

Some theoretical issues in heavy flavour physics

AMOL DIGHE

Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Mumbai 400 005, India
E-mail: amol@theory.tifr.res.in

Abstract. Some of the recent developments in heavy flavour physics will be reviewed. This will include an update on some of the Standard Model predictions, and a summary of recent measurements that may indicate the presence of new physics (NP). The focus will be on selected models of NP that are indicated by the anomalies in the current data. Observables that can potentially yield signatures of specific physics beyond the Standard Model will be pointed out.

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1. Introduction

Flavour physics has played a crucial role in the development of the Standard Model (SM). It was the τ - θ puzzle from kaon decays that led to the realization that parity may not be conserved in all interactions [1]. The Cabibbo angle [2] reconciled the universality of weak couplings with the mixing between quark flavours. The Glashow–Iliopoulos–Maiani (GIM) mechanism [3] further predicted the charmed quark to explain the lack of flavour-changing neutral currents (FCNC). The violation of charge-parity (CP) symmetry was observed in neutral kaon decays [4], and was accounted for in the Kobayashi–Maskawa (KM) paradigm [5] that also predicted the existence of the third generation of quarks. The large mixing observed in the neutral B system led to the inference that the top quark must be quite a bit heavier than the rest [6,7], and the rate of radiative B decays [8] helped predict the correct top quark mass before its direct discovery.

With the SM now well-established, flavour physics provides precision tests for the SM. The KM paradigm – that all the CP violations in the quark sector arise from the single phase in the Cabibbo–Kobayashi–Maskawa (CKM) matrix – has been tested from numerous directions, mainly from the decays of K and B mesons. Signals of NP, that normally introduces new heavy particles, can be obtained in the low-energy observables if the heavy particles contribute through Feynman diagrams involving loops. Rare decays of K , D and B mesons that occur through such loop processes are therefore good candidates to look for signals of NP. While the theoretical calculations of many such processes are plagued

with hadronic uncertainties, judicious selection of observables – like asymmetries that cancel out most of the hadronic uncertainties – can lead to precision tests of the SM.

While most of the flavour physics data till now have been consistent with the SM predictions, there have been a few tantalizing hints of NP in recent times. The difference in the direct and indirect measurements of $\sin(2\beta)$ [9], the anomalous CP-asymmetry in the like-sign dimuons [10], the lifetime difference and CP-violating phase measured in $B_s - \bar{B}_s$ mixing [11], the abnormally large observed branching fraction of $B \rightarrow \tau\nu$ [12], the forward–backward asymmetry in $B \rightarrow K^*\mu^+\mu^-$ [13], difference between the direct CP asymmetries of $D \rightarrow K^+K^-$ and $D \rightarrow \pi^+\pi^-$ [14]: all these measurements, singly or in combination, have indicated the presence of NP. If these anomalies are confirmed with more data and independent measurements, we shall be able to not only ascertain the presence of NP, but also discern some of its characteristics. A substantial part of this paper will be devoted to discussing these measurements and what characteristics of NP they may be indicating.

Independently of the NP signals in the data, there are still quite a few questions raised by flavour physics which have been around for a few decades and which may not have quick answers. Why are there three generations (if indeed, there are only three)? Why is there such an extreme hierarchy of fermion masses? What is the source of CP violation? How is the observed baryon asymmetry generated? These are questions in search of a framework that may be broader than the SM in its current state. These are not the questions this talk will dwell on.

Some caveats, excuses and apologies are in order. I shall deal mainly with B decays, and only partially with decays of D mesons, which have given us many interesting results in recent times. I shall emphasize the new theoretical and experimental results in the last couple of years, and by necessity, shall be biased towards the issues that I understand something about. The focus will naturally be on measurements at the border of SM and beyond, which could be a bit unfair to all those beautiful measurements that are consistent with the SM and have played a crucial role in strengthening its foundations. I shall start by describing in §2, the status of some of the SM predictions and related measurements. In §3, some of the anomalies recently observed in the recent data will be summarized. Section 4 will further explore some selected NP models that are indicated by the above anomalies, study the current constraints on them and point out how they may be tested in future experiments. Section 5 will present my personal concluding observations.

2. Standard Model predictions

The effective Hamiltonian for a typical B -decay process may be written in terms of the operator product expansion

$$\mathcal{H}_{\text{eff}}^{\text{SM}} \sim G_F \sum_i \lambda_i^{\text{CKM}} C_i(\mu) O_i(\mu), \quad (1)$$

where G_F is the Fermi constant, λ_i^{CKM} is some combination of the CKM matrix elements, and C_i are the Wilson coefficients corresponding to the operators O_i . While the CKM matrix elements are determined from a fit to the data, the Wilson coefficients at a scale

μ are calculated through renormalization group running, starting from their values at the scale M_W . The calculation of the decay rate simply follows

$$\Gamma(B \rightarrow f) = \int [\text{phase space}] |\langle f | H_{\text{eff}}^{\text{SM}} | B \rangle|^2. \quad (2)$$

Apart from the information on masses of quarks and mesons that is obtained from data, the calculation of the hadronic matrix element involves some non-perturbative inputs on quantities like decay constants and bag parameters that need to be obtained from lattice calculations.

2.1 Decay constants and bag parameters

Lattice results for the decay constants are now available with the number of sea-quark flavours $N_f = 2 + 1$. The recent results give $f_{B^+} = (196.9 \pm 8.9)$ MeV, $f_{B_s} = (242.0 \pm 9.5)$ MeV, $f_{D^+} = (218.9 \pm 11.3)$ MeV, $f_{D_s} = (260.1 \pm 10.8)$ MeV [15]. The individual decay constants are thus known to an accuracy of about 5%. The $SU(3)$ flavour-breaking ratios of these decay constants are however predicted to a much better accuracy: $f_{B_s}/f_B = 1.229 \pm 0.026$ and $f_{D_s}/f_D = 1.188 \pm 0.025$ [15]. The quantities that involve the ratios of decay constants are therefore much more well-known now, and the errors due to this ratio form only a minor part of the total error on the observables.

The bag parameters B_q ($q = d, s$) that parametrize the non-perturbative contribution to the neutral B mixing are also now calculated on the lattice for $N_f = 2 + 1$. The predictions for these parameters are conveniently given in terms of the combination $f_{B_q} \sqrt{B_{B_q}}$ that appears in the expression for the mass difference in a neutral B_q system. The current predictions from the HPQCD Collaboration [16] are $f_{B_s} \sqrt{B_{B_s}} = 266 \pm 18$ MeV and $\xi_B \equiv (f_{B_s} \sqrt{B_{B_s}})/(f_{B_d} \sqrt{B_{B_d}}) = 1.258 \pm 0.033$. As a result of rapid progress in the lattice calculations, the non-perturbative error on the prediction of $\Delta M_s/\Delta M_d$, the ratio of mass differences in the B_d and B_s systems, has now come down to only about 2.5%.

For a review of lattice results, the reader is referred to a recent talk [17].

2.2 CKM matrix elements

The global fits to the CKM matrix elements [9,18] have already been described in the talk by Tim Gershon [19]. So we shall not repeat the details here. The constraints in the $\bar{\rho}-\bar{\eta}$ plane have now been calculated from the combinations of measurements of (i) the ratio $|V_{ub}/V_{cb}|$, (ii) the parameter ϵ_K from $K \rightarrow \pi\pi$, (iii) the mass differences ΔM_d and ΔM_s , (iv) the angles α, β, γ (or equivalently, ϕ_2, ϕ_1, ϕ_3) of the unitarity triangle. Since almost all of the above measurements are consistent with each other, the KM paradigm of there being only one CP-violating phase in the SM is mostly vindicated. However, there are still a few open issues that we would like to reiterate here.

The value of $\sin 2\beta$ obtained from the time-dependent CP asymmetry in $b \rightarrow c\bar{c}s$ decays differs from the best fit obtained from the combination of all other measurements. The measurements of $|V_{ub}|$ from inclusive and exclusive semileptonic decays are different from each other. On top of it, the measured decay rate of $B \rightarrow \tau\nu$ yields a value for $|V_{ub}|$ that is too high to be consistent with either of these, and in combination with the $\sin(2\beta)$

measurements, gives an even greater deviation from the SM [9]. The measurements of $|V_{cs}|$ from semileptonic K decays and hadronic τ decays are also in tension with each other. While these discrepancies are still less than 3σ , they need to be resolved before the CKM picture may be said to be complete.

The unitarity of the CKM matrix plays a major role in the fit to the CKM elements, in the absence of which the magnitudes of V_{td} and V_{ts} would not be well-measured. The unitarity cannot be simply verified by testing whether the angles of the unitarity triangle sum up to π , since $\alpha + \beta + \gamma = \pi$ trivially by their definitions. What is needed is a non-trivial check of unitarity, like testing for the relation [20]

$$\sin \beta_s = \left| \frac{V_{us}}{V_{ud}} \right|^2 \frac{\sin \beta \sin(\gamma + \beta_s)}{\sin(\beta + \gamma)} [1 + \mathcal{O}(\lambda^4)], \quad (3)$$

that needs the measurement of the CP-violating phase β_s in the B_s mixing. We should soon be close to testing this important self-consistency feature of the KM paradigm: the anomalous CP-asymmetry in like-sign dimuons [10], as well as the measurements of the lifetime difference and CP-violating phase in $B_s \rightarrow J/\psi\phi$ [11] are some of the measurements that do not go in the current CKM fit. However, they will turn out to be crucial to get a consistent picture.

2.3 Mass differences and width differences in neutral meson systems

The effective Hamiltonian for $B_q-\bar{B}_q$ mixing may be written as $H = M - (i/2)\Gamma$, where M and Γ are 2×2 Hermitian matrices in the $(B_q\bar{B}_q)^T$ basis, that correspond to the dispersive and absorptive parts of the Hamiltonian, respectively. The diagonal elements are related as $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$ due to CPT-conservation, while the off-diagonal elements M_{12} and Γ_{12} are responsible for the mass difference and lifetime difference, respectively. As long as $|\Gamma_{12}| \ll |M_{12}|$, a condition that is satisfied in the B_d as well as B_s system, $\Delta M = 2|M_{12}| + \mathcal{O}(m_b^4/m_t^4)$ and $\Delta\Gamma = -2\text{Re}(M_{12}^*\Gamma_{12})/|M_{12}| + \mathcal{O}(m_b^4/m_t^4)$.

The origin of M_{12} is in the box diagram that gives rise to $B_q-\bar{B}_q$ mixing, where the contribution due to the top quark in the loop dominates compared to that due to the charmed and up-quark loops. In the SM,

$$M_{12}^q = \frac{G_F^2 M_W^2}{12\pi^2} m_{B_q} B_{B_q} f_{B_q}^2 \eta_{\text{QCD}} (V_{tq}^* V_{tb})^2 S(x_t), \quad (4)$$

where η_{QCD} is the QCD correction factor, $x_t = m_t^2/m_W^2$, and $S(x_t)$ is the Inami-Lim function [21]. The mass differences are measured to be [22] $\Delta M_d/\Gamma_d = 0.771 \pm 0.008$ and $\Delta M_s/\Gamma_s = 26.92 \pm 0.15 \pm 0.10$. These mass differences, when combined with the values of $f_{B_q}\sqrt{B_{B_q}}$ obtained from the lattice, yield the values of $|V_{td}|$ and $|V_{ts}|$, respectively.

The origin of Γ_{12} is in the intermediate on-shell states in the box diagram that leads to mixing. These states are thus the common final states that both B_q and \bar{B}_q decay to. Clearly these intermediate states involve only u and c quarks. In the $B_d-\bar{B}_d$ system, the contribution from intermediate $c-c$ quarks dominates, and $\Gamma_{12} \propto (V_{cb}V_{cd}^*)^2$. In the $B_s-\bar{B}_s$ system on the other hand, the contributions from intermediate $u-c$ quarks and $u-u$ quarks are significant as well, and since even the $c-c$ quark contribution gives terms proportional to $(V_{cb}V_{cs}^*)^2$, the value of $\Delta\Gamma_s$ is expected to be much larger than $\Delta\Gamma_d$, though $\Gamma_s \approx \Gamma_d$. The SM predictions are [23] $\Delta\Gamma_d/\Gamma_d = (0.42 \pm 0.08)\%$ and $\Delta\Gamma_s/\Gamma_s = (13.7 \pm 2.7)\%$.

Because of its small value, $\Delta\Gamma_d/\Gamma_d$ is often neglected in theoretical calculations, which is fine as long as the accuracy of experiments is worse than a percent level. However, in this precision era of flavour physics, a measurement of this quantity is possible and may turn up new surprises [19,24,25]. The average of measurements for $\Delta\Gamma_d$ from BABAR and DELPHI gives [12] $\Delta\Gamma_d/\Gamma_d = (1.1 \pm 3.7)\%$. Thus, the accuracy of these measurements has to improve by almost an order of magnitude before they can confirm the SM predictions. Of course, if NP can increase $\Delta\Gamma_d$ to a few percent (as it may, for a third-generation scalar leptoquark models [26]), its measurement would be a clear indication of NP which has flavour-dependent couplings with charged leptons.

On the other hand, $\Delta\Gamma_s/\Gamma_s \sim \mathcal{O}(10\%)$, and hence much more amenable to measurements. Indeed we already have a $\sim 2\sigma$ measurement of a non-zero $\Delta\Gamma_s/\Gamma_s$ through the $B_s \rightarrow J/\psi\phi$ decay [12]: $\Delta\Gamma_s/\Gamma_s = 0.154^{+0.067}_{-0.065}$. It may be noted that values of $\Delta\Gamma_s$ that are a factor of two larger than the SM prediction are easily possible with the current data. Such an enhancement is impossible [27] in a large class of models, including, for example, the minimal flavour violating (MFV) ones. However, models that have flavour-dependent couplings of charged leptons with NP, especially those that would cause an enhancement of the $B_s \rightarrow \tau\tau$ decay rate while keeping $B_s \rightarrow \mu\mu$ and $B_d \rightarrow \tau\tau$ constrained by the current limits, would be able to increase the value of Γ_{12} in the B_s system. Some such models could be the third-generation scalar leptoquark models [26] or the left-right symmetric models [28].

In the $D-\bar{D}$ system, the value of M_{12} is small since the quarks in the box diagram loop are not heavy. The SM prediction is difficult since the long distance contributions are significant and cannot be calculated reliably at the moment [29,30]. One expects $\Delta M_D/\Gamma_D \sim \mathcal{O}(10^{-3})$, while the measured value is $\Delta M_D/\Gamma_D = (0.63 \pm 0.2)\%$ [12]. The value of $\Delta\Gamma_D/\Gamma_D$ is also expected to be $\sim \mathcal{O}(10^{-4})$, since there are not many common final states for D and \bar{D} decays. The measurements give $\Delta\Gamma_D/\Gamma_D = (1.5 \pm 0.24)\%$ [12].

3. Some recent important flavour-physics measurements

In this section, we shall point out some of the recent measurements in heavy flavour physics. Details of most of these measurements have already been presented [19,31–33]. Some of these measurements are interesting because they are anomalous, i.e., deviate slightly from the SM expectations, in which case we shall indicate the features of NP they will point to, if they indeed pass the test of time. The other measurements are consistent with the SM. However, with the foreseen increase in accuracy, these have the potential of revealing signals of NP.

3.1 Tension among the measurements of $\sin 2\beta$

The direct measurement of $\sin 2\beta$ through the time-dependent CP-asymmetries of the form $b \rightarrow c\bar{c}s$ and $b \rightarrow s\bar{s}s$ are now consistent with each other, and give $\sin(2\beta)^{(\text{fit})} = 0.691 \pm 0.020$ [9]. However, the global fit to this quantity including all the flavour-physics data except the above time-dependent asymmetries – ϵ_K , $|V_{ub}/V_{cb}|$, Δm_d , Δm_s – yields $\sin(2\beta)^{(\text{direct})} = 0.830^{+0.013}_{-0.033}$ [9]. Such a difference may be caused by most of the NP

models that involve heavy particles contributing to either B_d - \bar{B}_d mixing or $b \rightarrow s$ decay through loops.

3.2 Like-sign dimuon asymmetry

The direct semileptonic decay $B \rightarrow \mu X$ always involves μ^- , while $\bar{B} \rightarrow \mu X$ always involves μ^+ . In general, therefore, the decay of a $B\bar{B}$ pair would be expected to produce muons of opposite signs. The like-sign dimuon signal $B\bar{B} \rightarrow \mu^\pm \mu^\pm X$ is obtained when one of the B and \bar{B} oscillates to the other before decaying, resulting in two muons with the same charge. The CP asymmetry in this channel is therefore the same as the semileptonic asymmetry. Since the like-sign dimuon sample at the Tevatron does not distinguish between B_d vs. B_s as the original meson, what one observes is a weighted average of the semileptonic asymmetries a_{sl}^d and a_{sl}^s , in B_d and B_s systems, respectively: $A_{sl}^b = (0.506 \pm 0.043)a_{sl}^d + (0.494 \pm 0.043)a_{sl}^s$ [10].

The SM prediction for this quantity is $A_{sl}^b = (-0.023_{-0.006}^{+0.005})\%$, while the measurement [10] yields $A_{sl}^b = (-0.787 \pm 0.172 \pm 0.093)\%$. This is a substantial, 3.9σ deviation from the SM. Given that a_{sl}^d has independently been measured to be $(-0.47 \pm 0.46)\%$ [12], most of the observed A_{sl}^b has to be contributed by a_{sl}^s . This gives $a_{sl}^s = (-1.81 \pm 1.06)\%$, which is about 2σ deviation from its SM prediction of $(0.0021 \pm 0.0006)\%$. Now

$$a_{sl}^s = (\Delta\Gamma_s/\Delta M_s) \tan \phi_s^{sl}, \quad (5)$$

where $\phi_s^{sl} = \text{Arg}(-M_{12}/\Gamma_{12})$ in the B_s system. Thus, in order to enhance a_{sl}^s to large values, one needs NP that will enhance either $\Delta\Gamma_s$, or ϕ_s^{sl} , or both.

3.3 Angular distribution in $B \rightarrow J/\psi\phi$

The angular analysis of $B_s \rightarrow J/\psi\phi$ [34,35] is another avenue for the measurements of $\Delta\Gamma_s$ and a related CP-violating phase, $\phi_s^{J/\psi\phi} \equiv \text{Arg}[-M_{12}/(V_{cb}V_{cs}^*)^2]$. Note that $\phi_s^{sl} \neq \phi_s^{J/\psi\phi}$. Indeed even in the SM, $\phi_s^{sl} = 0.0041 \pm 0.0007$ and $\phi_s^{J/\psi\phi} = -0.038 \pm 0.002$. Though both of them are small, they have opposite signs and their magnitudes differ by almost an order of magnitude. This is a consequence of the fact mentioned earlier, that in Γ_{12} the contribution of c - c intermediate states – that would have been proportional to $(V_{cb}V_{cs}^*)^2$ – does not dominate [23,36].

The results from $B_s \rightarrow J/\psi\phi$ have been in tension with the SM [11], though recent data indicate that they are becoming consistent with the SM [33]. Note that values of $\Delta\Gamma_s$ that are about a factor of two more than the SM are still possible.

3.4 Enhanced branching ratio of $B^+ \rightarrow \tau^+\nu_\tau$

The branching ratio of $B^+ \rightarrow \tau^+\nu_\tau$ in the SM is

$$\mathcal{B}_{\tau\nu} = \mathcal{B}(B^+ \rightarrow \tau^+\nu_\tau)_{\text{SM}} = \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B, \quad (6)$$

where τ_B is the B^+ lifetime. With $f_B = 192.8 \pm 9.9$ MeV [37] and $|V_{ub}| = (3.52 \pm 0.11) \times 10^{-3}$ [9,38], one gets the SM prediction, including higher-order corrections, to be $\mathcal{B}_{\tau\nu}(\text{SM}) = (0.81 \pm 0.15) \times 10^{-4}$. The measurement [12], on the other hand, gives

$\mathcal{B}_{\tau\nu} = (1.68 \pm 0.31) \times 10^{-4}$. This enhancement is more than 2σ , and is difficult to explain simply by the non-perturbative input on f_{B_d} .

There is a strong correlation between this branching ratio and the measurement of $\sin 2\beta$ [9], and the deviation from SM in the $(\sin 2\beta - \mathcal{B}_{\tau\nu})$ parameter space is rather striking. The measurements of $B \rightarrow D\tau\nu$ and $B \rightarrow D^*\tau\nu$ also show similar (1.8σ) excess [19]. Though none of the deviations by themselves are very strong, the fact that all of them are leaning towards one direction may be an indication of some NP that changes the effective value of V_{ub} .

3.5 Branching ratio of $B_s \rightarrow \mu^+\mu^-$

The SM prediction for the branching ratio is $\mathcal{B}_{\mu\mu} \equiv \mathcal{B}(B_s \rightarrow \mu\mu) = (0.32 \pm 0.02) \times 10^{-8}$ [39]. Recently, CDF has given a positive measurement of this decay, with $\mathcal{B}_{\mu\mu} = (1.8_{-0.9}^{+1.1}) \times 10^{-8}$ [40], while CMS and LHCb have given a combined upper limit of $\mathcal{B}_{\mu\mu} < 1.1 \times 10^{-8}$ [41]. These results already put strong bounds on the scalar and pseudoscalar contributions to NP. Moreover, the reach of LHC for this quantity is as low as 10^{-9} and therefore precision measurements of NP quantities may be expected from this channel. Section 4.5 will discuss NP in this mode in more detail.

3.6 Forward-backward asymmetry in $B \rightarrow K^*\mu^+\mu^-$

The forward-backward asymmetry $A_{\text{FB}}(q^2)$ in $B \rightarrow K^*\mu^+\mu^-$, where q^2 is the invariant mass of the $\mu^+\mu^-$ pair, is a result of the interference between photonic and Z -penguin contributions and is sensitive to NP. The zero of $A_{\text{FB}}(q^2)$ is a particularly clean prediction of the SM, because at this point the form-factor dependence cancels at the leading order (LO), and a relation

$$\text{Re}[C_9^{\text{eff}}(q_0^2)] = -(2m_B m_b / q_0^2) C_7^{\text{eff}} \quad (7)$$

between the short-distance coefficients is obtained [42]. Here q_0^2 is the point where $A_{\text{FB}}(q_0^2) = 0$. Next-to-leading-order (NLO) contributions shift the position of this zero to a higher value: $q_0^2 = 3.90 \pm 0.12 \text{ GeV}^2$ [43]. A substantial deviation from this zero crossing point would thus be a robust signal for NP.

The measurement of this asymmetry at BELLE [13] showed a deviation from the SM: the data showed positive values of A_{FB} throughout the q^2 range, and gave no sign of a zero crossing. The recent data from LHCb [44], however, do indicate zero crossing in the expected region. A deviation from the SM prediction may occur if the NP affects C_7^{eff} and/or C_9^{eff} , or if it changes the relation in eq. (7) itself, such as by introducing new Wilson coefficients.

3.7 CP asymmetry in $B \rightarrow K\pi$ decays

In the SM, the difference between the direct CP asymmetries in $B^+ \rightarrow K^+\pi^0$ and $B^0 \rightarrow K^+\pi^-$ is expected to vanish in the limit of isospin symmetry. This quantity is termed as ΔA_{CP} . Currently, experiments show [12]

$$\Delta_{K\pi} = A_{\text{CP}}(B^+ \rightarrow K^+\pi^0) - A_{\text{CP}}(B^0 \rightarrow K^+\pi^-) = 0.121 \pm 0.022, \quad (8)$$

which is a 5.8σ deviation from the expectation. It is not clear whether this deviation is due to NP, or due to some isospin-breaking hadronic effects that are not well understood. For

example, the SM prediction mentioned above neglects the contribution from electroweak penguin (P_{EW}) and colour-suppressed tree (C) diagrams, which is the naive expectation. Even with improved schemes of calculating these quantities, like QCD-improved factorization, ΔA_{CP} would be hard to explain as it would require large imaginary values for P_{EW} and C [45]. Moreover, if P_{EW} were the explanation, one would have been able to have some evidence from ratios of the form $B(B^+ \rightarrow \pi K)/B(B^0 \rightarrow \pi K)$ or $B(B^+ \rightarrow \rho K)/B(B^0 \rightarrow \rho K)$, which is not found. On the other hand, if C were the explanation, it would imply a breakdown of the power-counting in the context of soft collinear effective theory (SCET), which has been observed to hold in other modes [46]. The perturbative QCD framework for calculating decay rates allows an explanation, however only through an additional non-perturbative input [47]. Thus, the real explanation of this $\Delta A_{K\pi}$ anomaly still eludes us. There is a recent claim that Pauli blocking is responsible for the difference between the two channels: the tree diagram $\bar{b} \rightarrow \bar{s}u\bar{u}$ is Pauli-suppressed for a spectator u -quark in B^+ , but not for a spectator d -quark in B^0 decay [48].

3.8 Longitudinal polarization f_L in $B \rightarrow \phi K^*$

Longitudinal polarization (the fraction of events wherein the final-state particles ϕ and K^* are longitudinally polarized) in $B \rightarrow \phi K^*$ is expected to be close to unity, since one should have $1 - f_L = f_T \propto m_{K^*}^2/m_B^2$ [49,50]. However, experiments seem to prefer $f_T/f_L \simeq 1$ [51–53]. In the framework of the SM, one can think of two possible mechanisms that could be responsible for this puzzle: penguin annihilation or rescattering effects. The jury is still out on this.

4. Some NP models relevant to the anomalies

In this section, we shall explore some NP models that may be indicated by the anomalies in the previous section. The original motivation of these models may not lie in the observed anomalies. Indeed these models address a multitude of issues in low-energy flavour physics as well as high-energy collider physics. Hence in turn, these models are constrained by the data from a variety of decay modes. Here we present the current status of these models in the light of other data, and comment on whether these indeed can be viable explanations of the anomalies.

4.1 Fourth generation of quarks

While the inconsistency in the direct and indirect determination of $\sin 2\beta$ as described above, as well as any deviation in the measurement of $\phi_s^{J/\psi\phi}$ or ϕ_s^{sl} , can be caused by almost any model that has heavy particles in the loops contributing to $b \rightarrow s$ processes, extra quark generations is perhaps the most natural NP candidate. Indeed there is no fundamental reason why the number of generations in the SM should be restricted to three, and even a single extra generation provides three more mixing angles and two more CP-violating phases that can account for the deviation. The presence of a fourth generation clearly will have important phenomenological consequences [54–57].

Direct searches for the fourth-generation quarks have yielded lower bounds on their masses: $m_{t'} \geq 335$ GeV [58] and $m_{b'} \geq 372$ GeV [59] to 95% CL. Moreover, since these

quarks would greatly enhance the $gg \rightarrow h^0$ cross-section at the LHC, the limit on this cross-section gives an indirect lower bound of ~ 500 GeV on their masses [60]. On the other hand, the theoretical requirements from the perturbativity of the Yukawa couplings and the perturbative unitarity of the fermion–fermion cross-section [61] implies that these masses cannot be larger than ~ 600 GeV. Electroweak corrections, as parametrized by the oblique parameters S, T, U [62] further constrain the difference in the masses of t' and b' [63], which also depends on the masses of the fourth-generation charged leptons, the mass of the Higgs boson [64], and the mixing of the fourth generation with the other three [65]. The masses of the fourth-generation quarks are thus highly constrained and correlated, but SM4 still remains a viable scenario [66].

With four quark generations, the quark-mixing matrix is a 4×4 matrix, CKM4, whose parametrization requires six real parameters and three phases. Recently, a fit has been performed [67] to the flavour-physics data that includes the measurements of (i) R_{bb} and A_b from $Z \rightarrow b\bar{b}$, (ii) ϵ_K from $K_L \rightarrow \pi\pi$, (iii) the branching ratio of $K^+ \rightarrow \pi^+\nu\bar{\nu}$, (iv) the mass differences in the B_d and B_s systems, (v) the time-dependent CP asymmetry in $B_d \rightarrow J/\psi K_S$, (vi) γ from tree-level decays, (vii) the branching ratios of $B \rightarrow X_s\gamma$ and $B \rightarrow X_c e\bar{\nu}$, and (viii) the branching ratio of $B \rightarrow X_s\mu^+\mu^-$ in the high- q^2 and low- q^2 regions. It was found that all five NP parameters are consistent with zero, and the mixing of the fourth generation with the other three is constrained to be small. In particular, $|V_{ub'}| < 0.06$, $|V_{cb'}| < 0.027$, $|V_{tb'}| < 0.31$ at 3σ . Still, it was noted that NP signals in B_d^0, B_s^0 and rare K decays are possible.

4.2 Models that contribute to absorptive part of $B_s-\bar{B}_s$ mixing

From the discussion of anomalous CP-asymmetry in like-sign dimuons (§3.2) and the measurements of lifetime difference and CP-phase in the B_s system (§3.3), NP that contributes to the absorptive part of $B_s-\bar{B}_s$ mixing is strongly indicated. A recent model-independent fit [68] to these two measurements shows that the scenario with no NP contribution to Γ_{12} is severely disfavoured. Also, note that even if the tension of $\phi_s^{J/\psi\phi}$ measurements with the SM reduces, as long as the dimuon asymmetry forces ϕ_s^{sl} to be large, the reconciliation of these measurements would need a large NP contribution to Γ_{12} . Furthermore, this contribution will have to have a large complex part in such a scenario.

The models in which NP contributes significantly to Γ_{12} could include third-generation scalar leptoquarks that couple only to τ among the leptons [69], models with extra Z' bosons, or those with R -parity violating supersymmetry [70]. The only effective 4-Fermi operator that is unconstrained enough to be able to contribute significantly to Γ_{12} is $b \rightarrow s\tau\tau$ [71]. Therefore, all such viable NP models predict enhanced $B_s \rightarrow \tau\tau$ and related decays. Note that current data still allow the branching fraction $\mathcal{B}(B_s \rightarrow \tau\tau) \sim 5\%$.

4.3 MFV models with charged Higgs

If $B \rightarrow \tau\nu$ is indeed enhanced, it can be explained in MFV models with charged Higgs, which can lead to an enhancement of

$$R_{\tau\nu} = \frac{\mathcal{B}(B^+ \rightarrow \tau^+\nu_\tau)}{\mathcal{B}(B^+ \rightarrow \tau^+\nu_\tau)_{\text{SM}}} = \left(1 - \tan^2\beta \frac{m_B^2}{M_+^2}\right)^2, \quad (9)$$

where $|V_{ub}|$ used for the SM prediction should be determined only through the measurements that are not influenced by NP: $|V_{ub}/V_{cb}|$, $\Delta m_s/\Delta m_d$, and $\sin 2\beta$. This is indeed what is done in the fit to the ‘universal unitarity triangle’, the so-called UUTfit [38]. The 2σ range of the above ratio is $0.99 < R_{\tau\nu} < 3.14$. This implies that [38] (i) if the charged Higgs is heavy, only small $\tan\beta$ can survive, and that too barely, while (ii) if the charged Higgs is light, large $\tan\beta$ can explain the anomaly.

The constrained MSSM model (cMSSM) is one of the most studied models of the MFV-type, which also has important implications for collider physics. It turns out that the charged Higgs in cMSSM is necessarily heavy and as a result cMSSM cannot explain the anomaly, though a small region in its parameter space can survive it to 2σ [72], even after combining the constraints from other low-energy data from the branching ratios of $B_s \rightarrow \mu^+\mu^-$, $B \rightarrow K^*\gamma$ and the anomalous magnetic moment ($g-2$) of muon. Interestingly, this ‘golden’ region is still consistent with neutralino as the lightest supersymmetric particle that will account for most of the dark matter in the Universe in the R -parity conserving limit.

Note that though cMSSM survives the constraints from the low-energy data mentioned above, it cannot explain the anomalously high branching ratio of $B^+ \rightarrow \tau^+\nu$. Models with a non-minimal Higgs sector, like the non-universal Higgs model (NUHM), can increase the predicted value of $R_{\tau\nu}$ from unity [72]. However, given that the related decay $K^+ \rightarrow \mu^+\nu$ has a rate consistent with the SM, one would also need an extra ingredient that breaks universality of lepton couplings.

4.4 NP with new vector/axial vector operators

In the language of operator product expansion, it is convenient to parametrize NP in terms of the coefficients of operators with different Lorentz structures. Then one can talk about the contributions of vector/axial vector (VA), scalar/pseudoscalar (SP), or tensor (T) type of NP. This allows us to explore the NP in a model-independent manner. While SM already provides VA operators, the SP and T operators are completely new contributions.

The FCNC decays involving the $b \rightarrow s\mu\mu$ transition allow us to express multiple observables in terms of the coefficients of SP, VA and T operators. For example, one may study the branching ratios of $B_s \rightarrow \mu^+\mu^-$, $B \rightarrow X_s\mu^+\mu^-$, $B \rightarrow \mu^+\mu^-\gamma$, $B \rightarrow K\mu^+\mu^-$, and the forward–backward asymmetries in all of them (except $B_s \rightarrow \mu^+\mu^-$, which has none). In addition, for $B \rightarrow K^*\mu^+\mu^-$, apart from the branching ratio and A_{FB} , one also has the longitudinal polarization fraction f_L , and the angular asymmetries $A_{\text{T}}^{(2)}$, A_{LT} . The CP-asymmetries in the above observables, and the triple-product (TP) asymmetries: $A_{\text{T}}^{(\text{im})}$, $A_{\text{LT}}^{(\text{im})}$, are also potential harbingers of NP [73–77].

Although SM already has VA operators, new VA operators as well as their interference with the VA operators in the SM can give rise to significant deviations of the measurements of some of the observables mentioned above. Indeed, if the indication for an anomalous forward–backward asymmetry described in §3.6 is confirmed, it will mostly be due to new VA interactions [78]. These interactions can interfere with the SM terms constructively or destructively, thus enhancing or suppressing the differential branching ratios by up to factors of two. They also are able to enhance almost all the forward–backward asymmetries in the $b \rightarrow s$ channels mentioned above [74], the notable exception being A_{FB} in $B \rightarrow K\mu^+\mu^-$, where they cannot contribute.

The CP asymmetries and the triple-product asymmetries mentioned above are vanishingly small in the SM. These can be enhanced only if the new VA operators contribute, and even given the current constraints, they can enhance these asymmetries to a few percent, thus bringing them in the domain of observability [76].

4.5 NP with scalar/pseudoscalar/tensor operators

As mentioned in §3.5, the branching ratio of $B_s \rightarrow \mu^+ \mu^-$ is a potential candidate for observing a NP signature. In addition, it can even identify the Lorentz structure of the NP. In terms of the new SP, VA and T operators mentioned in §4.4, the branching ratio $\mathcal{B}_{\mu\mu}$ is given as

$$\mathcal{B}_{\mu\mu} = \frac{G_F^2 \alpha_{em}^2 m_{B_s}^5 f_{B_s}^2 \tau_{B_s}}{64\pi^3} |V_{tb} V_{ts}^*|^2 \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}} \times \left\{ \left(1 - \frac{4m_\mu^2}{m_{B_s}^2}\right) \left| \frac{R_S - R'_S}{m_b + m_s} \right|^2 + \left| \frac{R_P - R'_P}{m_b + m_s} + \frac{2m_\mu}{m_{B_s}^2} (C_{10} + R_A - R'_A) \right|^2 \right\}, \quad (10)$$

where C_{10} is the relevant SM Wilson coefficient and $R(R')$ s are the NP coupling coefficients. Clearly, only scalar, pseudoscalar and axial vector couplings can contribute to this quantity. The measurement of this branching ratio then is a direct probe of these NP Lorentz structures, and especially of the SP contributions, since their contribution is not suppressed in eq. (10) like that of the NP axial vector operators.

The upper bound on $\mathcal{B}_{\mu\mu}$ from the data constrains the contribution of SP operators to the other related decay modes mentioned in §4.4, like $B \rightarrow X_s \mu^+ \mu^-$, $B \rightarrow \mu^+ \mu^- \gamma$, $B \rightarrow K \mu^+ \mu^-$ and $B \rightarrow K^* \mu^+ \mu^-$. Therefore, the contributions of SP operators to these decay modes are often not large enough to stand apart from the SM background. The couplings of the T operators, on the other hand, are not as suppressed as those of the SP operators. Therefore, they typically contribute significantly to the branching ratios of these modes. However, the interference terms of these operators with the SM operators often suffer from the m_μ/m_b helicity suppression, and hence they tend to suppress the magnitudes of the asymmetries.

The observable where SP and T operators should make their presence felt is the forward–backward asymmetry in $B \rightarrow K \mu^+ \mu^-$ [79]. This quantity identically vanishes in the SM, since the hadronic matrix element for the $B \rightarrow K$ transition does not have any axial-vector contribution. As a consequence, the presence of only new VA operators is not enough to give rise to a non-zero A_{FB} . Both the SP and T operators, however, can perform this task. Moreover, while the SP operators can lead to an enhancement in the low- q^2 region, the T operators can do so in the high- q^2 region. In the optimistic scenario, it may be possible to identify the source of NP as effective SP or T operators.

5. Concluding remarks

Flavour physics has always been a window to the particle physics beyond the known, and has a track record of predicting new particles as well as interactions during the construction

phase of the SM. It has also served as a magnifying glass, in precision measurements of the SM and in trying to find cracks in the edifice, looking for NP. SM has survived the scrutiny so far, and the indirect bounds obtained on NP through flavour data are now getting significant enough to compete with the direct bounds from searches at the high-energy colliders.

In the last few years, there have been some tantalizing results that showed a deviation from the SM. Most of them have been within the 2σ level, that is, consistent with a statistical fluctuation, taken individually. When combined, they sometimes give a stronger signal, as in the case of $\sin 2\beta$ and $\mathcal{B}(B^+ \rightarrow \tau^+\nu)$, or in the case of dimuon asymmetry and the angular distribution in $B_s \rightarrow J/\psi\phi$. The former, when combined with the data on K decays, could point to non-universal leptonic coupling that will contribute to $b \rightarrow d\tau\nu$, but not to $s \rightarrow d\mu\nu$. Of course all these speculations would be practically relevant only when the deviations from the SM are confirmed to a much higher statistically significant level.

The anomalies that stand out are the isospin asymmetry in the CP-violation in $B \rightarrow K\pi$ decays, which is about 5.8σ away from the SM prediction, and the CP-asymmetry in like-sign dimuons, which is about 3.9σ . While the former anomaly may be attributed to uncertainties in the theoretical calculations, it is more difficult to dismiss the latter one. If this anomaly is indeed confirmed, it points to a very specific kind of NP models: those that contribute to the absorptive part of the $B_s-\bar{B}_s$ mixing, which can come only from effective $b \rightarrow s\tau\tau$ operators. Note that here again one needs non-universal leptonic couplings, since the effective $b \rightarrow s\mu\mu$ operators are already highly constrained. The measurements of modes like $B_s \rightarrow \tau\tau$ are crucial in this context.

Hadronic uncertainties in heavy-flavour physics have been the bane of many a hint of NP. On the other hand, the strength of heavy-flavour physics is the availability of many complementary channels that probe the same parameters in the effective low-energy theory. This is amply exemplified in the observables involving the FCNC process $b \rightarrow s\mu\mu$. The ever-increasing quantity of data should enable such correlated analyses of multiple related modes, even those that are even now referred to as ‘rare’.

The hero of the flavour-physics saga has been data, always data. We are at the mercy of data. May we live in exciting times.

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