Heat Transfer Characteristics of Non-Newtonian Fluid Flows in Narrow Confinements Considering the Effects of Streaming Potential

Ranabir Dey[1], Jeevanjyoti Chakraborty[2] and Suman Chakraborty[1,2*]
[1]Mechanical Engineering Department, Indian Institute of Technology Kharagpur, Kharagpur – 721302, India
[2]Advanced Technology Development Centre, Indian Institute of Technology Kharagpur, Kharagpur – 721302, India

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Abstract
Thermal characteristics of pressure-driven, power-law fluid flows through narrow confinements are analyzed following a semi-analytical approach, by taking into consideration the electrokinetic effects beyond the Debye-Hückel limit. The influence of the induced streaming potential on the flow velocity, temperature and Nusselt number are delineated for parametric variations of the ionic Peclet number and the flow behaviour index. The effect of viscous dissipation is also incorporated into the thermal analysis. The effects of the streaming potential on the hydrodynamic and thermal characteristics are found to be appreciably different for fluids exhibiting different rheological characteristics.

1. INTRODUCTION

As a consequence of the rapid advancements in the field of micro and nanotechnology, the last decade witnessed numerous research works aimed towards analyzing microscale hydrodynamic and thermal characteristics. Very recent instances of such efforts can be found in the works of Chen [1], and Sadeghi and Saidi [2]. However, majority of these studies delineate the microscale flow characteristics for Newtonian fluids. Hence, such models fail to address the underlying flow physics pertinent for complex fluids, such as blood samples, bacterial cell suspensions, protein or antibody solutions, polymer melts, and oils. Such flows are evidenced in majority of the 'lab-on-a-chip' based analytic microsystems, especially in those implemented in the field of bio-medical applications. Furthermore, taking into consideration the state-of-the-art microfluidic devices which analyse ionic non-Newtonian fluids (charged colloids, biological macromolecules), it is imperative to critically study the microscale transport phenomenon for non-Newtonian fluids, considering the electrokinetic effects.

In this regard, Das and Chakraborty [3] analytically derived closed-form solutions for velocity, temperature and solute concentration distributions for purely electroosmotic flows of non-Newtonian fluids. They also analysed the transport characteristics of a blood sample as a pertinent case study. In a later work, Chakraborty [4] delineated the microcapillary filling dynamics of non-Newtonian fluids, taking into purview the electrokinetic effects. Zhao *et al*. [5] performed a meticulous study focussed on the effects of electrical double layer (EDL) thickness, flow behaviour index, and applied electric fields on the hydrodynamic behaviour of non-Newtonian electroosmotic flows. In another extensive work, Vasu and De [6] analyzed the hydrodynamic characteristics of pressure-driven and electroosmotic non-Newtonian flows in closed micro-conduits. The novelty of this work stems from the fact that the

* Corresponding author, email: suman@mech.iitkgp.ernet.in

authors took into consideration the effects of the induced streaming potential while addressing the pressure-driven flow. They delineated the effects of various parameters, like flow behaviour index, channel geometry, on the different aspects of the underlying flow dynamics, for each type of actuation mechanism. However, the last two mentioned works did not address the thermal characteristics associated with such non-Newtonian microflows. In that regard, very recently, Chen [7] derived closed form solutions for velocity, friction coefficient, temperature, and Nusselt number for electroosmotic flows of non-Newtonian fluids. This study explained the effects of flow behaviour index, geometry of the micro-confinement, Joule heating and other electrical parameters on the fully-developed Nusselt number. In all of the above mentioned works, the non-Newtonian behaviour was modelled considering power-law fluids which obey constitutive relations in the form of power-laws.

For pressure-driven microflows, with electrokinetic effects, exclusion of the streaming potential induced convective transport of ions from the flow hydrodynamics model results in erroneous predictions of the flow characteristics, and is one of the major limitations of traditional electroviscous analysis [8]. Hence, the prime focus of the present work is to analyse, probably for the first time, the alterations, in the associated thermal characteristics of ionic power-law fluid flows through closed narrow confinements, triggered by the modification in the underlying flow physics due to included streaming potential effects. Temperature and Nusselt number profiles are derived semi-analytically considering the electrokinetic effects, by going beyond the Debye-Hückel linearization, as well as the effect of viscous dissipation, which further broadens the scope of the present work. A parametric study is performed to delineate the interesting features of the thermal characteristics.

2. MATHEMATICAL MODELLING

2.1 Flow modelling

Pressure-driven transport of a power-law fluid, containing symmetric electrolytes ($z^+ = -z^- = z$), through long, parallel-plate, narrow confinement of height $2H$ and width W ($W \gg 2H$) is considered here. The flow is actuated by a constant axial pressure-gradient, $P_x = -\frac{dP}{dx}$. For modelling the present physical situation, the following simplifying assumptions and approximations are made: steady, incompressible, and laminar flow of an ionic power-law fluid is considered, the thermophysical properties of the fluid are independent of temperature variations, the ionic species behave as point charges, the generated zeta potential (ζ) is uniform throughout the channel walls, and the Boltzmann distribution of ionic concentration remains valid as the developed EDLs do not overlap and the flow Peclet number is appreciably small ($Pe \le 1$).

Combining the Boltzmann distribution of ionic concentrations and the Poisson's equation, which relates the net free charge density in the diffuse layer of the EDL (ρ_c) and the electrokinetic potential (ψ), the Poisson-Boltzmann equation can be written as:

$$\nabla^2\psi = 2\frac{n_0 ze}{\varepsilon\varepsilon_0}\sinh\left(\frac{ze\psi}{k_B T}\right) \tag{1}$$

where e is the electronic charge, n_0 is the average number of positive or negative ions in the non-Newtonian electrolytic solution, ε is the dielectric constant of the medium, ε_0 is permittivity of free space, k_B is the Boltzmann constant and T is the absolute temperature of the solution. On solving Eq. (1) with the appropriate boundary conditions for the more general case of high ζ-potential (≈75 mV) (i.e. going beyond the simplifying assumption of Debye-Hückel linearization), the electrokinetic potential can be determined as:

$$\begin{aligned} \bar{\psi} &= \tanh^{-1}\left[e^{-\kappa\bar{y}}\tanh\left(\bar{\zeta}\right)\right], && 0 \le \bar{y} \le 1 \\ &= \tanh^{-1}\left[e^{-\kappa(2-\bar{y})}\tanh\left(\bar{\zeta}\right)\right], && 1 \le \bar{y} \le 2 \end{aligned} \tag{2}$$

where $\bar{\psi} = \frac{ez\psi}{4k_B T}$ is the non-dimensionalized electrokinetic potential, $\bar{\zeta} = \frac{ez\zeta}{4k_B T}$, $\bar{y} = \frac{y}{H}$ is the non-dimensionalized transverse co-ordinate and $\kappa = \frac{H}{\lambda_D}$ is the half-channel height to the Debye length ratio. The Debye length is the characteristics length scale for the EDL thickness and can be estimated by the Debye-Hückel parameter $\omega = \frac{1}{\lambda_D} = \sqrt{\frac{2n_0 e^2 z^2}{\epsilon k_B T}}$. It is to be noted here that for the present analysis, the origin of the pertinent co-ordinate system is considered to be fixed at the bottom plate of the channel.

Once the EDL is established, imposition of an external pressure-gradient results in the downstream migration of the free ions in the diffuse layer of the EDL. This generates a current, known as the streaming current (I_s), in the direction of the imposed pressure gradient. However, this pressure-driven convective transport of the ions results in an accumulation of ions in the downstream section of the flow conduit which manifests as an induced electric potential called the streaming potential (E_S). This induced field in turn generates a current, known as the conduction current (I_C), which establishes a counter-flow against the direction of the pressure-driven transport. At steady state, for a purely pressure-driven actuation, the streaming current and the conduction current balance each other, implying that the net ionic current in the flow is zero:

$$I_{ionic} = I_s + I_c = 0 \tag{3}$$

The streaming potential induced back flow appreciably alters the flow dynamics, and hence influences the associated heat transfer characteristics.

2.2 Hydrodynamic considerations

The non-Newtonian fluid obeys a power-law constitutive relation in the form of

$$\tau_{xy} = \eta \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \tag{4}$$

where η is the flow consistency index, n is the flow behaviour index, and where absolute values are used to ensure the real magnitude of the shear stress values. Under the present assumptions, along with the consideration of low Reynolds number flow, the Cauchy momentum equation reduces to the form:

$$\frac{d}{dy}\left(\eta \left| \frac{du}{dy} \right|^{n-1} \frac{du}{dy} \right) = \frac{dp}{dx} + \varepsilon E_S \frac{d^2\psi}{dy^2}, \tag{5}$$

where the last term represents the electrical body force due to the induced streaming potential. Now using the previous definitions of $\bar{y}$ and $\bar{\psi}$ and following the non-dimensionalization scheme:

$$\bar{u} = \frac{u}{U}, \quad \bar{E}_S = \frac{ezDE_S}{k_B TU}, \tag{6}$$

with $U = \left\{ -\frac{H^{n+1}}{\eta} \left(\frac{dp}{dx} \right) \right\}^{1/n}$ being a reference velocity, and D the coefficient of diffusivity, we express

Eq. (5) in dimensionless form as:

$$\frac{d}{d\bar{y}}\left\{\left|\frac{d\bar{u}}{d\bar{y}}\right|^{n-1}\frac{d\bar{u}}{d\bar{y}}\right\}=1-\bar{R}\frac{d^2\bar{\psi}}{d\bar{y}^2}\bar{E}_s \tag{7}$$

where $\bar{R}=\frac{4\epsilon k_B^2T^2}{e^2z^2\eta D}\left(\frac{H}{U}\right)^{n-1}$. Integrating eq. (7) once with respect to $\bar{y}$, and using the condition that at $\frac{d\bar{u}}{d\bar{y}}=\frac{d\bar{\psi}}{d\bar{y}}=0$, at $\bar{y}$ = 1we get:

$$\left|\frac{\partial\bar{u}}{\partial\bar{y}}\right|^{n-1}\frac{\partial\bar{u}}{\partial\bar{y}}=1-\bar{y}+\bar{R}\frac{\partial\bar{\psi}}{\partial\bar{y}}\bar{E}_S \tag{8}$$

Eliminating the absolute value expressions, we have:

$$\frac{d\bar{u}}{d\bar{y}}=\begin{cases}\left[1-\bar{y}+\bar{R}\frac{d\bar{\psi}}{d\bar{y}}\bar{E}_S\right]^{1/n}, & \frac{d\bar{u}}{d\bar{y}}\geq 0\\ -\left[-\left\{1-\bar{y}+\bar{R}\frac{d\bar{\psi}}{d\bar{y}}\bar{E}_S\right\}\right]^{1/n}, & \frac{d\bar{u}}{d\bar{y}}<0\end{cases} \tag{9}$$

Again integrating from 0 to a general location $\bar{y}$ and using the no-slip boundary condition, $\bar{u}$ = 1 at the wall, $\bar{y}$ = 0, we get:

$$\bar{u}=\int_0^{\bar{y}}\frac{d\bar{u}}{d\bar{y}}d\bar{y}=\begin{cases}\int_0^{\bar{y}}\left[1-\bar{y}+\bar{R}\frac{d\bar{\psi}}{d\bar{y}}\bar{E}_S\right]^{1/n}, & \frac{d\bar{u}}{d\bar{y}}\geq 0\\ -\int_0^{\bar{y}}\left[-\left\{1-\bar{y}+\bar{R}\frac{d\bar{\psi}}{d\bar{y}}\bar{E}_S\right\}\right]^{1/n}, & \frac{d\bar{u}}{d\bar{y}}<0\end{cases} \tag{10}$$

The streaming potential field, $E_{S,}$ is evaluated from Eq. (3), by incorporating the Boltzmann distribution of ionic concentrations, and can be written as:

$$\bar{E}_s=\frac{E_s}{E_0}=\frac{\int_0^2\bar{u}\sinh\left(4\bar{\psi}\right)d\bar{y}}{\int_0^2\cosh\left(4\bar{\psi}\right)d\bar{y}} \tag{11}$$

It is important to note that Eq. (7) is not an explicit expression for $\bar{E}_S$ because $\bar{u}$ (inside the integral in the numerator) subsumes within it a dependence on $\bar{E}_S$. Hence, it is evaluated through a fixed point iteration scheme with a carefully chosen relaxation parameter for convergence.

2.3 Thermal transport

Taking into consideration the effects of axial conduction, and viscous dissipation, the energy equation for the present physical situation can be written as:

$$\rho c_p u \frac{\partial T}{\partial x} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tau_{xy}\left(\frac{\partial u}{\partial y}\right), \tag{12}$$

where ρ is the density of the fluid, c_p is the specific heat capacity at constant pressure, T is the temperature of the fluid, and k is the thermal conductivity. For a thermally fully developed flow, the fundamental consideration of axially invariant Nusselt number ($Nu_H = \frac{hH}{k}$, where h is the convective heat transfer co-efficient), leads to the classical dimensionless temperature $\theta = \frac{T - T_W}{T_M - T_W}$ being also independent of the axial coordinate, where T_W is the channel wall temperature and T_M is the bulk mean temperature. Utilizing this in conjunction with the constant heat flux boundary condition ($q''_W = h(T_W - T_M)$ = constant), it can be shown that $\frac{\partial T}{\partial x} = \frac{dT_W}{dx} = \frac{dT_M}{dx} =$ constant. This implies that $\frac{\partial^2 T}{\partial x^2} = 0$. Hence, for a thermally fully developed flow with constant wall heat flux boundary condition, the axial conduction term is identically equal to zero. Moreover, an overall energy balance of an elemental fluid control volume yields

$$\frac{dT_M}{dx} = \frac{q''_W}{\rho u_a c_p H} + \frac{1}{2\rho u_a c_p H}\int_0^{2H} \eta \left|\frac{du}{dy}\right|^{n+1} dy \tag{13}$$

where u_a is the average fluid velocity and where for the viscous dissipation term inside the integral, use has been made use of the fact that $\left(\frac{du}{dy}\right)^2 = \left|\frac{du}{dy}\right|^2$ after expressing the shear stress in terms of eq. (4). On the basis of the aforementioned discussions, substituting eq. (13) in eq. (12), a dimensionless form of the energy equation is obtained as:

$$\frac{d^2\theta}{d\bar{y}^2} = Nu_H\left[-\frac{\bar{u}}{\bar{u}_a} + \frac{1}{2}Br_M\left\{\left|\frac{d\bar{u}}{d\bar{y}}\right|^{n+1} - \frac{\bar{u}}{\bar{u}_a}\int_0^2 \left|\frac{d\bar{u}}{d\bar{y}}\right|^{n+1} d\bar{y}\right\}\right] \tag{14}$$

where $\bar{u} = u_a / U$, and $Br_M = \eta U^{n+1} / H^n q''_w$ can be treated as the viscous dissipation parameter which is usually referred to as the modified Brinkman number in case of non-Newtonian flows. The non-dimensional temperature profile is obtained by solving eq. (14), subjected to the boundary conditions: at $\bar{y} = 0$, $\theta = 0$; and at $\bar{y} = 1$, $d\theta / d\bar{y} = 0$. However, Nu_H, which is a constant for thermally developed flows, occurring in eq. (14) is evaluated by utilizing the basic definition of bulk mean temperature, and can be written as:

$$Nu_H = \frac{2\bar{u}_a}{\int_0^2 \bar{u}\tilde{\theta}} \tag{15}$$

where $\tilde{\theta} = \theta / Nu_H$.

3. RESULTS AND DISCUSSIONS

In this section, the hydrodynamic and thermal transport characteristics of pressure-driven flows of solutions of ions, obeying non-Newtonian constitutive behaviour, through narrow confinements are delineated, by considering the electrokinetic effects as manifested through the generation of the streaming potential. The hydrodynamic characteristics are depicted by the variations in the non-dimensional flow velocity ($\overline{u}$) distribution along the channel cross-section; while the heat transfer characteristics are represented by the consequential variations in the non-dimensional temperature profile (θ) and the Nusselt number (Nu_H). The non-dimensional parameters intrinsic to this study are the half-channel height to Debye length ratio (κ), the non-dimensional zeta potential ($\overline{\zeta}$) at the walls of the nano-scopic flow conduit, and the flow behaviour index (n). It is to be noted here that the values of the parameter $\overline{R}$, which signifies the relative strengths of ionic advection and ionic diffusion in relation to the induced streaming field, used in the present analysis, are evaluated in a manner physically consistent with the values of the other involved parameters like flow consistency index ($\eta = 1 \times 10^{-3}$ Pa·s^n), co-efficient of diffusivity ($D = 1 \times 10^{-9}$ m^2/s), permittivity ($\varepsilon = 80 \times 8.85 \times 10^{-12}$ F/m), applied pressure gradient (-10^8 Pa/m), and the appropriate flow behaviour index which dictates the fluid rheology.

The variations in the cross-sectional flow velocity, triggered by the progressively increasing effect of the streaming field induced counter-flow, are shown in Fig. 1 for pseudoplastic fluids (n<1) (Fig. 1(a)), Newtonian fluids (n=1) (Fig. 1(b)) and dilatant fluids (n>1) (Fig. 1(c)) for κ = 5. The aforementioned value of κ appropriately addresses the electrokinetic phenomenon in narrow confinements, and κ is considered to be 5 throughout the present analysis. Moreover, in the present formalism, the variations in the induced streaming field are realized by altering the non-dimensional wall zeta-potential.

For any fluid, the flow velocity for, $\overline{\zeta}$ = 0, i.e. without considering any electrokinetic effects, corresponds to the normal pressure-driven flow characteristics. Now, the development of the wall zeta-potential leads to the formation of the EDL. Under such circumstances, the imposition of an external pressure-gradient triggers the downstream advective transport of the free ions in the diffuse layer of the EDL. The accumulation of the counter-ions, in the downstream region, induces the streaming potential, which consequently triggers the counter-migration of ions. The streaming field induced 'back-flow' impedes the pressure-driven liquid transport, as coherently reflected by the reduction in the cross-sectional flow velocity for different fluid rheology (see Fig. 1), on considering non-zero values of $\overline{\zeta}$. With increasing wall zeta potential, the concentration of the mobile ions in the EDL further increases, which enhances the pressure-driven streaming of the ions. The enhanced streaming of ions culminates in the generation of higher streaming field, which consequently results in stronger 'back flow'. Hence, with increasing wall zeta-potential, the flow velocity progressively decreases, due to the enhancement in streaming field induced counter flow, irrespective of the fluid rheology, as can be perceived in Fig. 1. However, it must be noted here that for identical values of all the involved parameters, the flow velocity, stemming from the interplay of pressure-driven actuation and streaming field induced 'back flow', greater for higher value of the flow behaviour index, as can be inferred from a comparison of Figs. 1(a), (b) and (c).

The variations in the non-dimensional temperature due to the streaming field induced alterations in the underlying flow dynamics, for different fluid rheology, are shown in Fig. 2(a, b). It is worth mentioning here that the thermal characteristics delineated here, consistently accounts for the viscous dissipation effects which are significant for flows through narrow confinements, and cannot be precluded *a priori*.

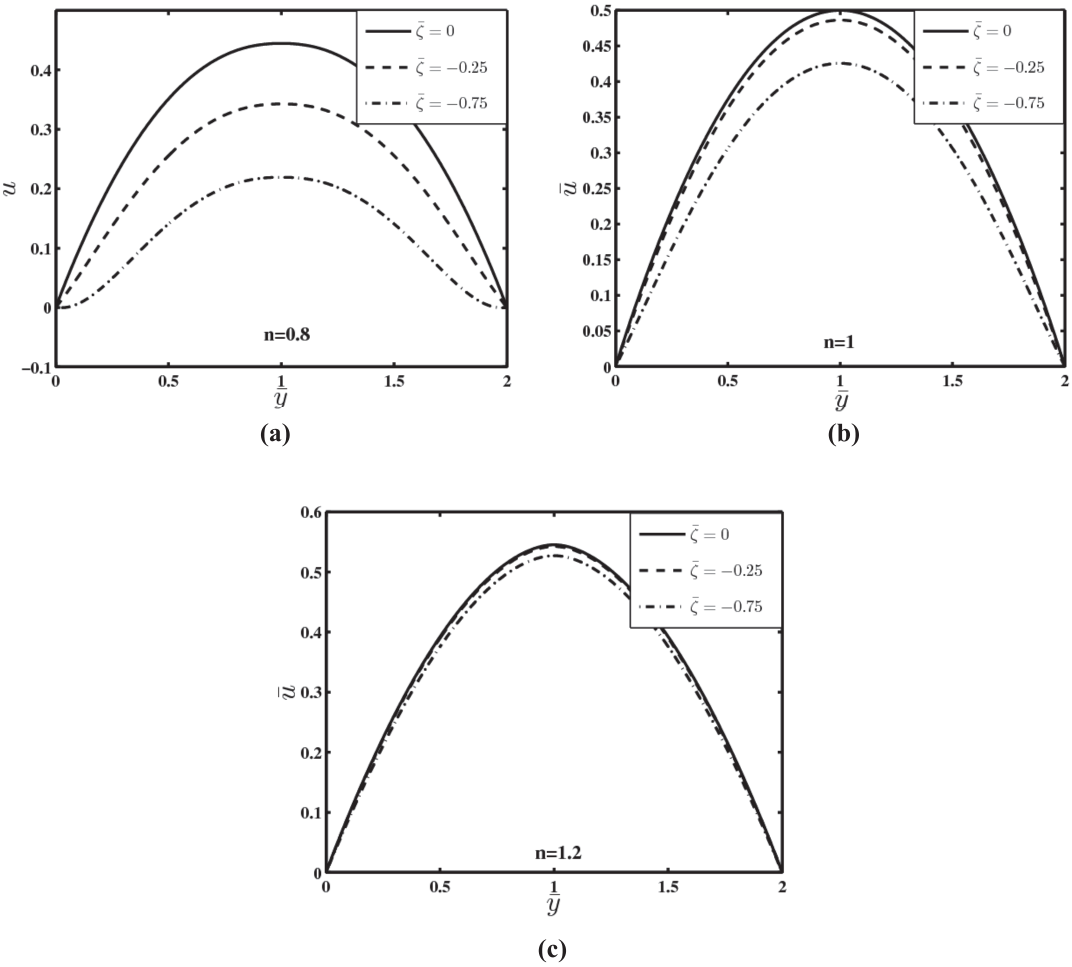

Fig. 1. Variations in the cross-sectional flow velocity with increasing wall ζ-potentials for different fluid rheology: (a) n=0.8 (b) n=1(c) n=1.2, for κ=5.

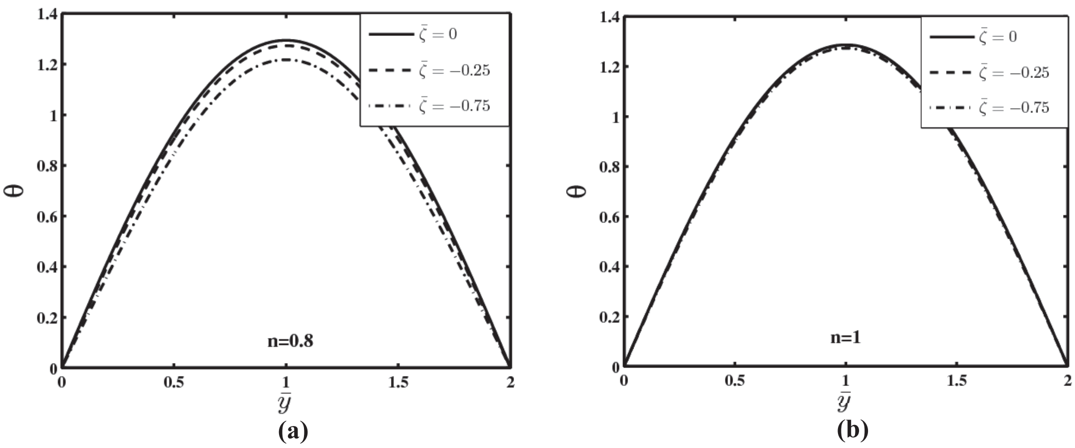

Fig. 2. Effects of the induced streaming potential on the non-dimensional temperature profiles for (a) n=0.8 (b) n=1.

For n<1, i.e. for pseudoplastic fluids, the non-dimensional temperature θ gradually decreases at every spatial location along the along the channel cross-section, with increasing influence of the induced streaming field (Fig. 2(a)). The streaming field induced counter migration of ions hinders the advective transport of the bulk liquid, and this consequently reduces the bulk advective transport of the thermal energy, transferred from the walls of the narrow confinement. The reduction in the rate of thermal energy transfer from the channel walls by the flowing liquid, due to the reduction in the flow velocity, increases the wall temperature (T_W), as the wall heat flux is still externally maintained at a constant level. Moreover, the substantial reduction in flow velocity simultaneously reduces the heat generated due to viscous dissipation effects, which is dependent on the velocity gradient. It is to be noted here that although viscous dissipation is a spatially non-uniform heating effect, for flows through narrow conduits its effect penetrates into the bulk of the flow. Hence, the reduction in viscous dissipation, due to reduced velocity gradients, reduces the liquid temperature over majority of the channel cross-section. Such reduction in the liquid temperature culminates in appreciable reduction in the bulk mean temperature (T_M). Hence, the simultaneous increase in T_W and decrease in T_M consequently increases the magnitude of the term ($T_M - T_W$), which appears in the denominator of the mathematical expression for θ. So, with increasing, the magnitude of ($T_M - T_W$) increases resulting in the gradual decrease of θ. Qualitatively identical trend in alterations of θ profiles, with increasing effect of streaming field induced 'back flow', are valid for Newtonian fluids (n=1) (Fig 2(b)) and dilatant fluids (n>1) (not shown here). However, quantitatively, the alterations in θ profiles with increasing $\bar{\zeta}$ are not as significant for $n \geq 1$, as compared to that observed for n<1. This can be attributed to the relatively less significant changes in the underlying flow dynamics, induced by the streaming field generated counter flow, for $n \geq 1$, as compared to that observed for n<1.

For complete comprehension of the heat transfer characteristics, it is imperative to have a clear idea about the associated Nusselt number variations. Hence, the variations of Nu_H with the magnitude of the wall zeta potential, for different fluid rheology, are shown in Fig. 3.

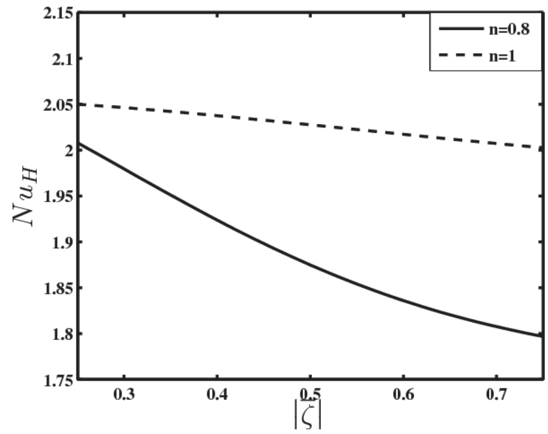

Fig. 3. Variation of Nusselt number with the magnitude of the wall zeta potential for n=0.8 and n=1.

For pseudoplastic fluids (n=0.8), increasing wall zeta potential appreciably decreases the pressure-driven flow velocity, due to the streaming field generated 'back flow', which culminates in significant enhancement of the term ($T_W - T_M$), as mentioned previously. Now, under such circumstances, in order to maintain the constant wall heat flux boundary condition at the channel walls, where the wall heat flux is given by $q''_W = h(T_W - T_M)$ (q''_W is considered to be always positive in accordance with the fluid heating condition), the convective heat transfer co-efficient (h) decreases. The gradual decrease in h

results in progressive reduction of the Nusselt number with increasing wall zeta potential. Similar trend in variation of Nu_H with $\bar{\zeta}$ is also valid for $n \geq 1$, as can be seen for $n=1$ in Fig. 3. However, for $n \geq 1$, the rate of decrease Nu_H with $\bar{\zeta}$ is relatively lower, as compared to that for $n<1$, because of the weaker streaming field induced flow field alterations, and hence smaller increase in $(T_W - T_M)$. But it is to be noted here that at a definite value of $\bar{\zeta}$, the value of Nu_H is more for $n=1$ than for $n=0.8$, because the flow velocity is always higher for the former case under identical conditions.

4. CONCLUSIONS

For pressure-driven flows of solution of ions, obeying non-Newtonian constitutive behaviour, through narrow confinements, the induced streaming field generated 'back-flow' progressively reduces the flow velocity, with increasing wall zeta potential. For any liquid flow, this reduction in the flow velocity culminates in gradual decrease of the heat transfer characteristics under the constant wall heat flux boundary condition, as reflected by the non-dimensional temperature profile variations and the Nusselt number variations with increasing wall zeta potential. However, for pseudoplastic fluids, the reduction in the non-dimensional temperature and the rate of decay of the Nusselt number, with progressively increasing electrokinetic effects, is much higher as compared to that observed for Newtonian and dilatant fluids.

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