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Research Paper

Stirling Like Engine Using Plasma Electric Fields

K AVINASH* and S CHAUDHARY
Department of Physics and Astrophysics, University of Delhi, Delhi 110 007, India

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Conversion of heat into mechanical work using electrostatic fields associated with charged particles in electron-ion plasma is discussed in the present article. Thermodynamic closed cycle, involving isochoric and isothermal processes similar to that in classical Stirling cycle, is constructed. The cycle shows that expansion and compression of charged particles under electrostatic pressure when the plasma is heated and cooled produces net work in cycle at the cost of plasma thermal energy. The efficiency of the cycle is calculated in the case where the electrostatic fields are of arbitrary magnitude. The possibility of practical power generation using this cycle is also shown in the article.

Key Words: Stirling Engine; Plasma Engine; Electrostatic Fields; Dusty Plasmas; Charged Particle Engine Cycle

Introduction

It is well known that the heat energy of gases can be converted into mechanical work and *vice-versa* by expansion/compression of hot gases caused by their kinetic pressure $P = nT$ where n and T are the density and temperature of the gas respectively. This paradigm has given rise to many devices of practical use e.g., heat engines, heat and cooling pumps, refrigerators etc.

Gases when heated above a sufficiently high critical temperature go into "plasma" state, where individual atoms are ionized or dissociated into negatively charged electrons and positively charged ions. In the plasma state the number densities of electrons and ions are almost equal in number i.e., $n_e = n_i = n$ (n_e, n_i are electron and ion densities), hence there are no large scale electric fields or potential plasma. In fact hot plasma behaves almost like an ideal gas with kinetic pressure $P = (n_iT + n_eT) = 2nT$ where we have assumed that ions and electrons have equal temperature (temperature is expressed in energy units). The gas also breaks down and results in a plasma state when a high voltage passes through it such as in fluorescent tubes and other high voltage discharge etc. Recent experiments (Thomas and Morfill 1996; Chu and Lin 1994; Melzer *et al*., 1994; Nosenko *et al*., 2004 and Barkan *et al*., 1994) have shown that when dust of micron sized particles of some material like aluminum or graphite etc are injected in a hot plasma, then these dust particles, which initially are electrically neutral, acquire a net negative charge which can be as high as 10^3 to 10^4 times the charge on an electron. Such plasma which contain electrons, ions and particles of micron/submicron sized dust are often called "Dusty plasma". The electrostatic (ES) repulsion between these negatively charged micron sized particles behaves like pressure called ES pressure P_E. It has been shown recently that plasma thermal energy can be converted into mechanical work and vice versa via ES pressure P_E (Avinash, 2010). Specifically, it was shown that the adiabatic/isothermal compression and expansion of these charged particles against P_E converts plasma thermal energy into work consistent with the laws of thermodynamics. A Stirling cycle (Organ, 1997)

**Author for Correspondence: E-mail: ak0005@uah.edu*

involving dusty plasma with hot plasma and micron sized particles was constructed and its efficiency was calculated (Avinash, 2010). The main practical advantage of plasma engine, if and when made, is its light weight. It involves plasma as fuel, which is much rarer and hence much lighter than gasoline and hence will be useful in space applications where weight is a severe constraint.

In earlier calculations (Avinash, 2010) the efficiency of the Stirling like cycle using plasmas was calculated using the approximation that the electrostatic potential φ due to charged particles is much smaller than the plasma temperature T i.e., $\varphi << T/q$ where q is the electronic charge. This approximation is not always true; especially if the concentration of micron sized particles in the plasma is high. In this paper we generalize our calculation of the efficiency of the cycle. Specifically we calculate the general expression of efficiency which is valid for all values of electrostatic potential and particle density.

The paper is organized as follows. In Sec II we describe, in some detail, "dusty plasmas" and the origin of ES pressure P_E. We also give an equation of state for P_E. In Sec III we describe the first law of thermodynamics involving P_E. In Sec IV we describe the plasma Stirling cycle and calculate its efficiency.

Model and Formulation of the Problem

Our model consists of hot plasma consisting electrons carrying a charge – q and singly charged ions of charge + q contained in a volume V in thermal equilibrium. Let number density and temperature of electrons and ions be n_e, T and n_i, T respectively. Within this plasma there are N_d micron sized charged dust particles each carrying a negative charge $-q_d$. Let the number density of these dust particles be denoted by n_d. Because of the negative charge the dust particles will repel each other. Let these mutually repelling dust particles be confined within a mesh of volume V_d within the plasma ($V_d < V$). The holes of the mesh are small enough not to allow the dust particles pass through and hence they are confined within the mesh. The electrons and ions however can freely go in and out of the meshed volume V_d. It should be noted that if one considers two negatively dust charged particles in vacuum where there are no other charges, then as is well known they repel each other via Coulomb potential, which at point r is given by $\psi = -q_d/r$. If, however, these charges are within the plasma then the potential is screened by electrons and ions surrounding the dust particles and in this case the potential is given by $\psi = -q_d \exp(-r/\lambda_d)/r$ where λ_d is the Debye length (expression given later). The condition of charge neutrality in the volume outside V_d where there are no dust particles is given by

$$qn_i = qn_e = qn_0 \tag{1}$$

where n_0 is the electron and ion densities without dust particles. And within the volume V_d where there are negatively charged dust particles, in addition to electrons and ions, this condition is given by

$$qn_i - qn_e - q_d n_d = 0 \tag{2}$$

Then, because of screening, electrostatic potential ψ is zero outside V_d and non-zero inside. Because plasma is in thermal equilibrium, the electron and ion densities are given by Boltzmann's relations $n_e = n_0 e^{-q\psi/T}$, $n_i = n_0 e^{q\psi/T}$ (temperature will be expressed in the units of eV in this paper; 1eV = 1.6×10^{-19} J). Hence the condition of charge neutrality in Eq. (2) is given by

$$q_d n_d = q_d \frac{N_d}{V_d} = qn_0[\exp(q\psi/T) - \exp(-q\psi/T)] \tag{3}$$

where we have used $n_d = N_d/V_d$. Next we define the ES pressure P_E. As stated earlier, negatively charged dust particles repel each other hence on the walls of the mesh a pressure will be exerted by the dust particles. This pressure, which arises due to electrostatic repulsion, is called the ES pressure P_E and can be calculated as follows. The total electrostatic force density on dust particles within V_d is

$$F = q_d n_d E = -q_d n_d \nabla\psi = -\nabla\int q_d n_d d\psi = -\nabla P_E \tag{4}$$

This equation clearly shows that the force density can be expressed as negative of the gradient of scalar P_E. This scalar P_E has the characteristics of pressure and since it arises due to ES energy it is called ES pressure. Substituting Eq. (3) in (4) and performing the integral gives

$$P_E = 2n_0 T\left[\cosh\left(\frac{q\psi}{T}\right) - 1\right] \tag{5}$$

where ψ is given by Eq. (3). Hence for given plasma density n_0 and plasma temperature T, the ES pressure P_E can be calculated in terms of dust density n_d or volume V_d (for given N_d), via Eq. (3) and (5). In fact Eqs. (3) and (5) constitute equation of state for $P_E = P_E(V_d)$. The thermal energy of the dusty plasma is given by $U = (3/2)NkT$ where $N = N_e + N_i$. Because of shielding, the contributions of electrostatic energy to U are negligible while the thermal energy of dust particles is zero as temperature of dust particles is assumed to be zero i.e., $T_d \approx 0$. This assumption is justified as dust in thermal equilibrium with back ground neutral gas which is at room temperature $T_d \approx T_n \approx 300$ K = 0.027eV. This is much smaller than the plasma temperature which is in the range T = 10-20 eV. It should be noted that the P_E is controlled by the plasma temperature T (dust temperature is taken to be zero). Hence by controlling the heat input to the plasma given by Q we can manipulate the ES pressure P_E by the dust particles on the mesh. Now, if under the action of P_E the volume V_d changes by amount δV_d then work done is $P_E \delta V_d$. Further, if during this process, the heat input/output to the plasma is δQ and change in the plasma thermal energy is δU, then the energy conservation requires

$$\delta Q = \delta U + P_E \delta V_d \tag{6}$$

where P_E as a function of V_d is defined by the equation of state given by Eqs. (3) and (5). This equation clearly shows that plasma thermal energy may be converted into mechanical work or vice versa via the ES pressure P_E exerted by the dust particles on the walls of the bounding mesh. We will demonstrate this by constructing closed engine cycles $P_E - V_d$ space using Eqs. (6) along with (3) and (5).

Engine Cycles

The simplest engine cycle which can be constructed using Eqs. (3), (5) and (6) is a Stirling like cycle. Like classical Stirling cycle this cycle consists of isochoric heat input and exhaust coupled with isothermal expansion and compression. We first construct this cycle and calculate its efficiency in the limit of small dust particle density where $\frac{q\psi}{T} << 1$.

Plasma Engine Cycle in Weak ES Potential Limit

In the limit $\frac{q\psi}{T} << 1$, the exponentials in Eq. (3) and the Cosh function in Eq. (5) can be expanded in the small argument limit and combined to give following simple equation of state for P_E

$$P_E = \frac{q_d^2 N_d^2}{2q^2 n_0} \frac{T}{V_d^2} \tag{7}$$

This equation of state shows that for constant plasma temperature T, $P_E \propto V_d^{-2}$. This is the isothermal equation of state for ES pressure P_E. It is interesting to compare this with isothermal equation of state for usual kinetic pressure $P \propto V^{-1}$. Clearly at constant temperature, ES pressure falls much more rapidly than the kinetic pressure with volume. In Fig. 1 we show a schematic of closed cycle involving isochoric and isothermal processes. Dust particles expand when the plasma is heated and compressed when plasma is cooled producing net work in the cycle. This is described as follows. Let dust particles be initially confined in a volume V_{d1} having temperature T_1.

(a) Along AB

The plasma is heated at constant volume raising the plasma temperature from T_1 to T_2 and the ES pressure of dust particles increases from P_{E1} to P_{E2}. If C_V is the specific heat of the plasma at constant volume, then the amount of heat added to the plasma along AB is

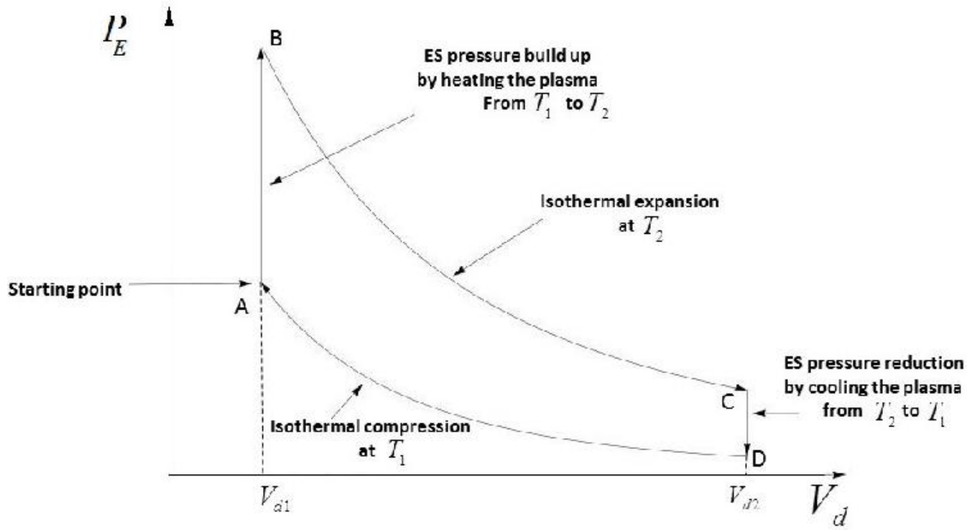

Fig. 1: Stirling-like cycle based on ES pressure in the plasma

$$\Delta Q_{AB} = C_V (T_2 - T_1) \tag{9}$$

(b) Along BC

Under the action of the ES pressure $P_{E2,}$ the dust particles are allowed to expand from the initial volume V_{d1} to V_{d2} at constant plasma temperature T_2 and mechanical work W_{BC} is done by dust. The internal energy U of the plasma remains constant. The amount of the heat added to the plasma equals the work done W_{BC} given by

$$\Delta Q_{BC} = W_{BC} = \int_{Vd1}^{Vd2} P_E dV_d = \frac{q_d^2 N_d^2 T_2}{2q^2 n}\left(\frac{1}{V_{d1}} - \frac{1}{V_{d2}}\right) \tag{10}$$

(c) Along CD

The plasma is cooled from T_2 to T_1 at constant volume and the ES pressure of dust particles decreases from P_{E2} to P_{E1}. The amount of heat removed from the plasma is

$$\Delta Q_{CD} = \Delta Q_{AB} = C_V (T_2 - T_1) \tag{11}$$

(d) Along DA

Under the action of P_{E1} dust particles are compressed isothermally at constant plasma temperature T_1 from volume V_{d2} to V_{d1}. The heat removed from the plasma along DA and work done on dust particles is given by

$$\Delta Q_{DA} = W_{DA} = -\int_{Vd1}^{Vd2} P_E dV_d = -\frac{q_d^2 N_d^2 T_1}{2q^2 n}\left(\frac{1}{V_{d1}} - \frac{1}{V_{d2}}\right) \tag{12}$$

The net heat input in the cycle is $\Delta Q_{AB} + \Delta Q_{BC}$ while the net work done is $W_{BC} - W_{DA}$. Hence the efficiency of the cycle which is the ratio of net work done to net heat input is given by

$$\eta = \frac{W_{BC} - W_{DA}}{\Delta Q_{BC} + \Delta Q_{AB}} \tag{13}$$

Next we consider an efficient heat regenerator (Organ, 1997) which ensures that the heat which is used to raise the plasma temperature from T_1 to T_2 along AB can be taken from the heat recovered while cooling the plasma from T_2 to T_1 along CD. In this case the heat input is just ΔQ_{BC} and the efficiency of the this cycle approaches that of the ideal Carnot cycle i.e., $\eta = 1 - (T_1/T_2)$.

This result is consistent with the fact that with efficient heat regenerator the efficiency of the Stirling cycle approaches that of ideal Carnot engine (Organ, 1997). This shows that it is possible to convert plasma thermal energy into mechanical work via ES pressure generated by the presence of the dust particles. Next we generalize our calculation of efficiency to arbitrary dust density and arbitrary ES potential limit.

(B) Plasma Engine Cycle in Arbitrary ES Potential Limit

From Eq. (5) it can be seen that in the limit $\frac{q\psi}{T} > 1$ the ES pressure P_E can be comparable to or even larger than the plasma kinetic pressure $P = 2n_0T$. Hence for same volume compression ratio the ES pressure P_E will do more work than ordinary kinetic pressure of gases/plasmas. The general expression for the pressure is given by Eq. (5) where P_E is a function of ψ which in turn is a function of V_d via Eq. (3) which may be expressed as

$$q_d \frac{N_d}{2V_d} = qn_0 \sinh\left(\frac{q\psi}{T}\right) \tag{14}$$

Hence the general expression for the work done for arbitrary ES potential ψ is

$$W = \int_{Vd1}^{Vd2} P_d dV_d = 2n_0 T \int_{Vd1}^{Vd2} [\cosh\{\sinh^{-1}\left(\frac{q_d N_d}{2qn_0 V_d}\right)\} - 1] dV_d \tag{15}$$

It can be seen that the integral is a function of volume V_d only. Applying this formula for work done in the cycle described earlier gives

$$W_{BC} = 2n_0 T_2 \int_{V_{d1}}^{V_{d2}} \left[\cosh\left\{\sinh^{-1}\left(\frac{q_d N_d}{2qn_0 V_d}\right)\right\} - 1\right] dV_d = \Delta Q_{BC} \tag{16}$$

$$W_{DA} = 2n_0 T_1 \int_{V_{d2}}^{V_{d1}} \left[\cosh\left\{\sinh^{-1}\left(\frac{q_d N_d}{2qn_0 V_d}\right)\right\} - 1\right] dV_d = \Delta Q_{DA} \tag{17}$$

Since the volume integral involved in path BC and DA as shown in Eq. (16) and (17) are same the efficiency of the cycle with efficient heat regenerator is again

$$\eta = \frac{W_{BC} - W_{DA}}{\Delta Q_{BC}} = 1 - \frac{T_1}{T_2} \tag{18}$$

This completes our general calculation of the efficiency of the plasma Stirling cycle for arbitrary plasma potential. In the case $\frac{q\psi}{kT} > 1$, $\sinh(q\psi/T) \approx \cosh(q\psi/T) \approx \exp(q\psi/T)/2$. In this case the expressions for P_E and the work done W are

$$P_E = \frac{q_d T N_d}{q V_d} \tag{19}$$

$$W = \int P_E dV_d = \frac{q_d}{q} T N_d \ln \frac{V_{d1}}{V_{d2}} \tag{20}$$

4. Power Generation

In this section the possibility of practical power generation using Plasma Stirling Engine for some typical plasma and dust parameters which are possible within present day plasma technology is shown.

In Fig. 2 a possible layout of an engine based on plasma Stirling cycle discussed earlier is shown. The hot plasma is contained in the big chamber of volume V. Within the plasma there is metallic cylinder. One end of the cylinder is closed by fixed mesh with holes which are small enough not to let dust particles pass through. The plasma electrons and ions however can freely pass through mesh. The other end of the cylinder has similar mesh which is movable within the cylinder like a piston and is connected to an external load. The plasma thus is contained in entire volume V; inside as well as outside the cylinder. The dust is confined in the cylinder in the volume V_d between the movable and fixed mesh as shown in the Fig. 2. Let the dust be confined in initial volume V_{d1}. The presence of dust creates electrostatic field and the electrostatic pressure inside in the cylinder which is responsible for conversion of heat into work.

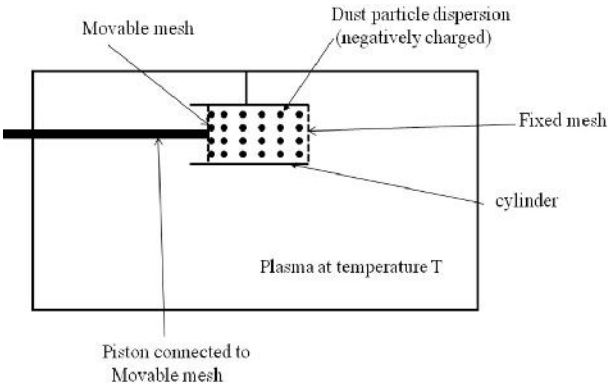

Fig. 2: A possible layout of plasma Stirling like engine

The cycle is started by quickly raising the plasma temperature from T_1 to T_2 at fixed volume V_{d1}. Due to this input of heat, the ES pressure P_E on the dust rises. As a result the dust expands from V_{d1} to V_{d2} at T_2 and work is done by the engine. Later the plasma is cooled from T_2 to T_1 and the dust is compressed at T_1 to complete the cycle. As shown earlier expansion of dust at higher plasma temperature and compression at lower temperature produces net work in the cycle. To show the possibility of practical power generation, following typical parameters of plasma and dust are considered: $n = 10^{22}$ m^{-3}, $T_1 = 10$ eV, $T_2 = 20$ eV, diameter of dust particle = 0.2 µm, $n_d = 10^{20}$ m^{-3}, V_{d1} = 100 cc, V_{d2} = 1000 cc. In this case, the charge on a dust particle as compared to that on an electron is $q_d / q \approx 10^3$ and $\frac{q\psi}{T} \approx 3$. Due to the presence of charged dust the electrostatic potential inside the cylinder will be 30 V. This is the case of high ES potential and Eq. (19) and (20) will be used for calculating the ES pressure and work done. For these parameters, the kinetic pressure of the plasma nT = 16 k Pa while P_E = 160 k Pa. $W_{BC} \approx 40$ J and $\eta' = 0.5$, where we have assumed efficient heat regenerator (Organ, 1997) and Carnot efficiency given by Eq. (18). Hence the net work produced in a cycle is 20 J. If this cycle is repeated at 50 Hz, which roughly is the RPM of modern IC engine, the power produced by the plasma Stirling engine per cylinder is 2 kW.

It should be noted that in the calculation given above various parameters are not optimized. Higher power can be produced by carefully choosing parameters e.g., plasma temperature, density, dust size, dust number density, volume V_{d1}, etc. The main aim of the calculation given here was just to show the possibility of practical power generation.

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