq. (5) for this case is $N_d(r) = N_c c_1\sqrt{2}\sin(r/c_1\sqrt{2})/r$, where N_c is the peak value of N_d at the center of the dust cloud. From this solution we see that the density goes to zero where $r/c_1\sqrt{2} = \pi$, and we thus have the radius of the cloud $R = c_1\pi\sqrt{2} \approx 5.6$.

For large densities, the equilibrium solutions are obtained numerically. Fig. 3 displays the distributions of the dust density (left panels) and the gravitational potential (right panels) for different masses M of the dust cloud, where the total dust mass is obtained as the integral of the dust density over the volume of the dust cloud, $M = \int_0^R N_d r^2\, dr$. Outside the dust cloud ($|r| > R$) the gravitational potential has the exact solution $\psi = -M/|r| + \text{const}$. The dust density is largest in the center of the dust cloud, where the gravitational potential has its minimum. At a radius $r = R \approx 5$ we see that the dust density falls to zero (indicated in the upper left panel). We observe from Fig. 3 that the density in the central core of the dust cloud becomes more peaked for larger masses, while the radius of the dust cloud remains almost constant, $R \approx 5$, and we can conclude that the typical radius of the dust cloud can be estimated by its linear value for $R = c_1\pi\sqrt{2}$. The mass $M = 4.9$ is close to a maximum value, which we will denote the *critical mass* of the dust cloud, above which we could not find equilibrium solutions. In Ref. [19], the time independent equations were solved and the limiting value of the normalized mass was computed to be 0.85. To convert this into the mass limit obtained in this Letter we must multiply 0.85 with $(\sqrt{2}c_1)^3$ on account of different length normalization. For $c_1 = 1.26$ we have $0.85(\sqrt{2}c_1)^3 = 4.8$. This confirms the numerically evaluated mass limit (≈ 4.9) in the present Letter.

Let us discuss a few scenarios with parameters typical for a molecular cloud and the interstellar medium. The radius and mass of the dust grains are taken to be $a = 3.0 \times 10^{-5}$ cm and $m_d = 2 \times 10^{-13}$ g, respectively. In our model, we have for simplicity assumed spherical dust grains that are equally sized and have the same mass—in reality observations have revealed that the size distribution obeys a power law in the sub-μm range [21] and that they are irregularly shaped [22]. Typical parameter values for a molecular cloud is [23–25] $n_0 = 10^{-3}$ cm^{-3} and $T = 100$ K, giving a typical length scale $L = 5 \times 10^{10}$ cm, time scale $t' = 10^{11}$ s (≈ 3000 years) and mass scale $M' = 10^{17}$ g. For the interstellar medium, typical values are $n_0 = 0.1$ cm^{-3} and $T = 10^4$ K, giving the length scale $L = 5 \times 10^{12}$ cm, time scale $t' = 10^{11}$ s and mass scale $M' = 10^{23}$ g. For the parameters relevant to a molecular cloud, $R = 5$ corresponds to a radius of $\approx 2.5 \times 10^{11}$ cm (≈ 3.5 solar radii), while for the parameters relevant to a interstellar gas, $R = 5$ corresponds to a radius of $\approx 2.3 \times 10^{13}$ cm (≈ 1.6 AU). The masses of the dense dust clouds range from $\sim 10^{17}$ to $\sim 10^{23}$ g, which is comparable with a satellite or a small planet.

The time-dependent dynamics for a dust cloud of total mass $M = 1.95$ is illustrated in Fig. 4. For this case, the dust cloud exhibits damped pulsations, and after some time it reaches a stable equilibrium. The pulsations have a periodicity in time of $T_p \approx 33$ corresponding to $\approx 10^5$ years for the parameters of the interstellar medium and molecular cloud. The collapse of a dust cloud illustrated in Fig. 5, where we have taken the mass $M = 4.9$, which is slightly below its critical mass. Here, the dust cloud starts contracting slowly. At $t \approx 9$, the core of the dust cloud starts collapsing and we see a rapid increase of the dust density in the center of the cloud. Due to the inertia of the contracting dust cloud, it collapses even though its mass is slightly below the critical mass. Our scenario for the formation of planetesimals is that the interstellar grains first undergo a gravitational instability [16,18]. This instability saturates to form tenuous, stable dust clouds of the type constructed here with masses below the critical mass M_{AS}. As more and more charged dust falls into the dust clouds, some of them reach their mass limits where they collapse and form planets with hard surfaces. We can thus see the dust clouds as prestates of planetesimals or satellites.

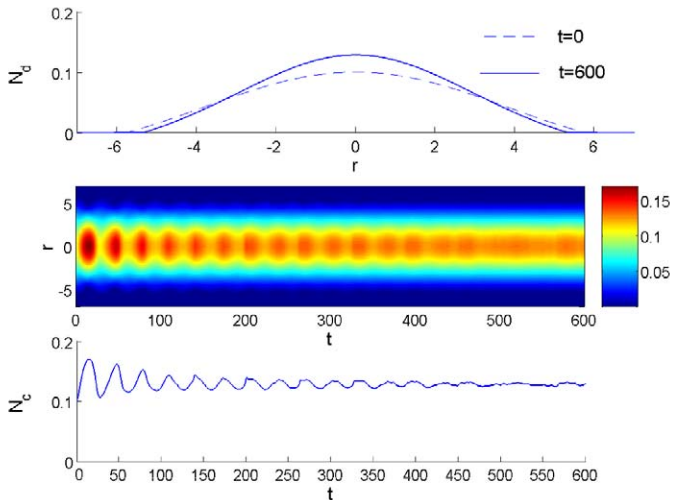

Fig. 4. The time-dependent dynamics of a dust cloud, showing the initial and final particle density distribution of the dust cloud (upper panel), the radial dust particle density distribution N_d as a function of time (middle panel) and the value of the particle distribution at the center of the cloud, $N_c = N_{d,r=0}$, as a function of time (lower panel). The initial condition was taken to be $N_d = 0.181\sin(r/1.81)/r$, giving the total mass $M = 1.95$.

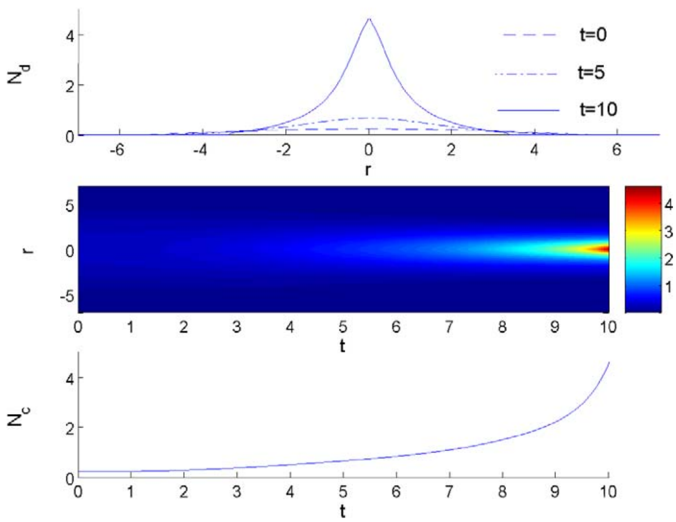

Fig. 5. The collapse of a dust cloud. The radial density distribution of the dust cloud at different times (upper panel), the radial dust particle density distribution N_d as a function of time (middle panel), and the value of the distribution at the center of the cloud, $N_c = N_{d,r=0}$, as a function of time (lower panel). The initial condition was taken to be $N_d = 0.475\sin(r/1.81)/r$, giving the total mass $M = 4.90$.

In summary, we have here presented a study of the dynamics of self-gravitating astrophysical dust clouds. For low-mass dust clouds, the attractive gravitational force is balanced by the repulsive electric force inside the dust cloud and a stable equilibrium can be reached. However, the dust clouds have a critical mass limit above which they collapse and form planetesimals with hard surfaces. The physics of this mass-limit is similar to Chandrasekhar's mass-limit for black-holes and white dwarfs [26]. Typical masses of the dust clouds are comparable with the ones of satellites or small planets, while the sizes can be larger than our Sun. Even though our model is simplified (we have assumed only spherical symmetry, neglected that the dust has a size distribution, etc.) it constitutes a first step towards understanding the dynamics and collapse of self-gravitating dust clouds where the electrostatic force is as important as the gravitational force.

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